JOURNAL OF ENVIRONMENTAL ECONOMICS AND MANAGEMENT **36,** 314–323 (1998)
ARTICLE NO. EE981050

Sequencing and Valuing Public Goods

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Received October 28, 1997; revised August 10, 1998

This paper establishes several propositions concerning the importance of context in valuing public goods. It first provides necessary and sufficient conditions for the value of a public good to be independent of context. Utility-constant valuation sequences are considered where public goods are systematically made available or taken away. For the case of strict Hicksian substitutes, willingness to pay for an increase in one of the public goods is strictly decreasing the farther out in a sequence it is valued. For the destruction of public goods the reverse is true for willingness to accept compensation. Sequencing has opposite implications for the assessment of the benefits of providing public goods than for the assessment of the damage from destroying them. © 1998 Academic Press

1. INTRODUCTION

The issue of valuing public goods, particularly environmental amenities, has moved to the forefront of policy discussions.[1] One disturbing aspect of valuation efforts is the observation that if one summed the public's estimated values for individual environmental amenities, the sum may exceed disposable income. Hoehn and Randall [8] point out that the problem lies in the adding together of individually derived willingness to pay estimates. There will typically be policy interactions that are missed when analysis is limited to individually-derived benefit estimates. Hoehn and Randall conclude that ''conventional procedures [independent valuations] systematically overstate net benefits but also define a valid benefit cost approach.'' Focusing on substitution and complementarity in valuation, Hoehn [7] later concludes that independent valuation may either understate or overstate net benefits.[2]

[1] The term public good is used to describe goods that are collectively provided, but may not strictly satisfy nonexcludability and nonrivalry in consumption.

[2] Substitution in valuation refers to a decreasing value for provision of one good as more of another public good is made available. Complementarity in valuation refers to an increasing value under the same conditions.

0095-0696/98 $25.00

The analysis provided in this paper takes a somewhat different approach in addressing several related questions. Rather than assuming that policies are substitutes or complements in valuation, the focus is on local and global properties of preferences that offer potential explanations for complementarity or substitution in valuation. The first question addressed is in general, regardless of value classifications as substitutes or complements, should the valuation context (levels of other public goods) generally affect the value of a change in a particular public good? This question is affirmatively answered using a rationed goods representation of public goods[3] and drawing inferences from the relation between the rationed and unrationed consumption spaces. Necessary and sufficient conditions for context independence are characterized and shown to be very restrictive. The second question addressed is do there exist local conditions on preferences that imply value substitution or complementarity? It is established that the entire set of public goods being classified as Hicksian substitutes in the unrationed consumption space provides valuation substitutes. In contrast, if the entire set of public goods are classified as unrationed complements, this will ensure complementarity in valuation only in the limited case of two public goods. If there are three or more public goods, then the unrationed complementarity condition is insufficient to produce a set of public goods which are all complements in valuation.

In order to address multidimensional policy changes, utility-constant valuation sequences are used to establish that if a global substitutability condition is satisfied, sequencing has opposite implications for the assessment of provision benefits than for losses from destruction or injury. Willingness to pay for a particular public good decreases when valued later in a willingness to pay sequence, while willingness to accept compensation increases when valued later in a willingness to accept sequence.

2. A MODEL OF PUBLIC GOODS CONSUMPTION

Suppose that individuals have strictly convex preferences over n market goods and k public goods. Preferences are representable by a strictly quasiconcave utility function that is increasing in all arguments. It is further assumed that the utility function is twice continuously differentiable. The objective of consumers is to maximize utility in market goods subject to market prices, the budget constraint, and the level of public goods. Letting the vector X denote the levels of market goods, the vector Q the levels of public goods, and y the level of income, the problem is as follows:

$$\max_{X} U(X, Q) \qquad \text{s.t. } p \cdot X \leq y \text{ and } Q = \overline{Q}. \tag{1}$$

Maximization of utility yields the indirect utility function, $v(p, Q, y)$. Letting $U = v(p, Q, y)$ and assuming the regularity conditions outlined in Diewert [5] are satisfied allows for the dual expenditure minimization problem

$$\min_{X} p \cdot X \qquad \text{s.t. } U(X, Q) = \overline{U} \text{ and } Q = \overline{Q}. \tag{2}$$

[3] Examples of the rationed goods approach can be found in Neary and Roberts [13], Deaton [4], Madden [11], and Cornes [3].

The minimization problem produces a set of Hicksian demands, $X^h(p, Q, U)$, that depend on market prices, the level of public goods, and the level of utility. Expenditure minimization yields the expenditure function, $e(p, Q, U) = p \cdot X^h(p, Q, U)$. The marginal value vector for each of the k public goods equals the negative of the gradient of the expenditure function (Mäler [12]).

$$-\nabla_Q e(p, Q, U) = p^v(p, Q, U) \in \mathbb{R}^k. \tag{3}$$

Potentially each public good's marginal value, $p_i^v(p, Q, U)$, is influenced by the level of that public good as well as the level of all other public goods, a fact that figures prominently in this paper. The vector of marginal values is often referred to as virtual prices, a term adopted in this paper.[4] Suppose that two levels of public good i are under consideration with $q_i^0 < q_i^1$ and define the following reference utility levels, $U^0 = v(p, q_i^0, Q_{-i}, y)$ and $U^1 = v(p, q_i^1, Q_{-i}, y)$ where Q_{-i} represents the remaining $k - 1$ public goods. As shown by Mäler [12] and Loehman [10], the virtual prices have a direct link to willingness to pay and willingness to accept. Using the relationship provided in (3), willingness to pay and willingness to accept can be represented as the integral of the virtual price over the change in the public good. The two measures differ in the reference level of utility. The relationships between virtual prices and willingness to pay/willingness to accept facilitate an analysis of discrete changes using local properties of preferences. Imposing various preference conditions allows the determination of the substitute/complementary classification between public goods.

As noted above in footnote 4, the virtual prices satisfy the condition that if the consumer were minimizing expenditures on both market and public goods subject to prices p and p^v, respectively, the same level of market and public goods would be chosen as what occurs under rationing. Let $X_v^h(p, p^v, U)$ and $Q_v^h(p, p^v, U)$ represent the unrationed demands. By definition, the following identity is satisfied

$$\begin{bmatrix} X^h(p, Q, U) \\ Q \end{bmatrix} = \begin{bmatrix} X_v^h(p, p^v, U) \\ Q_v^h(p, p^v, U) \end{bmatrix}. \tag{4}$$

Madden [11, lemma 1], establishes the relationship between the price derivatives of unrationed demands for Q and the quantity derivatives of virtual prices. Using the implicit function theorem, the identity from (4) can be differentiated with respect to p and Q[5]

$$\begin{bmatrix} \dfrac{\partial X^h}{\partial p} & \dfrac{\partial X^h}{\partial Q} \\ \mathbf{0}_{\mathbf{kxn}} & \mathbf{I}_{\mathbf{kxk}} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial X_v^h}{\partial p} + \dfrac{\partial X_v^h}{\partial p^v}\dfrac{\partial p^v}{\partial p} & \dfrac{\partial X_v^h}{\partial p^v}\dfrac{\partial p^v}{\partial Q} \\ \dfrac{\partial Q_v^h}{\partial p} + \dfrac{\partial Q_v^h}{\partial p^v}\dfrac{\partial p^v}{\partial p} & \dfrac{\partial Q_v^h}{\partial p^v}\dfrac{\partial p^v}{\partial Q} \end{bmatrix}. \tag{5}$$

[4]The term virtual prices (see Neary and Roberts [13] and Madden [11]) is common in the rationed goods literature and refers to the fact that for a given level of rationing, the virtual prices satisfy the following condition. If the consumer were minimizing expenditures on all goods, including the public goods, facing prices p for market goods, p^v for public goods, and subject to the same level of utility, the same level of market and public goods would be chosen as in the original problem where Q is rationed.

[5]The derivation of this relationship can also be found in Samuelson [15] and Chavas [2].

Using the lower right-hand corner set of differential equations and assuming nonsingularity of the matrix $\partial Q_v^h / \partial p^v$, the following relationship exists[6]

$$\frac{\partial p^v}{\partial Q} = \left[\frac{\partial Q_v^h}{\partial p^v} \right]^{-1}. \tag{6}$$

Thus, there exists an inverted relationship between the matrix of quantity derivatives of virtual prices and matrix of cross-price derivatives for the compensated demands of Q. Knowing this relationship gives rise to several propositions that provide insight into the substitute/complement classification of values for Q.

3. PROPOSITIONS

Recently a great deal of attention has been focused on what has been referred to as embedding. Consider the following statement by Kahneman and Knetsch [9]:

> The present article reports an experimental investigation of what is perhaps the most serious shortcoming of CVM [contingent valuation method]: that the assessed value of a public good is demonstrably arbitrary, because willingness to pay for the same good can vary over a wide range depending on whether the good is assessed on its own or embedded as part of a more inclusive package.

Smith [16] provides an analysis of embedding in the case of a single public good and argues that embedding should be expected on account of substitution. Randall and Hoehn [14] demonstrate embedding for sequences of commodity price changes, as opposed to changes in the levels of public goods, based on estimates for consumer demand in the Dominican Republic reported by Yen and Roe [18].

In an earlier version of this paper, Carson, Flores, and Hanemann [1] use a general characterization of context independence and show that under the class of preferences considered in this paper, context independence is unlikely to be satisfied.[7] That paper develops a definition of context independence of virtual prices, provides necessary and sufficient conditions for virtual price context independence, extends these results to preference context independence, and then shows that the within the set of considered preferences, the subset of preferences exhibiting context independence is a closed set with no interior which is best understood using analogy. In a Euclidian space, as opposed to a functional space, we have a subset of measure zero. The definitions and propositions are provided here without proof.[8]

DEFINITION. p_1^v is independent of context if any change in any of the other public goods leaves p_1^v unchanged. Formally, p_1^v is *context independent* if $\partial p_1^v(p, Q, U)/\partial q_j = 0$, $j \neq 1$ at all levels of Q.[9]

[6] This result is from Madden [11, lemma 1].

[7] Recall these preferences are those that can be represented by an increasing, strictly quasiconcave, twice differentiable utility function.

[8] Interested readers are referred to the earlier version of this paper, Carson, Flores, and Hanemann [1].

[9] Note then that by symmetry, the marginal value of q_1 is context independent if and only if the derivative of p_j^v with respect to q_1 will also be zero for all $j \neq 1$.

PROPOSITION 1. *Assume preferences can be represented by an increasing, strictly quasiconcave, twice differentiable utility function. The virtual price of q_1 is context independent if and only if*

$$\frac{\partial q_{v,1}}{\partial p_j^v} = \frac{\partial q_{v,1}^m}{\partial p_j^v} + \frac{\partial q_{v,1}^m}{\partial y} q_j = 0 \quad \forall j \neq 1, Q \in \mathbb{R}_{++}^k,$$

where the left-hand side of the equation is the $(1, j)$ entry of the unrationed substitution matrix and the m terms designate the uncompensated demands.

DEFINITION. q_1 is context independent if for all $Q^0, Q^1 \in \mathbb{R}_{++}^k$,

$$e\big(p, q_1^0, Q_{k-1}^0, U\big) - e\big(p, q_1^1, Q_{k-1}^0, U\big) = e\big(p, q_1^0, Q_{k-1}^1, U\big) - e\big(p, q_1^1, Q_{k-1}^1, U\big).$$

The left-hand side of the equation is the value for any change in q_1 with other public goods at Q_{k-1}^0 and the right-hand side is the value for the same change, but with other public goods at Q_{k-1}^1.

PROPOSITION 2. *q_1 is globally context independent if and only if the virtual price of q_1 is context independent.*

Let E denote the set of expenditure functions generated from preferences that can be represented by strictly increasing, strictly quasiconcave, twice differentiable utility functions.

PROPOSITION 3. *Let $S \subset E$ such that for each $e \in S$, e satisfies the condition of global context independence. Then S is a closed set with no interior.*

Propositions 1–3 are based on the class of preferences that can be represented by strictly increasing, strictly quasiconcave, twice differentiable utility functions, a class that basically dominates applied microeconomic analysis. The three propositions establish that economic theory predicts that values of public goods will depend upon the levels of the public goods in a very general sense. The conclusion is that contextual valuation effects are the norm rather than the exception.

By imposing additional conditions, more precise predictions are possible. Suppose that $q_{v,i}$ and $q_{v,j}$ are (strict) Hicksian substitutes for all $i, j = 1, 2, \dots, k$, where $i \neq j$. Recall that two goods are (strict) Hicksian substitutes if $(\partial q_i / \partial p^{v_j})$ $(>) \geq 0$.

Note that the matrix $\mathbf{H_Q} = [\partial Q_v^h / \partial p^v] = [\partial p^v / \partial Q]^{-1}$ possesses the following properties:

(i) $\mathbf{H_Q}$ is negative definite given the assumption of nonsingularity and the property of negative semidefiniteness of the entire unrestricted substitution matrix;

(ii) All off diagonal elements of $\mathbf{H_Q}$ are strictly positive; and

(iii) Properties (i) and (ii) imply all elements of $\mathbf{H_Q}$ are nonzero and from this, it follows that $\mathbf{H_Q}$ is indecomposable, where a square matrix $\mathbf{A}$ is indecomposable if there does not exist a permutation matrix $\mathbf{P}$ such that

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{0} & \mathbf{A}_{22} \end{bmatrix}.$$

Matrices with off diagonals of like signs have been studied extensively and Takayama [17] has summarized many results pertaining to such matrices. Of particular interest in this case is the result that given the three conditions above, $\mathbf{H}_{\mathbf{Q}}^{-1} = \partial p^{v}/\partial Q < \mathbf{0}$. That is, all elements of the matrix made up of quantity derivatives of the virtual prices are strictly negative.

By considering the weaker condition of Hicksian substitutes (versus strict), condition (iii) above is lost. However, a further result from Takayama [17] is that given conditions (i) and (ii), then $\mathbf{H}_{\mathbf{Q}}^{-1} = \partial p^{v}/\partial Q \leq \mathbf{0}$. The consequences of Hicksian substitutes in the set of rationed goods on $\partial p^{v}/\partial Q$ can be summarized as follows:

$$\left[\frac{\partial p^{v}}{\partial Q}\right](<) \leq 0 \quad \text{if all } q_i, q_j \text{ are (strict) Hicksian substitutes.}$$

From this result it follows that when all rationed goods are (strict) Hicksian substitutes, $p_1^{v}(p, Q, U)$ is decreasing (nonincreasing) in the levels of $q_j = 1, 2, \dots, k$. If we assume that the Hicksian substitution condition holds over the neighborhood in $\mathbb{R}^k$ in which we are considering a finite set of changes, the following propositions easily follow.

PROPOSITION 4. *Assume that all rationed goods are (strict) Hicksian substitutes. Then willingness to pay for an increase in q_1 (WTP_1) is a nonincreasing (decreasing) function in the levels of $q_j = 1, 2, \dots, k$.*

Proof. For simplicity, let $j = 2$ and suppress the price and utility notation. The change in *WTP* for q_1 at different levels of q_2 is written as follows:

$$\begin{aligned} WTP_1\left(q_j^0\right) - WTP_1\left(q_j^1\right) &= \int_{q_j^1}^{q_j^0} \frac{\partial WTP(s)}{\partial s}\, ds \\ &= \int_{q_j^1}^{q_j^0} \left[\int_{q_1^0}^{q_1^1} \frac{\partial p_1^{v}(t, s)}{\partial q_j}\, dt\right] ds. \qquad (7) \end{aligned}$$

Under the Hicksian substitute assumption, the partial derivative is everywhere nonpositive (negative) over the path integral in q_1 making the term in brackets nonpositive (negative). If $q_2^1 > q_2^0$, the entire expression will be nonnegative (positive) which implies that $WTP_1(q_2^0) > WTP_1(q_2^1)$. ∎

Now let us consider the case of utility-constant valuation sequences, maintaining the Hicksian substitute assumption. These sequences are important because willingness to pay or willingness to accept for any multidimensional change can be decomposed into a sequence of single-dimensional changes which are easily analyzed using the framework from above.

PROPOSITION 5. *Assume that all rationed goods are (strict) Hicksian substitutes. Suppose that we are interested in the willingness to pay for increases in all k goods and we look at a valuation sequence where goods are valued successively. If we permute the order of sequencing, then the willingness to pay for the change in q_1 will be (strictly) greatest when valued first in the sequence and (strictly) smallest when valued last in the sequence.*

Proof. This proposition follows as a direct consequence of Proposition 4. When valued first in the sequence, the levels of $q_j = 1, 2, \dots, k$ are at the initial level. Any permutation where the change q_1 is valued later will result in higher levels of substitute goods and thus a lower virtual price and willingness to pay. Valuing the change in q_1 last in the sequence will result in the highest levels of substitute goods, and hence, the lowest willingness to pay for the change in q_1. ∎

Proposition 5 is important because it describes the effect of bundling and gives an economic reason why the sum of independent valuations for each item in the bundle is more than the value of the bundle. As more substitutes are added to the bundle of changes, the value of the bundle will be less than the independent valuations because of successively higher levels of other public goods.

PROPOSITION 6. *Assume that all rationed goods are (strict) Hicksian substitutes. Willingness to accept compensation for a reduction in* q_1 *is also a (strictly) decreasing function in* q_j, $j = 2, 3, \dots, k$.

Proof. Willingness to pay and willingness to accept differ only by the reference utility level. All arguments from the proof of Proposition 4 apply. ∎

PROPOSITION 7. *Assume that all rationed goods are (strict) Hicksian substitutes. Suppose that we are interested in the willingness to accept compensation for a reduction in all k publicly provided goods and permute the valuation order. Then the willingness to accept compensation for the reduction in* q_1 *will be (strictly) smallest when valued first in the sequence, and (strictly) greatest when valued last in the sequence.*

Proof. All conditions are similar to Proposition 5 with the exception that as the change in q_1 is placed later in the sequence, the level of substitute goods is decreasing. Decreasing levels of substitute goods imply an increasing virtual price and willingness to accept. Therefore, willingness to accept compensation for the reduction in q_1 increases when placed later in the sequence and is smallest when placed first in the sequence. ∎

The analysis has so far focused on situations of a generic level of utility for both the willingness to accept sequence and the willingness to pay sequence. In practice, the true object of interest is often the willingness to accept compensation for a prescribed change at the utility level before the change, but for various reasons, willingness to pay for the same change at the post-change utility level is used as an approximation. Let us assume that willingness to accept exceeds willingness to pay.[10]

Suppose $q_i^1 > q_i^0$ for all $i = 1, 2, \dots, k$. Let U^1 be the maximum attainable utility given prices p, income y and public good $q_i = q_i^1$ for $i = 1, 2, \dots, k$. Let U^0 be the maximum level of utility attainable given the same p, y, and levels of q_j, $j = 2, 3, \dots, k$, but $q_1 = q_1^0$. Then $U^1 > U^0$ and the following inequality follows:

$$WTP = \int_{q_1^0}^{q_1^1} p_1^v(p, Q, U^0)\, dq_1 \le \int_{q_1^0}^{q_1^1} p_1^v(p, Q, U^1)\, dq_1 = WTA. \tag{8}$$

[10] In the case of a single rationed good, Hanemann [6] shows that if the rationed good is an unrationed normal good (positive income effect), then the reference utility level will lead to a higher difference in expenditures. For multiple rationed goods this may not always be the case, but is assumed here.

Now consider willingness to pay versus willingness to accept for this same q_1 quantity change, but in the different valuation contexts described above. That is, we are interested in the implication of sequencing on the valuations using willingness to pay with utility level U^0 and the decreased level of q_1 versus willingness to pay with utility level U_1 and the higher level of q_1.

PROPOSITION 8. *Assume that all rationed goods are Hicksian substitutes. Willingness to pay for a good when valued first in a willingness to pay sequence is no greater than willingness to accept valued in any order, and strictly less in the case of strict Hicksian substitutes.*

Proof. Note inequality (8) above and consider a willingness to accept sequence. By applying Proposition 7, the valuation is increasing when placed farther out in the willingness to accept sequence. Similarly by Proposition 5, willingness to pay for the prescribed change is greatest when valued first in a willingness to pay sequence leading to the desired result. ∎

Proposition 8 has particular relevance in the area of natural resource damage assessment. In damage assessment analysis, willingness to pay is often used as a proxy for willingness to accept. Putting aside measurement errors due to the various valuation techniques, willingness to pay for restoration alone, as opposed to combining with other goods, will be the closest in magnitude.

Madden [11] provides what he refers to as R classifications of compensated demands which apply to the work presented above. The unrestricted Hicksian substitution condition presented above yields R substitutes within all of the rationed goods. Madden shows that the R classifications "favor substitutes" in that for every good i, rationed and unrationed, there must always be at least one other good j that is an R substitute for i. This statement has significance here since one can ask the question, what happens when all rationed goods are unrestricted Hicksian complements? Might it be the case then that, similar to the case where all rationed goods are Hicksian substitutes, the derivative of the implicit prices with respect to other quantities is positive (as opposed to negative)? This question is answered by the following proposition.

PROPOSITION 9. *Suppose there are only two rationed (public) goods, $k = 2$, and these rationed goods are (strict) Hicksian complements, then willingness to pay for an increase in q_i, $i = 1, 2$ is a nondecreasing (increasing) function in the levels of q_j, $j \neq i$. When $k > 2$ and the goods are Hicksian complements, willingness to pay for an increase in q_i may be either increasing or decreasing in the levels of q_j, $j \neq i$.*

Proof. Assume $k = 2$ and both goods are unrestricted Hicksian complements

$$\left[\frac{\partial p^v}{\partial Q}\right] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = (h_{11}h_{22} - h_{12}h_{21})^{-1} \begin{bmatrix} h_{22} & -h_{12} \\ -h_{21} & h_{11} \end{bmatrix}.$$

Since both goods are complements, $h_{12} = h_{21}$ are negative. By negative definiteness, $(h_{11}h_{22} - h_{12}h_{21})$ is negative. Thus the off-diagonals of the virtual price substitution matrix are positive. The rest of the proof uses the same arguments used in proving Proposition 4. This result will not always be true for $k > 2$ as the following example of a symmetric negative definite matrix indicates

$$\begin{bmatrix} -8 & -1.5 & -3 \\ -1.5 & -2 & -2 \\ -3 & -2 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} -.216 & -.1622 & .324 \\ -1.622 & -1.622 & 1.24 \\ .324 & 1.234 & -1.487 \end{bmatrix}.$$

Thus depending upon the relative sizes of the terms in $\mathbf{H_Q}$, the off-diagonals of $\partial p^v / \partial Q$ may or may not be all positive when all rationed goods are unrestricted Hicksian complements. ∎

4. CONCLUDING REMARKS

This paper provides a general treatment of the implications of context and sequencing in the valuation of public goods. The treatment is based entirely on economic theory and eschews the consideration of alternative theories grounded in moral or psychological considerations. The paper formalizes what to most economists is basic economic intuition: The value of a public good is dependent upon the context in which it is provided. Using a formal economic model while adopting reasonable assumptions, we produce results consistent with one description of embedding.[11]

Admittedly our analysis does not completely resolve the issue of embedding. Some will argue that observed differences in values under different contexts are too large to be plausible. While our analysis does not explicitly address this issue, the fact that economic theory leaves open the possibility of a wide range of circumstances suggests a cautionary approach when attempting to judge whether economic theory or some alternative theory is at work. Meaningful judgement will require a clear understanding of the realm of economic possibilities and we feel that our analysis is useful to this end.

There are several policy implications that follow from these results. In order to make well-informed policy decisions, careful attention should be paid to policy interactions. As shown in Proposition 9 for discrete values and discussed by Madden [11] for marginal values, R classifications favor substitutes. Thus summing independent values for increases in provision will more often overestimate the value of a package of changes; for decreases the opposite is true. Thus, the status quo, as interpreted by the agent, plays a key role in accurately assessing the impacts of multidimensional changes.

Our results do not make independent valuations obsolete. With some confidence we can infer that if the sum of independent valuations for increases is less than the costs of provision for a package of increases, then the package of increases should not be undertaken.[12] In cases when the sum of independent values exceeds the cost of providing the package, confidence should necessarily diminish. Again for decreases, things work in opposite directions. Using willingness to pay as an approximation of willingness to accept is by definition a conservative estimate. Any

[11] As pointed out by an anonymous referee, many issues are lumped under the rubric of embedding. This paper focuses on the description summarized in the Kahneman and Knetsch [9] quote found at the beginning of Sec. 3.

[12] It may still be beneficial to undertake a subset of the increases.

attempt to combine the increase that is valued and then used as the approximation will only serve to move the estimate further away from the desired willingness to accept.

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