A Theory of Auctions
From the Auctioneer's Perspective

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1. Introduction

The literature on auctions has grown rapidly in recent years,[1] and has focused on four main topics: (1) the design of optimal auctions from the sellers perspective (Harris and Riviv, 1981; Meyerson, 1981; Riley and Samuelson, 1981), (2) bidding strategies by potential buyers (Capen, Clapp, and Cambell, 1972; Reece, 1978; Milgrom, 1981), (3) the strategic relationships between different forms of auctions and different informational states (Vickrey, 1961; Ortega-Reichert, 1968; Milgrom and Weber, 1982), and (4) the verification of the theoretical propositions of the auction literature by experimental economists (Smith, 1976; Coppinger, Smith and Titus, 1980; Cox, Roberson, and Smith, 1982). The auctioneer, in most of these paper's, is largely ignored and at most plays a very limited and passive role. And yet, if one is to read Cassady (1967), the auctioneer is the dominant party in the auction exchange transaction, particularly in the widely used English auction. This paper attempts to take that observation into account by developing a theory of auctions from the auctioneer's perspective.

The tendency to ignore the auctioneer in most of the economic work on auctions can be directly traced to the assumption of one good belong to one seller being auctioned off to n potential buyers.[2] In this case, either the objective function of the auction house and the seller coincide or the auction house is indifferent to the rules desired by the seller.[3] As soon as the unrealistic

[1] A recent survey is Engelbrecht-Wiggans (1980). Stark and Rothkopf (1979) provide a fairly comprehensive bibliography. The work in economics on auctions has been recently reviewed and synthesized by Milgrom and Weber (1982). The standard source of anecdotal evidence on auctions is Cassady (1967).

[2] Schotter (1976) calls the problem of the auctioneer one of the more interesting in the theory of auctions and his paper is one of the few in the economic literature to recognize the considerable role of the auctioneer.

[3] For example, the objective functions of the seller and auction house coincide if the auction house receives a percentage of the sale price as its payment and the auction house is indifferent to the rules desired by the seller (assuming equal cost) if the seller pays a fixed fee.

assumption, that only a single object is sold during the course of the entire auction, is dropped, the auction house and the seller's objective functions generally conflict. It is worth noting at this point, that it is usually the auction house and *not* the seller that dictates the rules of the auction and that the auction house frequently enjoys, at least locally, monopolistic powers.[4]

This paper consist of five sections, and, in all but the last, the "standard" version of an English auctions (where the auctioneer initiates the bidding process with an initial offer to sell the good for a particular amount and up receiving the first offer to buy the good at that price then solicits higher bids from the group of potential bidders) is used exclusively.[5] In the next section, we examine the instruments available to the auction house during the course of an auction. This analysis is largely from the buyer's perspective although some of the static implications for the auction house and the sellers are drawn out. In the third section, we retain the assumption that the auction house only has control over the actual auction (and not the consignment of goods to the auction) and describe the optimal control problem which the auction house faces. This control problem is examined for four cases which differ by the amount of information available to the auction house. The first case is the

[4] These local monopoly powers are geographical for many auction houses which specialize in agricultural commodities and commodity specific for other other auction houses. Auction houses tend to be very specialized in the type of goods the auction. The reasons for this specialization will become apparent later. The temporal infrequency of most types of auctions tends to reinforce these local monopoly powers. In a few cases, such as high quality art (Christies and Sotheby, Parke, Bernet), an oligopoly exists. Even in that particular case, the two auction houses rarely ever hold temporally competing sales which feature art work of the same style or period.

[5] Note that much of the theoretical work on English Auctions (e.g., Milgrom and Weber, 1982) has used a very peculiar form of the English auction sometimes referred to as the Japanese auction (Cassady, 1967) where the bidders press down a button continuously, releasing it to signify that they are no longer active in the bidding process. After releasing the button the individual is not allowed to return to the bidding process. Adoption of this rarely used Japanese auction as representative of an English auction essentially removes the role of the auctioneer. The Japanese auction also has much different informational characteristics than does the standard English auction.

where the auction house knows each potential bidder's maximum willingness to pay for the good. While this situation is completely unrealistic, it reveals what the auction house is trying to accomplish and serves the purpose of introducing a fair amount of notation without the clutter of probability distributions. The second case is when the auction house only knows all of the individual values held by the bidders, but does not know which specific value is held by any individual bidder. The third case is when the auction house has only some information on the parameters of the distribution of the bidders maximum values, but not the individual values. In all three of these, cases the auction house is assumed not to influence a bidder's maximum willingness to pay for the good being auctioned. The final and more realistic is when the auction house has some knowledge of the distributions of values held by the potential bidders and can influence those values. In this and the preceding case, the auction house may have incentives to invest in information concerning the maximum values held by the potential bidders. In the fourth section, the auction house and the sellers can bargain over consignment conditions. In the final section, we draw some conclusions and make recommendations for further extensions. Our analysis shows that many of the institutional features common to English auctions are optimal from the auction house's vantage point, and that some of these features, that may discriminate against a single seller, may work to the advantage of the average seller at an auction.

2. Auction House Instruments

The instruments typically available to an auction house in the course of an English auction include: (1) the quick hammer (QH), (2) the bid increment (BI), (3) the starting bid request (SBR), and the entry fee (EF). These are all instruments that directly influence the behavior of potential buyers.[6] While

[6] Three other instruments which directly influence the behavior of potential sellers are also available to the auction house: (1) the auction house's acceptance or rejection of the good

the auction house's optimization problem is not directly considered until the next section, recognition that the auction house generally faces a time constraint may be helpful in intuitively understanding the appeal of some of the instruments used by the auction house.

Before characterizing these auction house instruments, some notation and initial assumptions are necessary.[7] First let there be n potential bidders represented by the set N,

$$N = [1, \ldots, i, \ldots, j, \ldots, n]$$

where typical bidders will be represented by i and j as well as those subscripts. Next let there be a maximum of m goods from the set Q which may be sold during the course of the auction,

$$Q = [1, \ldots, q, \ldots, r, \ldots, m]$$

where q and r (and those subscripts) denote a typical good which may be sold at auction.[8] Bidder i's maximum willingness to pay (MWTP) for the good q given his or her current state of information is t_{iq}. This amount is assumed to be known with certainty by i, and is assumed to be the outcome of an independent stochastic process. The auctioneer and the seller do not know for certain t_{iq}, but do know that it lies in the finite interval $[a_{iq}, b_{iq}]$. The auction house's uncertainty about t_{iq} is assumed to be representable by a continuous probability distribution function, $f_{iA}(\,)$; and the other potential bidder's uncertainty about t_{iq} by $f_{ji}(\,)$. It will simplify matters greatly, if, we make two very strong

the seller desires to have auctioned, (2) the size and nature of the payment required by the auction house, and (3) the order in which the auction house offers the good for sale. In this section, we will examine only those instruments which directly affect the buyers with consideration of those instruments primarily influencing the seller's decision on whether or not to consign a particular good to a particular auction deferred until the fourth section.

[7] Where possible the notation in this paper follows Meyerson (1981).

[8] In this paper, we assume that the goods must be sold in this order which is known to the auction house, and that the auction house, while having a choice over whether good becomes part of the set Q, doesn't have control over the order.

additional assumptions. The first is to assume that t_{iq} is independent from the valuation of any other good r and that i does not face a budget constraint due to earlier (or planned subsequent) purchases of other goods at the auction.[9] This assumption is maintained through out this paper. The second is to assume that there is no uncertainty about the quality of the good q being sold through the auction, but that there is uncertainty about the preferences of the other potential bidders so that bidder i does not know with certainty t_{jq}. The model outlined is known as the private valuation model (Milgrom and Weber, 1982) and under the stated conditions each potential bidder knows what the good being auctioned is worth to them and their t_{iq} is not influenced by what the other potential bidders' MWTP for q. However, the strategy of the seller, S, the auctioneer, A, and the other bidders may be influenced by knowing t_{iq}. This assumption can be viewed as the special case where the bidding process conveys no information about the quality of the good and it will be relaxed later.

There are a few institutional features which need sketched out. In a standard English auction, the auctioneer announces a starting bid request (SBR) of the amount $d > 0$ and if there is a positive response (i.e., some bidder j indicates a willingness to pay d); the auctioneer makes a new bid request (BR) of the amount d + s, where s is known as the bid increment (BI) and is assumed, for now, to be a fixed percentage (less than 100%) of d chosen by the auction house. The auctioneer makes a new bid request of d + 2s, and so on, with g in the term d + gs signifying how many bid request have been made. If no bidder

[9] This assumption rules out order effects (i.e., the amount for which a good is sold for in the auction is invariant to the order in which the good is sold. Order effects can occur for any number of reasons. Cassady (1967), Duncan (1958), and Schotter (1976) present some interesting examples and reasons. One way to rationalized this assumption is to allow bidders who have exhausted budgets to be replaced by new bidders drawn at random from the same distribution as the bidders being replaced.

responds affirmatively to the SBR, the auctioneer proposes a new SBR, d', less than d. This reduction in the SBR continues until a positive response is forthcoming.[10] An important institutional feature for a standard English auction is that only one bidder is recognized as having the active bid at any one particular point in time. This active bid will be denoted $BR_{(j,\, d\, +\, gs)}$ and is interpreted as bidder j had the active bid (*) at the amount (d + gs). If at any time, the auctioneer does not solicit or cannot obtain higher bid higher [i.e., d + (g + 1)s], then the bidder recognized at d + gs receives the good and pays the amount of the last active bid, d + gs. The winning bid is denoted by $BR^{*(j,d+s)}$ where j was the winning bidder and paid the amount d + s.[11]

One of the troublesome features of the economic literature on English auctions is its inability to explain the incentives for any bidder to respond affirmatively to a SBR greater than zero since if no positive response is forthcoming the auctioneer must reduce the SBR. Two instruments, the QH and the BI, provide that incentive. The sequence of bids in an English auction is important and provides a fair amount of room for strategizing, and thus an important amendment to the dominant strategy for the bidder to bid up to his or her MWTP.

The incentive provided by the QH is clear. The auction house, following some rule, which may from the bidder's perspective appear to be random, awards the q to the last recognized bidder without attempting to solicit a higher bid. Use of the quick hammer has all sorts of strange effects on the

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[10] Thus the English auction need not always be of the ascending format. The reduction in the SBR if necessary will be much larger than the bid increment, s. We show below why there are strong incentives to the auction house to chose a SBR low enough so that at least one bidder will respond affirmatively. SBR's in actual English auctions, however, are always greater than zero and we show below that there is in general an optimal point for setting the SBR for a particular good and group of potential bidders.

[11] There are no ties in a standard English auction since there is a unique order of recognition by the auctioneer.

outcome of the auction process. The bidder with the highest t_{iq}, need not acquire q, nor does the price at which the good is sold necessarily reflect any bidders' t_{jq}. What this means is that there is a positive probability at all times that the individual recognized by the auctioneer as having placed the last recognized bid will be able to purchase the good for that amount. As long as this is true, every potential bidder whose t_{iq} is above d + gs may have a positive incentive to bid. In particular, for g > 0, every bidder with a t_{jq} greater than d + gs has some positive incentive to attempt to be recognized as the active bidder by the auctioneer.[12] For g = 0, there is a positive incentive based on the QH, but the potential bidder j must balance this incentive against the likelihood that if he or she does not respond at d that no other bidder will respond affirmatively thus forcing the auctioneer to reduce the SBR to d', and that the bidding if restarted at d', does not subsequently rise above d.

The next instrument used by the auction house is the bid increment (BI), which for now is assumed to be a constant fraction of d chosen by the auction house. To see how the introduction of BI fundamentally changes the nature of the English auction, assume first that bidder j has the second highest value for q which will be denoted by t_{*q}. Then, assume that the auctioneer recognized j's bid of t_{*q} = d + gs. Bidder i, whose t_{iq} is the largest among all n potential bidders, now faces a choice. If t_{iq} is greater than d + (g + 1)s, then i should respond to the next bid request affirmatively and pay the price d + (g + 1)s. If t_{iq} < d + (g + 1)s, i will not bid further and the auctioneer will award the good to j at the price of d + gs. Thus, it is possible for i to pay more than t_{*q} up to the amount t_{iq}, and it is also possible for the good q to be sold to j, the bidder

[12] There are obviously more valuable times to be recognized as the active bidder than others and in part the auction house uses the quick hammer to make all positions equally desirable. The role of the last active bidder is also important for some of the other instruments available to the auction house. This issue is discussed through out this and the next two sections.

with the second highest value, t_{*q}, for the good.[13] There are other possibilities. Suppose now that i had been recognized with a bid of d + gs and that bidder j had a value for the good of d + gs $=< t_{iq} <$ d + (g + 1)s, in this case, i receives the good and pays less than t_{*q}. This analysis need not be limited to only the two bidders with the highest $t_j q$'s. While it is impossible to make i bid more than t_{iq} for q, with a sufficiently large bid increment, i may be paying the t_{jq} far down the list of order statistics t_{jq}. It is also possible to show that some bidder other than the the with the highest t_{jq}'s may receive the good. It is straight forward following Yamey (1972) to show that under certain conditions that the winning bidder pays an amount $t_{*q} \pm s$. This conclusion will be modified somewhat below.

The bid increment has two effects: (1) it presents potential bidders with the difficult problem of working out a strategy for when to try to become recognized as the active bidder by the auctioneer, and (2) it in general creates positive profits for the winning bidder since only in the case that $t_{iq} = t_{*q}$ does the highest bidder pay his or her MWTP.[14] The magnitude of these profits is dependent on the order of the recognition by the auctioneer and the size of the bid increment.[15] It is important to note that these positive profits accrue only to those bidders who perceive a positive probability of being the winning bidder on one or more items sold during the auction. In the next section, we

[13] If j has the recognized bid of d + gs (= t_{*q}) then t_{iq} - [d + (g + 1)s] approaches 0 as t_{iq} - t_{*q} approaches s. Coppinger, Smith and Titus (1980) have observed this phenomena in their experiments using English auctions. Yamey (1972) and Schotter (1974) appear to be the first in the economic literature on auctions to recognize some of the properties of the BI on determining the outcome of the auction transaction.

[14] The quick hammer rule also creates positive profits for the winning bidder, and introduces a random element into the planning strategy of the bidders.

[15] Bid increments are frequently quite large in English auctions, particularly if viewed in terms of the bidder's expected profit and not the immediate resale value of the good. In art and yearling thoroughbred horse auctions, BI's of ten to five hundred thousand dollars are not uncommon on items selling in the hundred thousand to ten million dollar range. Five to ten percent of the starting bid request is typical.

will show how the BI increases expected profits for the auction house and on average for the seller.

The auction house may capture these profits through an entry fee (EF), which potential bidders must pay before being allowed to bid on any item during the course of the auction.[16] Potential bidders are likely to see higher profits the more goods they believe they have a chance of purchasing, the more frequently the QH is used, the larger the bid increment, and the smaller the number of potential bidders for each good of interest to the particular bidder in question.[17] Because the auction house does not know the t_{jq}'s, a uniform entry fee or a fee based upon proximity to the auctioneer may be necessary unless a preliminary auction is held for the right to bid during the subsequent auction.

The four instrument available to the auction house, the SBR, has the little effect on potential bidders unless the private valuation assumption is relaxed and bids are allowed to convey information. If, however, one considers the time constraint on the auction house, the number of bid requests necessary to get the highest possible bid is a direct function of the choice of d, for any given s, and the usefulness of the SBR to the auction house should be clear even without its informational properties.

The private valuation model can be relaxed by introducing revision effects functions (Meyerson, 1981),

$$v_{iq} = t_{iq} + \sum_{j!=i=1}^{n} e_{ijq}(t_{jq}) + e_{iA}(d), \tag{1}$$

[16] Note that the auction house not the seller captures these entry fees. There is a striking similarity between this type of profit and the value of a seat on one of the stock or commodity exchanges.

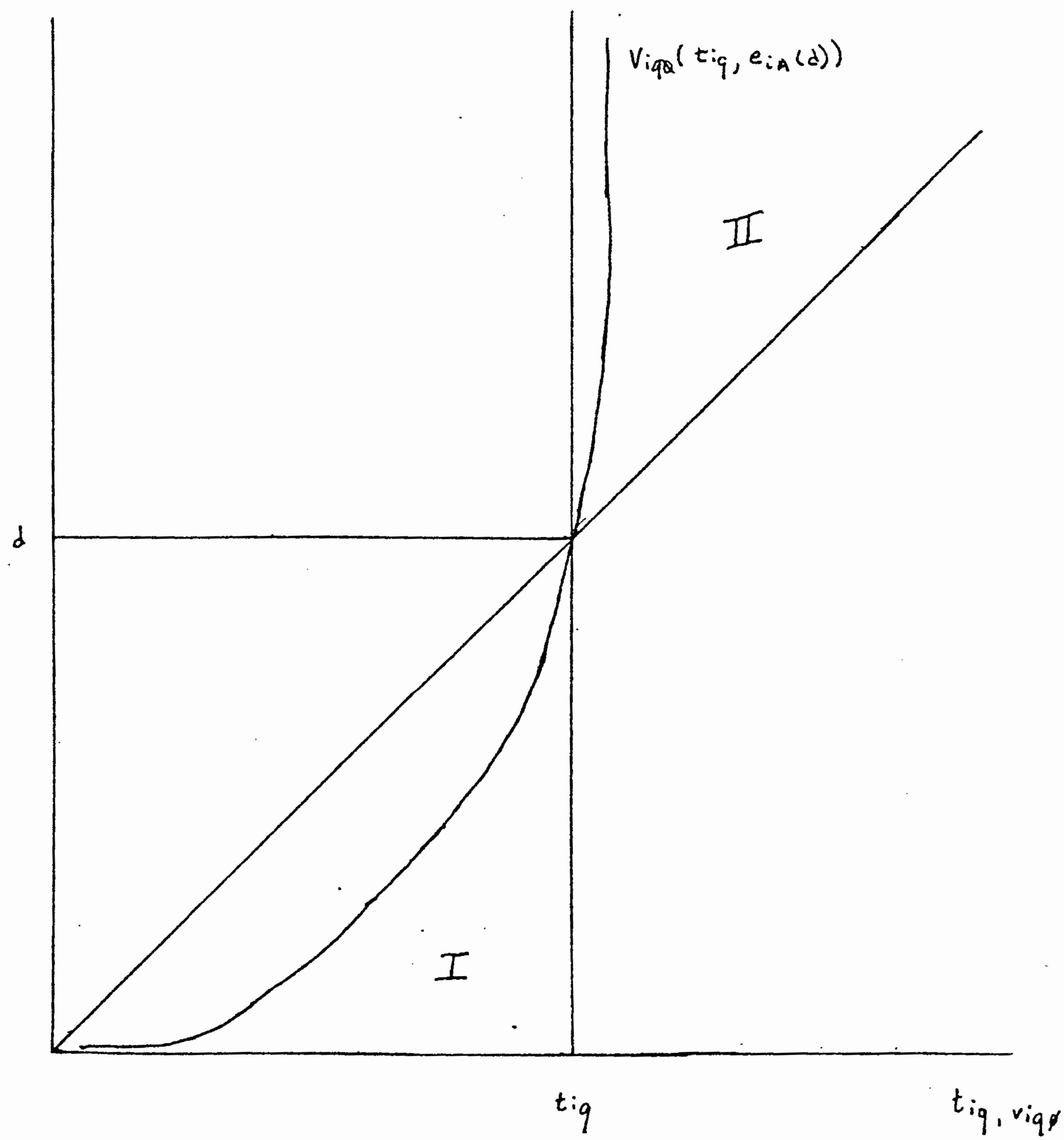
[17] The smaller the number of bidders the greater the chances of recognition by the auctioneer, the less chance that the bidder with the highest t_{iq} will have to exceed t_{*q} to win and the greater the chance of willing with a bid of less than t_{*q}.

where t_{iq} is i's initial MWTP for q, $e_{jq}(t_{jq})$ is a function which describes how much i would revise t_{iq} if i knew t_{jq}, and $e_{iA}(d)$ is a function which describes how much i revises t_{iq} *after* d is known. The new MWTP with the additional information is v_{iq}, which replaces t_{iq} as the maximum amount i is willing to bid rather than go without q. Revision of t_{iq} will occur in a sequential process and it will be useful at times to further subscript v_{iq} with g, v_{iqg} where g as before indicates how may bid requests have been made by the auctioneer with g = 0 at the time when only the SBR has been made.

Discussions with experienced auctioneers suggest very definite restrictions on the value of $e_{iA}(d)$ function. A SBR(d) less than t_{iq} tends to reduce v_{iq0} so that $t_{iq} \geq v_{iq0} > d$, while a SBR greater than t_{iq} tends to increase v_{iq0} so that $d > v_{iq0} \geq t_{iq}$. Thus v_{iq0} is restricted to lie in the area labeled I in figure 1 if $t_{iq} \geq d$ and the area labeled II if $t_{iq} \leq d$. The difference between t_{iq} and the 45 degree line passing through (0, 0) and (t_{iq}, d) is the value of the $e_{iA}(d)$.[18] A typical $v_{iq0}(e_{iA}|t_{iq})$ function is traced out in figure 1.

[18] For this function to be strictly true uncertainty about the quality of the good must be a continuous function. This rules out cases where i knows that if the auction house knows the quality of q and uses a SBR of d that the good is either authentic or a forgery.

Figure 1: v_{iq0} as a Function of t_{iq} and d



One way of thinking about v_{iq0} is as a weighted average of t_{iq} and d, $v_{iq0} = (1/w_t)(t_{iq}) + (1/w_d)(d)$ with the restrictions that $(1/w_t) + (1/w_d) = 1$; $(1/w_t) > 0$; and $(1/w_d) \geq 0$. If $(1/w_d) = 0$, then $v_{iq0} = t_{iq}$ and no revaluation takes place. The condition that $(1/w_d) = 0$ is necessary but not sufficient for the private valuation model to be applicable. The magnitude of the two weights will reflect i's perception of the relative quality of the two sources of information.

The shape of the *viq* function has some important implications. The SBR of the auctioneer can cause either an upward or downward revaluation by i but in no case can an upward revaluation cause a potential bidder whose t_{iq} was less than d to respond positively to a SBR of d.[19] This conclusion follows from the inequalities of the weights on t_{iq} and d. An immediate implication of this result is that the auction house is only concerned about the initial distribution of the order statistics, t_{jq}.

Since we are interested in the auctioneer's perspective, it is useful to define $e_{ijq}(t_{iq})$ to take account only of the information gained by i about t_{iq} during the course of the auction. Information gained about t_{jq} before the auctioneer's SBR is assumed to be incorporated into t_{iq} and the only avenue open to the seller to reveal information after the SBR is to become a bidder. The issue of the seller as a bidder and of reserve prices is taken up below but first we want to examine the properties of $e_{ijq}(t_{iq})$.

Sequentially, the auction house's SBR may cause a revaluation of i's MWTP, and then the $BR_{(j,d+s)}$ may influence i's MWTP. Thus in an English auction, there is a sequence of v_{iqg} and for g > 1. This sequence can be shown to be

[19] Although, in combination with information gained from observing bids of j, i may at some point enter a bid greater than d ($> t_{iq}$). Note that at times, A is a sense a super conveyor of information if i assigns zero weight to the auction house's opinion. This situation occurs when there is no positive response to the SBR of d which can be interpreted by i as implying that none of the other n - 1 bidders saw a profit of sufficient size to gamble on a quick hammer or to jockey for position by responding to the SBR.

non-decreasing.

There are two cases, the first occurs when i observes a $BR_{(j,d+gs)} < t_{iq}$ - s. This situation conveys no information which would cause i to revalue v_{iq} downward and provides no incentive to value v_{iq} upward so $e_{ijq}(BR_{(j,d+gs)}|BR_{(j,d+gs)} < t_{iq} - s) = 0$. Since only one bidder can be recognized as active at any particular BR, a $BR_{(j,d+gs)} < t_{iq}$ - s conveys only that at least one bidder had a $v_{iq} \geq d + gs$, information which gives no reason to revalue downward.[20] Bidder i's optimal strategy is to keep bidding until d + gs > $v_{iq} - s$ so i can wait until that point before deciding how to incorporate any new information gained from the bidding process. In particular, if i observes a $BR_{(j,d+gs)} \geq v_{iq} - s \geq t_{iq}$ - s, i knows that at least one other bidder values q more highly than the last BR position i was prepared to bid for, and i is faced with new knowledge *and* the need to act on it. The choice confronting i, at this point, is whether or not to raise v_{iq} to a point equal to or greater than $BR_{(j,d+gs)}$ and attempt to be recognized as the active bidder.[21] Ex ante, this sequence of v_{iqg} is always non-decreasing, however, ex post this may not be true. If i believed that several bidders were going to keep bidding and they stop, leaving i to purchase q far below where i believed that q would sell for, i may realize that his valuation of q was faulty.

In the typical English auction where the auction house receives a percentage (ϑ) of the final bid (BR^*) for q, there are two ways to handle reserve prices. In the first case, if q doesn't reach its reserve price, then the seller pays the auction house ϑ times the reserve price. In the second case, the seller pays ϑ times the last accepted bid request (BR^*).[22] This second case is

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[20] This is in contrast to a Japanese auction where the point at which bidders withdraw conveys information about the distribution.

[21] One reason that i may increase v_{iq} as the BR approaches t_{iq} is that a resale market may be created which lowers the risk to i of the purchase of q.

[22] A third and less frequent case is when the auction house charges a flat fee for a good which doesn't reach its reserve price. This fee is usually greater than ϑ times the reserve price. In all three cases setting a reserve price poses a potential cost to the seller in addition

more favorable to the seller, and is equivalent to the seller entering the bidding and bidding up to the reserve price.[23] Adopting this representation for reserve prices simplifies notation since the seller is just another bidder from the auction house's perspective and it gives greater weight to the auction house's assessment of the good $[e_{iA}(\text{d})]$ visa vi that of some other unknown bidder $[e_{ijq}(BR_{(j,d+gs)})]$ since bidder j may be an agent of the seller simply trying to run the price up. The seller of course runs an obvious risk in engaging in this practice.

3. The Auction House's Control Problem

In the simplest case, the auction house knows all of the order statistics (t_{iq}) and the bidder's which hold each and the auction house can not influence the t_{iq}'s through its actions. In making these assumptions, we have reverted back to the private valuation model. The optimization problem facing the auction house is,

$$\underset{T,d,s,g,z,d',EF,QH}{MAX} \sum_{q=1}^{T} \vartheta(BR_{(d+gs)q}) \text{ .} \tag{2}$$

subject to (for all q),

$$BR_{(d+gs)} \leq MAX\ (t_{iq}) \text{ ;} \tag{2a}$$

$$0 < d_q \leq MAX\ (t_{iq}) \text{ ;} \tag{2b}$$

$$0 < s_q \leq d \text{ ;} \tag{2c}$$

$$0 =< g_q \text{ ;} \tag{2d}$$

$$0 < d_q' < d_q - s_q \text{ ;} \tag{2e}$$

to the risk of not selling the good.

[23] Frequently the auction house employs agents who do this, and even in cases where the auction house forbids the seller from entering the bidding, the seller frequently employs an agent to bid for them.

$$T \geq \sum_{q=1}^{r} \varphi_{qd} + \varphi_{qd'} z + \varphi_{qd} g \; . \tag{2f}$$

where the last constraint specifies the total amount of time available to the auction house in which to hold the auction. The parameters φ_{qd}, $\varphi_{qd'}$, and φ_{qs} represent respectively, the amount of time necessary to introduce and set a SBR for the good q, the amount of time necessary to revise the first SBR if no response is forthcoming,[24] and the amount of time for the auctioneer to call for one BI.[25] In this section, we assume that the limit on the number of goods which the auction house has to sell, m, is not binding. If we make the assumption, that if d = MAX t_{iq} is asked i will respond affirmatively, an assumption which will be justified below, simple Kuhn-Tucker methods can be used to solve the auction house's optimization problem but it is obvious and stated here without proof.

The auction house sets d = MAX(t_{iq}) and sells r goods with the constraint T $\approx \sum_{q=1}^{r} \varphi_d$ binding.[26]

The number of bid increments made per good, g, is zero, and hence the size of the bid increment does not matter. In addition, the auctioneer never has to reset the SBR, so the percent of times d' is used, z, is zero, so the value of d' doesn't matter. The quick hammer is never used and since the auction house collects 100% of the possible revenue the entry fee it charges is zero.

[24] The choice variable z [0 < z] is the percent of times d is reset to d'. Multiple resets for one good simply increase z.

[25] This time constraint needs a further comment. In an English auction, there are two general ways in which a time constraint comes about. The first is when the auction house announces an opening and closing time for the auction. The second is when at some point in the auction, the potential buyers start to leave and the sellers remove their goods from the auction. It is generally this inability to keep the necessary buyers for more than a given amount of time which leads to the time constraint. If the auction house operationalizes this constraint by announcing beginning and ending times then they will try to estimate how many goods they should accept to sell in that period.

[26] The ≈ symbol here means that the integer nature of the problem is ignored. In practice auction house's close the sale at approximately the announced time.

The objectives of the seller and the auction house are identical in the case of perfect information. It is impossible for the buyer with the highest t_{iq} to avoid paying that amount. To see this, suppose that the auctioneer's SBR is d = MAX t_{iq} and that i who has the MAX t_{iq} refuses to respond to the SBR (d = MAX t_{iq}) where he is just indifferent to paying that amount and going without the good. The auction house simply announces a new SBR of d' much below d and recognizes some bidder other than i say j and then makes an BR of d' + s chosing s so that d' + s = MAX t_{iq}. Bidder i must now accept that new BR or j will receive the good. Bidder i can only delay but not avoid paying MAX t_{iq}. It is easiest to assume here that the threat of a QH prevents this sort of behavior, and it is not worth pursuing the potential bargaining power i gains from delaying since the assumption of perfect information is fairly implausible.

A slightly more realistic situation is to assume that the the auction house knows all of the t_{iq}'s but does not know to which bidder these order statistics belong. The key factor in allowing bidder i to avoid paying his maximum t_{iq} was the ability of the auction house to reduce d to d' , recognize j, and then jump the bid request to t_{iq}. Now the auction house cannot distinguish between i and j, and i will always have an incentive to shirk and not respond affirmatively to a SBR of d = MAX t_{iq}. To see this suppose i has not responded to the SBR of d = MAX t_{iq}. The auctioneer drops the SBR to d' and recognizes a bidder and proposes a BR of d' + s = MAX t_{iq}. If the auctioneer recognized j, then i accepts the BR equal to his MWTP. If the auctioneer recognized i then there will be no response to the BR = MAX t_{iq} and i will receive the good and pay d'. The expected amount paid by i is equal to,

$$[Prob\ BR_{(i,d')}](d') + [Prob\ BR_{(j=i,d')}](MAX\ t_{iq}). \tag{3}$$

This amount is strictly less than i's MWTP so i can expect positive profits and hence i will never respond to a SBR of MAX t_{iq}. Since opening with a SBR of d =

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[26] To get any type of solution, it is necessary to define a rule for when a bidder will respond

MAX t_{iq} incurs a time penalty and gains no revenue for the auction house, the optimal SBR for the auctioneer to set must be below MAX t_{iq}.

A number of features of the auction house's optimal strategy in this situation are readily apparent. If s is used, it is equal to MAX t_{iq} - d, which implies $g \leq 1$. The auction house knows the outcomes of distribution function $f(t_{iq})$ and its corresponding cumulative density function $F(t_{iq} < d)$ except for what subscripts (i, j) to attach to those outcomes.

The expected BR^{*} for a (d, s) pair to the auction house is:

$$E[BR^{*}] = (1/k)(d) + [1 - (1/k)](MAX\ t_{iq}), \tag{4}$$

if g = 1 and E[BR^{*}] = d if g = 0, where k equals the number of bidders who will respond affirmatively to a SBR of d. Let's take the first case where s = 1, the optimal d is found by solving,

$$\frac{\partial BR^{*}}{\partial d} = \frac{-1}{k^2}\frac{\partial k}{\partial d}d + \frac{1}{k} + \frac{1}{k^2}\frac{\partial k}{\partial d}MAX\,[t_{iq}] = 0\,, \tag{5}$$

which gives,

$$\frac{\partial k}{\partial d} = \frac{k}{d - MAX\,[t_{iq}]}\,, \tag{6}$$

or,

$$\eta_d = \frac{d}{k}\frac{\partial k}{\partial d} = \frac{d}{d - MAX\,[t_{iq}]}\,, \tag{7}$$

where η_d is the elasticity of opening bids with respect to the SBR. Now,

$$\frac{\partial k}{\partial d} = -n\,\frac{\partial F(d)}{\partial d} = -n\ f(d)\,, \tag{8}$$

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other bidders will open at any amount equal to or below their t_{jq}. Our reasoning is that t_{*q} is the lowest price on average that i can expect to pay and holding of the $BR_{(i,d)} = t_{*q}$ position has a number of desirable properties. For the other bidders, as soon as they see i attempting to be recognized they have no other choice if they are willing to pay the SBR than to do likewise.

which implies,

$$\frac{f(d)}{1 - F(d)} = \frac{1}{MAX\,[t_{iq}] - d}\,. \tag{9}$$

To illustrate, consider the case where η_d is approximately constant near the optimal d^*.[27] The optimal SBR is a proportion of the highest value,

$$d^* = MAX\,[t_{iq}]\,\frac{\eta_d}{\eta_d - 1}\,. \tag{10}$$

where that proportion is a function of the elasticity of opening bidders with respect to d. We are simply saying here that the further down the list of order statistics d is set, the less chance the bidder with the MAX t_{iq} will be recognized but the greater the loss if the highest bidder is recognized. Given the constraint that g = 1, there is obviously an optimal point to set d given all of the order statistics of the distribution.

To examine the case where g = 0 note that the gross revenue of the auction house is less than in the perfect information case for two reasons. First, the expected BR^* is lower for any q, and the second is that the auction house sells the same or fewer goods. This can be shown by noting in the case of prefect information,

$$T \approx \sum_{q=1}^{r} \varphi_{dq}\,,$$

while in this informational case,

$$T \approx \sum_{q=1}^{r'} \varphi_{dq} + \varphi_{sq}\,g\,. \tag{11}$$

Since we have implicitly assumed that φ_{qs} is not negligible, $r \geq r'$ with the equality holding only when g = 0.

[27] For a uniform distribution, d^* is near MAX t_{iq}; for a normal or a log normal distribution d^* moves near the peak as the variance decreases.

At some point as $(\varphi_{qs} / \varphi_{qd})$ becomes large, the auction house will set d = t_{*q} and always QH the good at that point thus setting g = 0 and thereby not using s. By doing this the auction house can guarantee itself,

$$\vartheta \sum_{q=1}^{r} BR_q^* = t_{*q} \ . \tag{12}$$

The auction house will make the discrete choice between g = 0 and g = 1 to maximize total sales revenue. Thus in this informational state, the auction house will on average do equal or better for the seller than a second price auction since the number of goods sold is r, the second price result, or r'. If the number of goods sold is r', then the expected revenue per good must be higher than when the auction house chooses to sell r goods. It is important to keep in mind, however, that while the expected revenue by the seller is equal to or higher than the second price auction, any individual seller may receive a much lower price than the second price auction.

While still maintaining the private valuation assumption, we will now assume that the auction house only has some information on the distribution of the bidder's t_{jq}'s. The auction house in setting the optimal d, s, and g, must now also worry about the possibility that the first d set will be too high and not draw an affirmative response. For a fixed r, the general representation of the expected proceeds from a sale to the auction house is now given by,

$$\underset{d_h}{MAX}, s \sum_{h=0}^{\infty} Prob\,(d_{h-1} > t_{*q} \geq d_h) \tag{13}$$

$$x \sum_{g=0}^{\infty} [\frac{1}{k\,(h,g)} Prob\,(d_h + (g+1)s > t_{*q} \geq d_h + gs \mid d_{h-1} > t_{*q} \geq d_h)$$

$$x \ [\vartheta(d_h + gs) - C(h\,\varphi_{d'}\,,\,g\,\varphi_s)]$$

$$+ (1 - \frac{1}{k\,(h,g)})\ Prob\,(d_h + (g+1)s > MAX\,[t_{iq}],\, t_{*q} \geq d_h + gs \mid d_{h-1} > t_{*q} \geq d_h)$$

$$x \ [\vartheta(d_h + gs) - C(h\,\varphi_{d'}, g\,\varphi_s)]$$

$$+ (1 - \frac{1}{k(h,g)})\ Prob\ (MAX\ t_{iq} \geq d_h + (g+1)s > t_{*q} \geq d_h + gs \mid d_{h-1} > t_{*q} \geq d_h)$$

$$x\ [\vartheta(d_h + (g+1)s) - C(h\varphi_{d'}, (g+1)\varphi_s)],$$

where d_h is the $h\underline{th}$ setting of the SBR, the first h - 1 SBR's failed to elicit an opening bid, k(h,g) $\approx N[1 - Prob\ (t_{iq} < d_h + gs)]$, and C( ) is the time-cost of the sale in terms of the number of bid increments and the number of resetting of d.[28] Equation (13) can be simplified by combining the probabilities,

$$\underset{d_h}{MAX,s} \sum_{h=0}^{\infty} Prob(d_{h-1} > t_{*q} \geq d_h) \tag{14}$$

$$x \sum_{g=0}^{\infty} Prob(d_h + (g+1)s > t_{*q} \geq d_h + gs \mid d_{h-1} > t_{*q} \geq d_h)$$

$$x\ [\vartheta(d_h + gs) - C()] + (1 - \frac{1}{k(h,g)})$$

$$x\ Prob(MAX[t_{iq}] \geq d_h + (g+1)s > t_{*q} \geq d_h + gs \mid d_{h-1} > t_{*q} \geq d_h)[\vartheta s - C^*()]$$

where $C^*(\,) = C\,(h\varphi_{d'}, g\varphi_s) - C(h\varphi_{d'}, (g+1)\varphi_s)$.

The term Prob $(d_{h-1} > t_{*q} \geq d_h)$ represents the probability that the $h\underline{th}$ setting of the SBR is the final setting (h = 0 for the initial SBR). The first term within the second summation is the probability that the bidder with the highest value receives the final bid of $d_h + gs$ (conditional on d_h being accepted). The second term is the conditional probability that some other bidder receives the final bid of $d_h + gs$. Finally, the third term is the conditional probability that the bidder with the highest valuation receives the final bid of d_h + (g + 1)s.

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[28] As the time-cost of changing the SBR decreases and the highest order statistics become numerous and tightly grouped, the use of decreasing SBR to reach the final bid rather that bid increments becomes more attractive. To see this, note that the third term of in the second summation of equation (13) decreases under these conditions; that is, the probability decreases that the bidder with the highest value will offer a final bid any higher than the bidders with high order statistics. The use of decreasing SBR's is merely a Dutch auction and may have cost advantages other than saving time, since the role of the auctioneer is very limited.

A few implications follow immediately from this general representation. First, the auction house's objectives diverge from the seller's due to the term $C(h\varphi_{d'}, g\varphi_s)$. As the time-cost of bid increments and resetting the SBR's become negligible, the optimal choice of d_h's and g for both the house and seller coincide. As the difference in values between the bidders with the two highest valuation decreases the optimal bid increment, s, decreases. As the contribution to the time-cost of resetting the SBR increases, the d_h decreases. As the contribution to the time-cost of each bid increases, the bid increment decreases. As the number of possible bidders near (i.e., between t_{*q} and t_{*q} - $(d_h + gs)$) the bidder with the second highest value increases, and the bid increment decreases in order to decrease the probability of the highest value falling below d_h + (g + 1)s.

Consider now our fourth informational case where the initial SBR may change the distributions of the bidders' highest valuations. Reputable auction houses may be able to inform bidders of the resale possibilities for the good or to assure potential bidders of the true value or authenticity of items being sold by setting the initial SBR above that expected by the bidders. Auction houses may also devalue items by setting the SBR below a bidder's expected value. However, an auction house's ability to manipulate a bidder's willingness to pay is limited. Setting a SBR that is so high that it is not accepted would tend to discourage bidders, and more importantly, reveal to the bidder with the highest valuation that there is no second highest bidder who considers the item to be worth the SBR.

The expected proceeds from an auction sale can be represented as in equation (14) above, but with the probability distribution of the valuation conditional on the initial SBR. The problem remains much the same. The new trade-off in the problem is between the risk of devaluing the good and and the risk of not getting an affirmative response to d and having to restart the the bidding at d'. The revaluation by the bidders after the SBR acts like an additional penalty for setting the SBR fairly low in order to avoid having use d'.

The auction house's problem is a variation on the knapsack problem, where the items chosen to be sold have stochastic values in terms of final bids and the times required for sale.[29] The auction house wishes to fill its allotted time with sold items that, a priori, have no fixed amount of time associated with them. The house may be thought of as attempting to adjust an inventory subject to a stochastic demand.[30] The solution to such problems is very time consuming and with a large number of potential bidders it can not generally be solved in a operationally feasible amount of time without putting a great deal more structure on the problem. One form of structure is to specify a priori the size of s and of d'. This is frequently done by auction houses and it is the rare, very-experienced auctioneer who can successful vary all of the choice variables open to the auctioneer.

4. The Auction House-Seller Relationship

In his or her relationship with the auction house, the seller, S, is assumed to be concerned only with the seller's share of the expected revenue from the sale of q, $E[BR_q^*]$ and the variance in that expected revenue, σ. The seller's expected revenue can be represented as a function of,

$$E[BR_S^*] = g(\vartheta, N, I, \tau_q, d, MAX\ t_{iq}, \delta), \qquad (15)$$

where ϑ is the auction house's share of the expected revenue, N as identified earlier is the number of potential bidders for q, I, is the information available

29 For a discussion of knapsack problems, see Dreyfus and Law (1977).

30 See Whittle (1982) for methods for solving stochastic inventory problems.

to the auction house, τ_q, is the amount of time spent by the auction house in selling the good, q, d is the SBR, and MAX t_{iq} is the initial MWTP of any of the bidder to pay for the good q, and δ is a measure of the distance between the high valued extreme order statistics.

Many of the derivatives of this function can be immediately signed. Dropping the S subscript:

$$\frac{\partial EBR}{\partial \vartheta} < 0\,;\ \frac{\partial E[BR]}{\partial N} > 0\,;\ \frac{\partial E[BR]}{\partial I} > 0\,;\ \frac{\partial E[BR^*]}{\partial MAX[t_{iq}]} > 0\,. \tag{16}$$

In the case of ϑ_q, d, and δ the derivatives with respect to E[BR] are indeterminate. The implication of these comparative static results is that the auction house can attract more sellers by lowering ϑ, attracting more potential bidders, N, or obtaining more information, I. It is interesting that in for particular types of goods, ϑ seems to be fixed, and the variation in ϑ across goods appears to represent differences in the fixed costs an auction house incurs to enter that particular line of auctioning.[31]

The role of the auction house in attracting potential bidders cannot be over emphasized. Wilson (1977) and Reece (1978) have shown the importance of the number of bidders. The auction house's strategy becomes particularly difficult in the presence of a small number of bidders. In a more precise sense, as N increases, the expected values of the extreme order statistics increase for almost all reasonable distributions (David, 1970).

The derivatives of σ with respect to N, I, d, and δ are:

$$\frac{\partial \sigma}{\partial N} < 0\,;\ \frac{\partial \sigma}{\partial I} < 0\,;\ \frac{\partial \sigma}{\partial d} < 0\,;\ \frac{\partial \sigma}{\partial \delta} > 0\,. \tag{17}$$

[31] It can be shown that in some cases the auction house does not invest in enough information and thus doesn't convey enough information through its actions. Sellers frequently take out ads which provide information (and hype) about their good and inform potential bidders that the item will be for sale at a particular auction.

For t_{iq}, the $\partial\sigma / \partial MAX[t_{iq}]$ is indeterminate. There is no reason why the auction house and the seller should have the same level of risk aversion so different auction houses can compete for sellers on the basis of the variance of the expected revenue.

The auction house will only accept goods to be sold that, on the margin, equate the expected cost of time with the expected proceeds of the sale. This practice by auction house has led to the common practice of bundling small items into larger lots and also the creation of specialty auction houses which sell small items. Even though these houses tend charge a larger ϑ they also tend to bundle in order to maximize the profits from the given amount of time in which it possible to attract a sufficient number of buyers.

The two extremes, from an industrial organization standpoint, are the auction house as a monopoly and the auction house as a competitive firm. If the auction house is a monopoly, it is probably a contestable monopoly since only the large fixed costs of opening up an auction house stand in the way. Since the business of the auction house is to bring together buyers and sellers and to provide authoritative information, the auction house's assets are those which are usually classified as good-will and education. If the auction house is a monopoly it stands to collect the profits resulting from the transactions made. If the auction house is a competitive firm any profits from the auction house will become bid away in the form of smaller ϑ's, or more money spent by the auction house on information or attracting buyers. There are a number of issues which could be explored here, but they take us far afield of our original purpose.

5. Concluding Remarks

We have obviously only begun to scratch the surface in our characterization of an English auction. Most particularly, the assumption of no order effects and of the bidders valuing each good independently of any other good being sold at the auction is unrealistic. Still we believe we have incorporated many of the features which are typical of an English auction, and shown why if the auction house's perspective was adapted, these features have many optimal characteristics. The role of information was shown to be instrumental in causing the divergence between the auction house's and the seller's objective functions. The ability of the auction house to use its rules and actions to acquire the highest possible bid is certainly in the interest of the seller. However, the additional costs to the house of searching for the highest possible bid (by adjusting the bid increment or obtaining information) works to the disadvantage of the seller. The equilibrium properties of our formulation are unclear at this time.

In this sense, the seller-auctioneer relationship is just a member of the larger class of principal-agent relationships (Hess, 1983). Recent work on principal agent relationships with incomplete information, particularly those which emphasize repeated interactions,[32] (Radner, 1981; Holmstrom and Meyerson, 1983; Meyerson, 1983) would appear to be useful in expanding the simple analysis of the last section. The industrial organization of large auction house is also an interesting avenue for future work. Auction houses may enjoy at least locally the benefits of being both a monopolist and a monopsonist. The sizable fixed cost for both the auction house and the buyers attending many auctions have been overlooked in almost all of the literature on auctions

[32] The repeated interactions come both during the auction itself and from the fact that many of the buyers and sellers tend to deal with the auction house on a long term basis.

except for some of the work on petroleum and mineral leases where the estimate of the resources on which the company's bid will be based is given some attention. The auction house is a unique institution which exists largely to gather together a critical mass of buyers and sellers. This aspect of auctions, too, has been overlooked and we have only begun to show that auction houses can compete with each other on that basis.

The main focus of this paper has been on the modifications which must be made to auction theory when the role of the auctioneer is recognized. One of the main results of this paper has been the divergence of the second price and English auction outcomes under the private valuation model.[33] We have also provided some evidence that suggests as the maximum values or order statistics get tightly grouped that a special case of our formulation of an English auction, which is equivalent to a Dutch auction, may become desirable in the face of a time constraint; and indeed almost all of the uses of Dutch auctions are when the goods are homogeneous and the bidders tightly bunched. The risk adverse seller may prefer the Japanese auction which guarantees a second price outcome but it will yield a lower expected return to the auction house and reduce auction house control. This suggests that the auction house will demand a higher percentage of revenue from goods sold in this manner. We believe it possible to show that the Japanese and Dutch auctions are particular degenerate cases of the English auction that are optimal from the auction house's perspective in only a limited number of circumstances. We are presently working on proofs along this line.

[33] Hopefully we have also emphasized that the form of an English auction matters, and while researchers such as Milgrom and Weber (1982) have been careful to specify the features of their stylized English (i.e., Japanese auction), their results have been too frequently quoted and used as applying to the standard English auction.

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