ECON 120A , SPRING 2003 --- Prof. Ramu Ramanathan

ANSWERS TO PROBLEM SET#2

I. This is a binomial case. n=25, and p=0.4. Let X=the number of patients who recover. The drug will be discredited if less than 10 patients recovered. The desired probability is

$P(X\leq 9)=\ 1-P(X\geq 10)=1-\sum_{10}^{25}(0.4)^{x}(0.6)^{25-x}\ =1-0.5754=0.4246.$

II. Let $\overline{X}$ be the average income. Then by property 2.10a in Handout#2,

$\overline{X}\sim N(\mu, \frac{\sigma^2}{n})=N(50,100/25)=N(50, 4).$

We need $P(47<\overline{X}<52)$. If we subtract the mean and divide by the standard deviation, the resulting random variable has the standard normal distribution Z.

Therefore, the probability is equal to

$$P(\frac{47-50}{4}\leq Z\leq\frac{52-50}{4}) = P(-0.75\leq Z\leq 0.5).$$

Using the symmetry of the distribution, we have

$P(-0.75\leq Z \leq 0.5)= P(0\leq Z \leq 0.75)+ P(0\leq Z \leq 0.5)=0.2734+0.1915=0.4649.$

III.

1. $\int_0^\theta f(x)dx=\int_0^\theta \frac{1}{\theta}dx=\frac{1}{\theta}\theta-0=1$

2. $E(X^m) = \int_0^\theta \frac{X^m}{\theta} dx = \frac{\theta^m}{m+1}$

3. E(X)= $\theta / 2$, $E(X^2) = \theta^2 / 3$, $E(X^3) = \theta^3 / 4$, $E(X^4) = \theta^4 / 5$.

4. Var(X) = $E(X^2) - (E(X))^2 = \theta^2 / 12$,

Var(Y) = $E(Y^2) - (E(Y))^2 = E(X^4) - (E(X^2))^2 = 4\theta^4 / 45$,

Cov(X,Y)=E(XY)-E(X)E(Y)= $E(X^3) - E(X)E(X^2) = \theta^3 / 12$

$$\rho_{XY}^2 = \frac{(Cov(X,Y))^2}{Var(X)Var(Y)} = 0.9375.$$

5. ρ_{XY}^2 is still equal to 0.9375 since it doesn't depend on θ.

IVV. 1.

X	Prob.	X^2	X^3	X^4	E(X)	E(X^2)	E(X^3)	E(X^4)
1	0.04	1	1	1	0.04	0.04	0.04	0.04
2	0.04	4	8	16	0.08	0.16	0.32	0.64
3	0.04	9	27	81	0.12	0.36	1.08	3.24
4	0.04	16	64	256	0.16	0.64	2.56	10.24
5	0.04	25	125	625	0.2	1	5	25
6	0.04	36	216	1296	0.24	1.44	8.64	51.84
7	0.04	49	343	2401	0.28	1.96	13.72	96.04
8	0.04	64	512	4096	0.32	2.56	20.48	163.84
9	0.04	81	729	6561	0.36	3.24	29.16	262.44
10	0.04	100	1000	10000	0.4	4	40	400
11	0.04	121	1331	14641	0.44	4.84	53.24	585.64
12	0.04	144	1728	20736	0.48	5.76	69.12	829.44
13	0.04	169	2197	28561	0.52	6.76	87.88	1142.44

14	0.04	196	2744	38416	0.56	7.84	109.76	1536.64
15	0.04	225	3375	50625	0.6	9	135	2025
16	0.04	256	4096	65536	0.64	10.24	163.84	2621.44
17	0.04	289	4913	83521	0.68	11.56	196.52	3340.84
18	0.04	324	5832	104976	0.72	12.96	233.28	4199.04
19	0.04	361	6859	130321	0.76	14.44	274.36	5212.84
20	0.04	400	8000	160000	0.8	16	320	6400
21	0.04	441	9261	194481	0.84	17.64	370.44	7779.24
22	0.04	484	10648	234256	0.88	19.36	425.92	9370.24
23	0.04	529	12167	279841	0.92	21.16	486.68	11193.64
24	0.04	576	13824	331776	0.96	23.04	552.96	13271.04
25	0.04	625	15625	390625	1	25	625	15625
					13	221	4225	86145.8

2. E(Y)= $E(X^2)$=221,

Var(Y)= $E(Y^2)-(E(Y))^2 = E(X^4)-(E(X^2))^2$ =37304.8

Cov(X,Y)=E(XY)-E(X)E(Y)= $E(X^3)-E(X)E(X^2)$ =4225-13*221=1352

3. $\rho^2_{XY} = \frac{(Cov(X,Y))^2}{Var(X)Var(Y)} = \frac{1352^2}{37304.8*52}$=0.943