Econ 120B, Fall 2003, Answers to Homework #1

Exercise 3.30

a. The coefficient for income is $\Delta\hat{Y}/\Delta X = 0.0556$. It is the marginal effect of income on travel expenses. For a one billion dollar increase in aggregate income, expenditure on travel is expected to increase, on average, by 0.0556 billions of dollars or 55.6 millions of dollars. This is quite reasonable.

b. For α, $H_0: \alpha = 0$, $H_1: \alpha \neq 0$. $t_c = 0.4981/0.5355 = 0.93$. Under the null hypothesis t_c has the t-distribution with 49 (51-2) d.f. For a 5% level, critical t^* is in (2.000, 2.021). Since $t_c < t^*$, we cannot reject $H_0: \alpha = 0$.
For β, $H_0: \beta = 0$, $H_1: \beta \neq 0$. $t_c = 0.0556/0.0033 = 16.85$. Under the null hypothesis t_c has the t-distribution with 49 d.f. For a 5% level, critical t^* is, as before, in (2.000,2.021). Since $t_c > t^*$, we reject $H_0: \beta = 0$. Thus the conclusion is that α is not statistically different from zero but β is.

c. $R^2 = 1 - \dfrac{ESS}{TSS} = 1 - \dfrac{417.110}{2841.330} = 1 - 0.147 = 0.853.$

d. Test statistic is

$$F_c = \frac{R^2}{1-R^2}(n-2) = \frac{0.853}{0.147} 49 = 284.33$$

Under the null hypothesis that the correlation between expenses on travel and income is zero, F_c has the F-distribution with one d.f. for the numerator and 49 d.f. for the denominator. Critical F^* for 1% level is in (7.08, 7.31). Since $F_c > F^*$, we conclude that correlation between travel expenses and income is significantly different from zero.

e. Let X^* be the new income variable and Y^* be the new expenditure variable. Then $X^* = 1000^2\, X$ and $Y^* = 1000^2\, Y$. Model is $Y = \alpha + \beta X + u$ and estimated model is $\hat{Y} = \hat{\alpha} + \hat{\beta} X$.

$$\frac{Y^*}{1000^2} = \alpha + \beta \frac{X^*}{1000^2} + u$$

$$Y^* = 1000^2 \alpha + \beta X^* + 1000^2 u$$

We therefore have, $\hat{\alpha}^* = 1000^2 \hat{\alpha} = 498100$, its standard error is 1000^2

0.5355 = 535500. R^2, $\hat{\beta}$, and its standard error, are unchanged. $ESS^* = 1000^4\ ESS = 1000^4\ 417.11$ and $TSS^* = 1000^4\ TSS = 1000^4\ 2841.33$.