

I. Using annual data for 31 years, the following model of timber harvest in Oregon was estimated:

$$\text{HARVEST} = b_1 + b_2\text{EXPORTS} + b_3\text{HOUSTART} + b_4\text{INDPROD} + b_5\text{TIMBPRIC} + b_6\text{PRODPRIC} + u$$

- HARVEST** = Total timber harvested in billion board feet
EXPORTS = Volume of timber exports to foreign countries in 100 million board feet
HOUSTART = Total housing starts in the United States in millions
INDPROD = Index of industrial production for paper and wood products
TIMBPRIC = Price of timber measured in dollars per 1,000 board feet
PRODPRIC = Producer price index for all commodities

The following is the unrestricted model results with standard errors in parentheses.

$$\begin{aligned} \text{(U) } \widehat{\text{HARVEST}} &= 3.913 + 0.108 \text{ EXPORTS} + 0.524 \text{ HOUSESTART} + 0.525 \text{ INDPROD} \\ &\quad (0.574) \quad (0.082) \quad (0.355) \quad (0.127) \\ &\quad - 0.018 \text{ TIMBPRIC} - 0.456 \text{ PRODPRIC} \quad \text{ESS} = 6.22273 \\ &\quad (0.011) \quad (0.087) \end{aligned}$$

Ia (3 points) To test whether individual coefficients are zero or different from zero, write down the degrees of freedom (d.f.) for this model, the critical value for a 10 percent level of significance, and the statistical distribution of the test statistic (don't ask whether the test is one-tailed or two-tailed; you decide what it is based on the information given).

d.f. 25 Critical value 1.708 Distribution under null t_{25}

Ib (15 points) Test each regression coefficient (except the constant term) for whether it is significantly different from zero or not, and whether or not the variable is a candidate for omission by making appropriate entries in the following table.

	Calculate test statistic	Significant/ Insignificant	Omit variable/ Retain variable
b_2)	$0.108/0.082 = 1.317$	Insignificant	Omit
b_3)	$0.524/0.355 = 1.476$	Insignificant	Drop
b_4)	$0.525/0.127 = 4.134$	Significant	Retain

	Calculate test statistic	Significant/ Insignificant	Omit variable/ Retain variable
b_5)	$-0.018/0.011 = -1.636$	Insignificant	Omit
b_6)	$0.456/0.087 = 5.241$	Significant	Retain

Ic (2 points) Which of the variables should be dropped first? Fully explain why.

Because b_2 has the smallest t-statistic in absolute value, EXPORTS should be dropped first.

A second model was estimated and the relation is given below (again standard errors are in parentheses).

$$(R) \quad \overbrace{\text{HARVEST}} = 3.602 + 0.618 \text{ HOUSTART} + 0.612 \text{ INDPROD} - 0.481 \text{ PRODPRIC}$$

$$(0.533) \quad (0.360) \quad (0.091) \quad (0.089)$$

$$ESS = 7.1265$$

Id (12 points) State the joint null hypothesis on the b s of Model U that will make Model R the restricted model. Then compute the test statistics, state its distribution under the null including d.f., and carry out the test at the 10% level. What is your conclusion in terms of the joint significance of the parameters in your H_0 ?

$$H_0: b_2 = b_5 = 0.$$

$$F_c = \frac{(ESS_R - ESS_U)/2}{ESS_U/25} = \frac{(7.1265 - 6.22273)/2}{6.22273/25} = \frac{0.452}{0.248} = 1.815$$

$$F^* = 2.53 \quad \text{Since } F_c < F^*, \text{ we cannot reject } H_0 \text{ and conclude that } b_2 \text{ and } b_5$$

are jointly insignificant.

II (9 points)

Indicate whether each of the following statements is valid and explain your reasons. In particular, state relevant assumptions and properties, if appropriate.

- a. “Although multicollinearity lowers t -statistics, all the variables with insignificant regression coefficients should not be dropped from the model in one swoop.”**

If we drop the variable with the least significant coefficient, it is possible that a variable it was correlated with might become significant whereas the latter was previously insignificant. If we drop both, we will miss this. The statement is therefore TRUE.

- b. **“Even though multicollinearity raises the standard errors of regression coefficients, the t - and F -tests are still valid.”**

t - and F - tests depend only on the normality of u_t . Multicollinearity does not violate that. Therefore, the tests are still valid and the statement is TRUE.

- c. **“High multicollinearity affects standard errors of estimated coefficients, but estimates are still efficient.”**

Efficiency requires that $E(u_t) = 0$, X_t is given and hence non-random, u_t has a constant variance, and zero covariance with all other errors. Multicollinearity does not violate these assumptions and hence estimators are still efficient. Statement is TRUE.

III (9 points) State three reasons why it is sensible to omit the variable with an insignificant coefficient. Justify your answers.

- 1) *A simpler model is easier to interpret than a more complicated model.*
- 2) *Omitting the variable increases the d.f. and improves the precision of the remaining coefficients, that is, reduces standard errors.*
- 3) *Omitting the variable increases the power of tests.*
- 4) *Omitting the variable reduces the width of a confidence interval.*
- 5) *Omitting the variable reduces any multicollinearity caused by it and hence improves precision of the remaining coefficients.*