

Using quarterly data for 10 years (40 observations), the following double-log model (Model A) was estimated:

$$(A) \quad LPCCARS = a + b LPRICE + g LPCINCOME + d LINTRATE + u$$

where PCCARS is per-capita new car sales, PRICE is new-car price index, PCINCOME is per-capita income, and INTRATE is the prime interest rate. The prefix L for each variable denotes the logarithm. The estimated coefficients and associated statistics are in the following table. (*Note: Values in parentheses are t-statistics NOT standard errors.*) In addition, three seasonal dummies have been defined—SPRING, SUMMER, and FALL—each of which takes the value 1 in the corresponding quarter and 0 otherwise. In Model B these seasonal dummies have been included and in Model C only the SPRING dummy variable is present.

Variable	Model A	Model B	Model C
CONSTANT	- 31.852 (- 5.8)	- 33.320 (- 7.6)	- 32.582 (- 7.4)
LPRICE	- 1.751 (- 7.6)	- 1.774 (- 9.8)	- 1.760 (- 9.6)
LPCINCOME	4.732 (6.7)	4.907 (8.8)	4.814 (8.5)
LINTRATE	- 0.193 (- 3.3)	- 0.197 (- 4.3)	- 0.196 (- 4.2)
SPRING		0.092 (2.8)	0.125 (4.6)
SUMMER		- 0.051 (- 1.6)	
FALL		- 0.049 (- 1.5)	

#### Model Selection Statistics

ESS	0.31044	0.17699	0.19341
SGMASQ	0.8626	0.5363	0.5526
AIC	0.9482	0.6279	0.6209
FPE	0.9488	0.6302	0.6217
HQ	1.0079	0.6987	0.6701
SCHWARZ	1.1226	0.8438	0.7668
SHIBATA	0.9316	0.5973	0.6644
GCV	0.9584	0.6501	0.6316
RICE	0.9704	0.6807	0.6447

1. (2 points) **What is the control season?** *The control season is Winter because no dummy variable has been defined for it.*
2. (18 points) **In Model B, state the expected signs for each coefficient (exclude the constant term) and your reasons for expecting them. Are any of the observed signs “wrong” or do they agree with your intuition ?**

#### **LPRICE**

*As prices increase, demand for cars will decrease and hence we would expect the sign to be negative and the actual sign is correct.*

#### **LPCINCOME**

*As income increases, demand for cars will increase also and hence we would expect the sign to be positive and the actual sign is correct.*

#### **LINTRATE**

*As borrowing rates increase, demand for cars will decrease and hence we would expect the sign to be negative and the actual sign is correct.*

#### **SPRING**

*Compared to the Winter season, we would expect car sales to pick up in the Spring and hence we would expect a positive here and the sign is correct.*

#### **SUMMER**

*Here also we would expect a positive sign because people will be going on vacations in the Summer and hence car demand should increase. The sign is negative and is counterintuitive. [However, since the coefficient is insignificant, the wrong sign is not worrisome and could be due to pure random chance.]*

#### **FALL**

*In the Fall, people return from vacation and schools would have started. So, the sign could go either way. Here too the coefficient is insignificant and hence the wrong sign is not worrisome and could be due to pure random chance.]*

3. (5 points) **In Model C, test the null hypothesis that the price elasticity of demand for new cars is numerically equal to 1, against the alternative that demand is either price elastic or inelastic (ignore the sign of the coefficient) . Use a 1% level, show all your steps, and state you conclusion as to whether demand is price elastic or not.**

*The null hypothesis is that  $b$  is -1 against the alternative that it is not -1. Compute  $|t_c| = (1.76 - 1)/s_{\hat{b}}$ , where  $s_{\hat{b}}$  is the standard error for  $\hat{b}$ . But the table has only the  $t$ -statistic. We can get  $s_{\hat{b}}$  as  $1.76/|t_c| = 1.76/9.6 = 0.183$ . Therefore the required  $t$ -statistic is  $t_c = (1.76 - 1)/0.183 = 4.15$ . Under the null hypothesis, this has a  $t$ -distribution with  $40 - 5 = 35$  d.f. For a 1% test,  $t^*$  is in the range (2.705=2.750).*

Because  $t_c > t^*$ , we reject the null hypothesis and, since  $|\hat{\mathbf{b}}| > 1.0$ , we conclude that demand is price elastic.

**You want to test the null hypothesis that there is no seasonal difference in the constant term across seasons. (Note: only the constant term difference tested here.)**

- 4. (3 points) Write down another econometric model (in symbolic terms with no numerical values but only  $\mathbf{b}$  's) that will enable you to test this hypothesis. This will be the unrestricted model (U).**

*Let  $\mathbf{a} = a_0 + a_1 \text{ SPRING} + a_2 \text{ SUMMER} + a_3 \text{ FALL}$ . Substituting this in the basic model we get the unrestricted Model (U) as  $LPCARS = a_0 + a_1 \text{ SPRING} + a_2 \text{ SUMMER} + a_3 \text{ FALL} + \mathbf{b} \text{ LPRICE} + \mathbf{g} \text{ LPCINCOME} + \mathbf{d} \text{ LINTRATE} + \text{error}$ . This is the same as Model B.*

- 5. (2 points) In this model, state the null and alternative hypotheses that will make Model A the restricted model.**

$H_0: a_1 = a_2 = a_3 = 0$ .  $H_1: \text{at least one of them is not zero.}$

- 6. (3 points) With the values given in the above table compute the test statistic.**

$$F_c = \frac{(0.31044 - 0.17699)/3}{0.17699/(40 - 7)} = 8.29$$

- 7. (3 points) Under the null hypothesis, what is its distribution and the degrees of freedom?**

$F_c$  is distributed as  $F(3, 33)$

- 8. (4 points) Actually carry out the test at the 1 percent level. What do you conclude about seasonal differences in the constant term?**

*From the F-table for 1 percent level look up  $F_{3,33}^*$  as a value between 4.31 and 4.51. Since  $F_c > F^*$ , we reject the null hypothesis and conclude that the constant term is significantly different across seasons.*

- 9. (10 points) Perform appropriate tests to decide which variables of Model B you think should be dropped. Explain fully your methodology for doing these tests and your rationale for omitting one or more variables.**

*To decide on the variables to exclude, we do a t-test. The degrees of freedom for the t-statistics in Model B in the table is 33. For a 1 percent level of significance, the critical value  $t^*$  is in the range (2.704, 2.750). Any variable with a corresponding t-statistics below this (numerically) is a candidate to be dropped. According to this rule, we would drop SUMMER and FALL. This would result in Model C.*

**10. (5 points) Which would you choose as the “best” model and why?**

*The model with the lowest model selection statistics is preferable as the best. Six out of the eight criteria choose Model C as the best. Another criterion to use is the significance of the coefficients. All the coefficients in Model C are highly significant. Also, the signs for the coefficients agree with our intuition. Thus, Model C would be chosen as the best model.*

**11. (10 points) You want to test whether the price elasticity ( $b$ ) is different across seasons. Starting from Model B, derive another model that will enable you to test for this. This model must be written in the standard form of a regression model with a dependent variable related to a constant plus a number of  $b$ 's multiplied by variables. Show your derivations.**

*Let  $\mathbf{b} = b_0 + b_1 \text{ SPRING} + b_2 \text{ SUMMER} + b_3 \text{ FALL}$ . Substituting this in Model B we get the unrestricted Model (D) as*

$$\text{LPCCARS} = a_0 + a_1 \text{ SPRING} + a_2 \text{ SUMMER} + a_3 \text{ FALL} + b_0 \text{ LPRICE} + b_1 (\text{SPRING} \times \text{LPRICE}) + b_2 (\text{SUMMER} \times \text{LPRICE}) + b_3 (\text{FALL} \times \text{LPRICE}) + \mathbf{g} \text{ LPCINCOME} + \mathbf{d} \text{ LINTRATE} + \text{error}$$

**12. (3 points) Write down the null hypothesis — in your model in question 11 — to test the hypothesis “there is no seasonal difference in the price elasticity.”**

$$H_0: b_1 = b_2 = b_3 = 0$$

**13. (12 points) Describe step-by-step how you would carry out this test using the Lagrange Multiplier (LM) test. (Note: You cannot actually carry out this test because there is not enough information.) In particular, state what new variables must be created, which regressions should be run, how to compute the test statistic, its distribution and d.f. under the null, and the decision rule at 1% level. Note that, wherever feasible, numerical values must be provided.**

*First regress LPCCARS against all the explanatory variables in Model B and save the residuals as*

$$\hat{u}_t = \text{LPCCARS} - \hat{a} - \hat{b} \text{ LPRICE} - \hat{g} \text{ LPCINCOME} - \hat{d} \text{ LINTRATE}$$

*Next create  $Z1 = \text{SPRING} \times \text{LPRICE}$ ,  $Z2 = \text{SUMMER} \times \text{LPRICE}$ , and  $Z3 = \text{FALL} \times \text{LPRICE}$ . Then regress  $\hat{u}_t$  against the variables in Model B plus the three Z's. Next compute  $LM = nR^2$ , where  $R^2$  is the unadjusted  $R^2$  in the above regression. Under the null, this has a  $\chi^2$  distribution with 3 d.f. At the 1 percent level previously used, the critical value  $LM^*$  is 11.3449. The null hypothesis is rejected if  $LM > LM^*$ .*