Econ 120A --- Spring 2003 --- Professor Ramu Ramanathan (10 %)
HOMEWORK #2

This homework is due no later than 10:10 pm on Wednesday, May 28, 2003. Papers turned in at the end of the class will get a 10% penalty and NO PAPERS WILL BE ACCEPTED AFTER I LEAVE THE CLASS ROOM.

I.

A physician knows that the probability that a patient will recover from a certain rare disease, without any drugs, is 0.25. To test the effectiveness of a new drug he gives it to 25 patients having this disease and decides, beforehand, to discredit the drug unless 10 or more of the patients recover. What is the probability that he will discredit the new drug on the basis of this experiment even though it raises the recovery rate (the probability that any one patient will recover) to 0.4?
[Hint: What you want to calculate is the probability that fewer than 10 patients recover even though the true recovery rate is 0.40.]

II.

Suppose that *X* denotes the annual incomes, in thousands of dollars, and that, for a particular group of people, *X* is normally distributed with mean 50 and variance 100. A random sample of 25 persons is drawn from the group. What is the probability that the <u>average income</u> ($\overline{X}$) is between 47 and 52 thousands of dollars?
[Caution: You need to look at the distribution of average income and not that of X.]

III.

Consider the random variable X, distributed as Uniform (0, θ) with f(x) = 1/ θ in the interval $0 \leq x \leq \theta$.

1. **First integrate f(x) from 0 to θ and show that it is 1.**
2. **Next derive an expression for $E(X^m)$ in terms of m and θ.**
3. **From that, derive E(X), $E(X^2)$, $E(X^3)$, and $E(X^4)$ in terms of m and θ.**
4. **Suppose $Y = X^2$. Derive expressions for Var(X), Var(Y), Cov(X,Y), and ρ_{xy}^2, the square of the coefficient of correlation between X and Y in terms of θ.**
5. **Assume that θ = 2 and compute the numerical value of ρ_{xy}^2. Note that even though X and Y have an exact relationship, the coefficient of correlation is not ± 1.**

IV.

The following question involves running Excel. You must submit a printout showing all the calculations.

The random variable X has the discrete uniform distribution with the outcomes being positive integers from 1 through 25. Note that they all have equal probabilities. Create an Excel spreadsheet and enter the numbers 1 through 25 in column A. In column B enter the correspond probabilities. Column C is the square of Column A, Column D is the third power of Column A (or the product of Columns A and C). Column E is the fourth power of Column A, the same as the square of Column C.

1. **Using the definition of expectation of a discrete random variable, numerically compute E(X), $E(X^2)$, $E(X^3)$, and $E(X^4)$.**
2. **Let the random variable Y be equal to X^2. Compute E(Y), Var(Y), and Cov(X,Y).**
3. **Use that to compute ρ_{xy}^2, the square of the coefficient of correlation between X and Y. Show that it is not equal to 1 even though there is an exact relation between X and Y.**