Econ 120B, Spring 2003 --- Prof. Ramu Ramanathan
Answers to Homework \#3

EX 6.3
a. We have, $\partial Y \partial X=\beta_{2}+\beta_{3}(1 / X)+\beta_{5}(Z / X)$. The elasticity follows as

$$
\eta=\frac{X}{Y} \quad \frac{\partial Y}{\partial X}=\frac{\left.X\left[\beta_{2}+\beta_{3}(1 / X)\right]+\beta_{5}(Z / X)\right]}{\left[\beta_{1}+\beta_{2} X+\beta_{3} \ln X+\beta_{4} Z+\beta_{5}(Z \ln X)\right]}
$$

b. Regress $Y_{t}$ against a constant and $X_{t}$ and obtain the residuals $\hat{u}_{t}=$ $Y_{t}-\hat{\beta}_{1}-\hat{\beta}_{2} X_{t}$. Next regress $\hat{u}_{t}$ against a constant, $X_{t}, \ln X_{t}, Z_{t}$, and $Z_{t} \ln X_{t}$. Then compute the unadjusted $R^{2}$. The test statistic is LM $=n R^{2}$, where $n$ is the number of observations.
c. Under the null hypothesis that $\beta_{i}=0$ for $i=3 \ldots 5$, LM has the $\chi^{2}$ distribution with 3 d.f.
d. Look up the $\chi^{2}$ table for 3 d.f. to obtain the ${ }_{*}$ critical value LM $^{*}$ at the $5 \%$ level. Reject the null if $\mathrm{LM}>\mathrm{LM}_{*}$ Alternatively, compute $p$-value $=$ the area to the right of $\mathrm{LM}^{*}$ in $\chi_{3}^{2}$ and reject the null if $p$-value is less than 0.05 .

EX 6.10
a. $\quad F_{c}=\frac{(E S S B-E S S A) / 2}{E S S A /(40-6)}=\frac{(0.311974-0.309293) / 2}{0.309293 / 34}=0.147$
b. Under the null hypothesis, this has the $F$-distribution with 2 d.f. for the numerator and 34 d.f. for the denominator.
c. $\quad F_{2,34}^{*}(0.10)=(2.44,2.49)$.
d. Since $F_{c}<F^{*}$, we cannot reject the null hypothesis.
e. Not rejecting the null implies that the coefficients for $\ln$ (UNEMP) and $\ln$ (POP) are jointly insignificant.
f. The $t$-statistic for $\ln$ (PRICE) is given by $(1.557-1) / 0.230=2.42$. For $\ln ($ INCOME $)$ it is $(4.807-1) / 0.708=5.38$. For $\ln ($ INTRATE $)$ it is $(0.208-1) / 0.058=-13.66$.
g. Under the null hypothesis, the $t$-statistics have the $t$-distribution with d.f. (for Model B) $40-4=36$.
h . The critical value $t^{*}$ for 36 d.f. and 5 percent level is (since the alternative is two-sided) in the range 2.021 to 2.042 .
i. Since all the $t$-statistics are numerically above this we reject the null hypothesis and conclude that the elasticities are significantly different from 1. Since the elasticities for price and income are numerically greater than 1 , they are elastic. For interest rate it is inelastic.

EX 6.17
a. First regress PRICE against a constant, SQFT, and YARD and obtain $\hat{\beta}_{1}, \hat{\beta}_{2}$, and $\hat{\beta}_{3}$. Then compute $\hat{u}$ as PRICE $-\hat{\beta}_{1}-\hat{\beta}_{2}$ SQFT $-\hat{\beta}_{3}$ YARD.
b. The test statistic is $\mathrm{LM}=n R^{2}=59 \times 0.115=6.785$. It is distributed as chi-square with 2 d.f.
c. $>$ From the chi-square table we have $\mathrm{LM}^{*}=5.99146$. Since LM LM*, we reject the null hypothesis and conclude that either $\ln (\mathrm{SQFT})$, or $\ln (\mathrm{YARD})$, or both belong in the model.
d. The rule of thumb for inclusion is any new variable with $p$-value less than 0.50 . By this rule, $\ln (\mathrm{SQFT})$ should be included. The new model is PRICE $=\beta_{1}+\beta_{2} \mathrm{SQFT}+\beta_{3} \mathrm{YARD}+\beta_{4} \ln (\mathrm{SQFT})+$ v .

