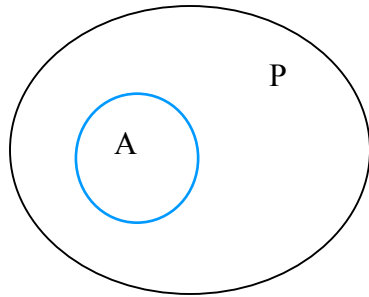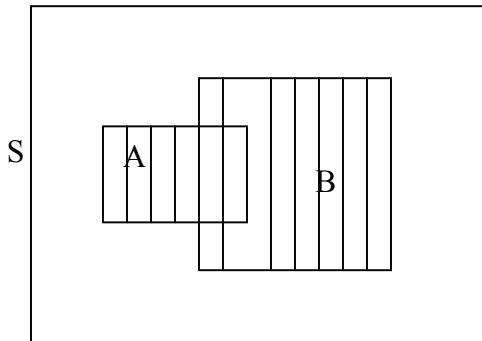**ECON 120A, SPRING 2003 --- ANSWERS TO HOMEWORK #1**

1.
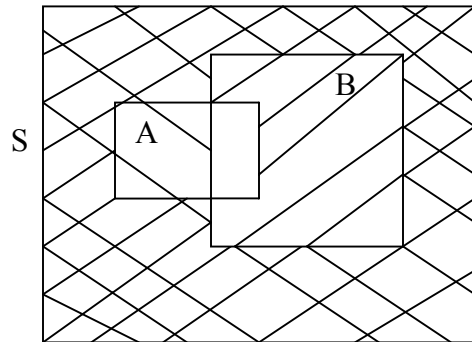Wealthy landlord $= B \cap C$. Republican non-public employee $= D \cap E$. Either of these groups is denoted by $P = (B \cap C) \cup (D \cap E)$. The statement is that A is a subset of this. Thus, we have $A \subset (B \cap C) \cup (D \cap E)$. Note that $A = P$ is not correct because there may be elements in P that are not in A (See Venn diagram).
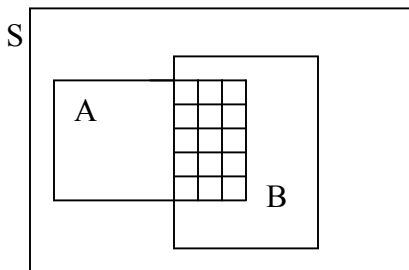

2.

$(A \cup B)^c$ is the area in the Venn diagram not lined.

A^c is the area with right slashes.
B^c is the area with left slashes.
$A^c \cap B^c$ is the area with both left and right slashes.

We note that the area not lined in the left diagram is identical to the area in the right diagram that has both left and right slashes. This proves that $(A \cup B)^c = A^c \cap B^c$.

The grid area is $A \cap B$ and the area without lines is $(A \cap B)^c$.

In the right hand diagram above, $A^c \cup B^c$ is the area with either a right or a left slash and is the same as the area without any lines in the left hand diagram just drawn.
Hence $(A \cap B)^c = A^c \cup B^c$.

3.
Let H = high blood pressure and A = drinks alcohol. We are given that
$P(H) = 0.05$, $P(A \mid H) = 0.25$, $P(A \mid H^c) = 0.75$. We want $P(H \mid A)$.

$$P(H \mid A) = \frac{P(H \cap A)}{P(A)} = \frac{P(A \mid H)P(H)}{P(A)} = \frac{0.0125}{P(A)}$$

$$P(A) = P(A \cap H) + P(A \cap H^c) = P(A \mid H)\ P(H) + P(A \mid H^c)\ P(H^c)$$

$$= (0.25 \times 0.05) + (0.75 \times 0.95) = 0.725.$$

Therefore $P(H \mid A) = 0.0125/0.725 = 0.017$.

4.
This is the binomial case. The probability of checking a correct answer at random is $1/k$.
Suppose the student got x out of the N-n answers correct and N-n-x answers wrong. Then the score S is $S = n + x - a\ (N$-n-$x)$. Also, $E(x) = (N$-$n)/k$. Hence,

$$E(S) = n + \frac{N-n}{k} - a(N-n)(1-\frac{1}{k})$$

For this to be equal to n, the condition is, $\frac{N-n}{k} - a(N-n)(1-\frac{1}{k}) = 0$

Solving this for a we get or $a = 1/(k$-$1)$. In the True/False case, $k = 2$. Therefore $a = 1$.

5.
(a) Total revenue in t years is tP. The total cost in t years is X^2. Therefore, net profit is
$\Pi(X, t, P) = tP - X^2$.

(b) $E(\Pi) = E(tP - X^2) = tP - E(X^2)$. Since $Var(X) = E(X^2) - [E(X)]^2$,
$E(X^2) = Var(X) + [E(X)]^2 = 5t + 25t^2$.
Therefore $E(\Pi) = tP - (5t + 25t^2) = (P$–$5)t - 25t^2$.

(c) The first order condition is $\partial E(\Pi)/\partial t = 0 = (P$–$5) - 50t$. Solving for t, we get
$t^* = (P$–$5)/50$. The second derivative is negative and hence the second order
condition is satisfied.

(d) For t^* to be 10, we need $P = 505$.

(e) For $t = 10$ and $P = 505$, $E(\Pi) = (P$–$5)t - 25t^2 = 5000 - 2500 = 2500$.