Econ 120B, Fall 2003, Answers to Homework #1

Exercise 3.11

The slope of the line connecting the first and second point is $(Y_2 - Y_1)/(X_2 - X_1)$. The slope of the line connecting the second and third points is $(Y_3 - Y_2)/(X_3 - X_2)$. By proceeding similarly, the slope of the line connecting point $t - 1$ with t is given by $(Y_t - Y_{t-1})/(X_t - X_{t-1})$. Therefore, $\tilde{\beta}$ is the average of the slopes of the straight lines connecting successive data points. The scatter diagram for this is given below.

The average of all the slopes in the scatter diagram is given by

$$\tilde{\beta} = \frac{1}{n-1} \sum_{2}^{n} \left[\frac{Y_t - Y_{t-1}}{X_t - X_{t-1}} \right]$$

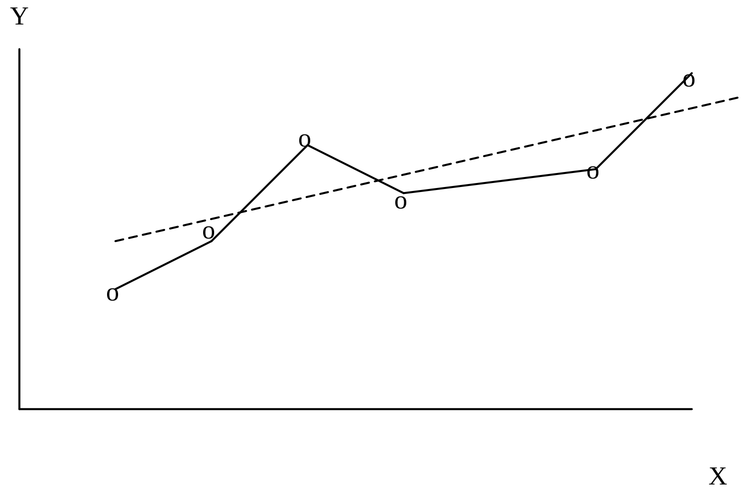
To compute the expected value of $\tilde{\beta}$, we have,

$$\frac{Y_t - Y_{t-1}}{X_t - X_{t-1}} = \frac{\alpha + \beta X_t + u_t - \alpha - \beta X_{t-1} - u_{t-1}}{X_t - X_{t-1}} = \beta + \frac{u_t - u_{t-1}}{X_t - X_{t-1}}$$

Hence,

$$\tilde{\beta} = \beta + \frac{1}{n-1} \sum_{2}^{n} \left[\frac{u_t - u_{t-1}}{X_t - X_{t-1}} \right]$$

X_t is nonrandom and $E(u_t) = E(u_{t-1}) = 0$. Therefore, $E(\tilde{\beta}) = \beta$, which means that $\tilde{\beta}$ is unbiased. By the Gauss-Markov Theorem, OLS estimates are most efficient among unbiased linear estimators. This implies that any other such estimator, in particular $\tilde{\beta}$, is inefficient (or at least is no more efficient) than the OLS estimate.


Y
o
o
o
o
o
o
X