Econ 120B **Ramu Ramanathan**
Spring 2003 **Answers to Exam 2**

Using data for the 58 counties in California, the following model was estimated:

$$\text{MEDINC} = \beta_1 + \beta_2 \text{FAMSIZE} + \beta_3 \text{HIGHSCHL} + \beta_4 \text{COLLEGE} + \beta_5 \text{UNEMP} + \beta_6 \text{URB} + u$$

MEDINC = Median family income in thousands of dollars
FAMSIZE = Number of persons per household
HIGHSCHL = Percent of the population (25 years and over) that had only a high school education
COLLEGE = Percent of the population (25 years and over) that completed four years of college or higher
UNEMP = Percent unemployment rate
URB = Percent of urban population

The estimated unrestricted model (U), with standard errors in parentheses, is given as follows:

$$\widehat{\text{MEDINC}} = \underset{}{-41.464} + \underset{(15.742)}{15.382}\,\text{FAMSIZE} + \underset{(3.033)}{0.335}\,\text{HIGHSCHL} + \underset{(0.126)}{1.021}\,\text{COLLEGE} - \underset{(0.132)}{0.403}\,\text{UNEMP} + \underset{(0.229)}{0.029}\,\text{URB} + \underset{(0.029)}{}$$

$$\text{ESS} = 578.7298 \quad \text{Adjusted } R^2 = \overline{R}^2 = 0.837$$

Let β_i be a typical regression coefficient, *i* = 1, 2, …, 6.

1. **(2 points) To test whether β_i is significantly different from zero, state the null and alternative hypotheses:**

   H_0: $\beta_i = 0$ **H_1:** $\beta_i \neq 0$

2. **(3 points) Write down an expression for the test statistic (don't compute it yet). Be sure to define symbols you use.**

   $t_c = \dfrac{\hat{\beta}_i}{s_{\hat{\beta}_i}}$ *where* $\hat{\beta}_i$ *is the regression coefficient and* $s_{\hat{\beta}_i}$ *is the standard error.*

3. **(2 points) State its distribution under the null and the <u>numerical value</u> of the d.f.**

   Under the null, t_c is distributed as t with 58 − 6 = 52 d.f.

4. **(3 points) Write down the range for the critical value for a 10% level of significance.**

(1.671, 1.684)

5. (15 points) In the table below, enter the numerical value of the test statistic for the above test, indicate whether you would reject the null or not, and whether the variable is a candidate for omission or retention from Model U.

Variable	Test statistic	Reject/ Not reject	Omit/ Retain
Constant	**Ignore**	**Ignore**	**Ignore**
FAMSIZE	*5.07*	*Reject*	*Retain*
HIGHSCHL	*2.66*	*Reject*	*Retain*
COLLEGE	*7.73*	*Reject*	*Retain*
UNEMP	*–1.76*	*Reject*	*Retain*
URB	*1.00*	*Not Reject*	*Omit*

A second model (Model R) was estimated and its coefficients and related statistics are given next.

$$\widehat{\text{MEDINC}} = \underset{(13.584)}{-61.766} + \underset{(2.746)}{16.971}\,\text{FAMSIZE} + \underset{(0.109)}{0.498}\,\text{HIGHSCHL} + \underset{(0.079)}{1.213}\,\text{COLLEGE}$$

$$\text{ESS} = 640.229 \quad \text{Adjusted } R^2 = 0.826$$

Use the first model as the unrestricted model and the second as the restricted model, and perform a relevant test at the 10 percent level by carrying out the following steps.

6. **(2 points) State the null and alternative hypotheses in terms of the β s of Model U.**

H_0: $\beta_5 = \beta_6 = 0$ $\qquad$ H_1: *At least one of them is not zero*

7. **(4 points) Compute the test statistic.**

$$F_c = \frac{(ESSR - ESSU)/(k-m)}{ESSU/(n-k)} = \frac{(640.229 - 578.7298)/2}{578.7298/52} = 2.76$$

8. (2 points) State its distribution including the d.f. $F_{2,52}$

9. **(3 points) State the range for the critical value.** *(2.39, 2.44)*

10. (2 points) Based on the above, would you reject the null hypothesis or not? Why or why not?

Because $F_c > F^*$, we reject the null hypothesis.

11. (3 points) What do you conclude in terms of the significance or not of the coefficients of the variables omitted in Model U?

At least one of β_5 and β_6 is not zero.

12. (3 points) Is there a contradiction between this test and the one you did in (5)? If yes, point out where.

There is no contradiction because we find from the t-test that β_5 is significantly different from zero.

13. (3 points) Model R had UNEMP and URB omitted from Model U. Suppose this was wrong and, say, UNEMP should have been kept. Does this omission affect any of the properties of unbiasedness, consistency, BLUE, and valid of tests? If yes, in what way? If not, why not?

Because β_5 was significantly different from zero, it should have been kept. The omission causes the estimates to be biased and inconsistent. Also hypothesis tests are invalid.

14. (3 points) Suppose some of the independent variables were highly correlated. Does it affect the properties of unbiasedness, consistency, BLUE, valid of tests? If yes, in what way? If not, why not?

The proofs of unbiasedness, consistency, BLUE, and the valid of tests do not make any assumption about multicollinearity. Therefore these properties still hold.