Econ 120A **Ramu Ramanathan**
Spring 2003 **First Midterm Answers**

I. (5+3+7 =15 points)
The continuous random variable *X* has the geometric distribution with $f(x) = a\, x^{a-1}$ with $a > 0$ and $0 < x < 1$.

(a) Derive an expression for the m^{th} moment $E(X^m)$.

$$E(X^m) = \int_0^1 x^m f(x)dx = \int_0^1 x^m a x^{a-1} dx = a\int_0^1 x^{m+a-1} dx$$

$$= a\left[\frac{x^{m+a}}{m+a}\right]_0^1 = \frac{a}{m+a}$$

(b) From that write down $E(X)$ and $E(X^2)$, in terms of a.

$$E(X) = \frac{a}{a+1} \qquad\qquad E(X^2) = \frac{a}{a+2}$$

(c) Use these to derive the variance of the distribution.

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{a}{a+2} - \left[\frac{a}{a+1}\right]^2 = \frac{a(a+1)^2 - a^2(a+2)}{(a+2)(a+1)^2}$$

$$= \frac{a(a^2+2a+1) - a^2(a+2)}{(a+2)(a+1)^2} = \frac{a}{(a+2)(a+1)^2}$$

II. (6+2+6+2 = 16 points)
In the TV show "Who wants to be a millionaire", a contestant has earned $64,000 and is trying for $125,000. However, she does not know which of the four answers is correct. If she guesses correctly, she will get $125,000. If she is wrong, she ends up with $32,000. Her dilemma is whether to walk away with $64,000 (Option A) or take a gamble and choose one answer completely at random (Option B) and with the equal probability of ¼.

(a) Compute the expected win for each option. Carefully explain.

Option A (walk away with $64,000):

Win $64,000 with probability 1 and hence the expected win is $64,000.

Option B (gamble for $125,000):

Probability of guessing correctly is ¼ and that of guessing wrong is ¾. If guess is wrong, payoff is $32,000 and if guess is correct, payoff is $125,000. Therefore,

$E(payoff) = 32000 \times 3/4 + 125000 \times 1/4 = 24{,}000 + 31{,}250$

$= 55{,}250$

(b) If the goal is to maximize the expected value of a win, what is your advice to her? Should she gamble or walk away? Why?

Because E(win for A) > E(win for B), she should walk away and not gamble.

Suppose, <u>instead of the above scenario</u>, she used the 50/50 lifeline and narrowed the answers to two and wants to guess at random with the equal probability of ½ among the remaining two answers (Option C).

(c) Compute the expected win for this option.

Probability of guessing correctly now is ½ and that of guessing wrong is ½. If guess is wrong, payoff is $32,000 and if guess is correct, payoff is $125,000. Therefore,

$E(payoff) = 32000 \times 1/2 + 125000 \times 1/2 = 16{,}000 + 62{,}500$

$= 78{,}500$

(d) Now what is your advice, based on maximizing the expected value of wins, between walking away with $64,000 and choosing Option C?

Because E(win for C) > E(win for A), she should gamble.

III. (4+3+3+4+5 = 19 points)
The probability of a male, age 60, dying within one year is 0.025 and the probability of a female, age 55, dying within one year is 0.01 and the two events are independent. If a man and his wife are ages 60 and 55 respectively,

(a) What is the probability that they both die?

Let M be male dying and F be female dying. We need $P(M \cap F)$*. Because of independence,* $P(M \cap F) = P(M)\,P(F) = 0.025 \times 0.01 = 0.00025$.

(b) What is the probability that they both survive?

M^c is male not dying and F^c is female not dying.

$P(M^c \cap F^c) = P(M^c)\,P(F^c) = (1-0.025)(1-0.01)$

$= 0.975 \times 0.99 = 0.96525$

(c) What is the probability that only one will die within a year?

$P(\text{only one will die}) = 1 - P(\text{both die}) - P(\text{both survive})$

$= 1 - 0.00025 - 0.96525 = 0.0345$

Another way of doing this is to note that only one dying is

$(M \cap F^c) \cup (M^c \cap F)$. *Probability is* $P(M \cap F^c) + P(M^c \cap F)$

$= 0.025 \times 0.99 + 0.975 \times 0.01 = 0.0345$

(d) Suppose you sell a one year term life insurance of $25,000 to each, that is, you pay $25,000 to the surviving spouse if only one of them dies, $50,000 to their children if both of them die, and nothing if both live through the year. Calculate the expected total payment.

$E(\text{payment}) = 50000 \times 0.00025 + 25000 \times 0.0345 + 0 \times 0.96525$

$= 862.50 + 12.50 = 875$

(e) If you want to make an expected total profit of $200, what premium should you charge each of them assuming that the premiums are <u>different and in proportion to the respective probabilities of death</u>?

To get a $200 profit, the total premium revenue should be 1,075. Prorated charges will be

Husband: $1075 \times \dfrac{0.025}{0.025+0.01} = \$767.86$

Wife: $1075 \times \dfrac{0.01}{0.025+0.01} = \$307.14$