Economics 245 — Fall 2021

International Trade

Problem Set 3

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1 Offshoring as a Rybzcynski Effect

There are two industries 1 and 2 and two factors of production: non-offshorable labor N and offshorable labor L. Non-offshorable labor earns a wage s and offshorable labor earns a wage w. Each industry i's production function $Q_i = AF_i(N_i, L_i)$ is homogeneous of degree one. The foreign country's production functions are identical up to a Hicks-neutral productivity parameter: $Q_i = A^* F_i(N_i, L_i)$. Suppose $A^* < A$. Throughout this question, assume that factor-price equalization occurs: $w/w^* = s/s^* = A/A^*$ and that A and A^* are constant.

Introduce one additional type of trade: firms can have L-tasks performed offshore. Onshore and offshore tasks are not perfect substitutes. They are combined through a Cobb-Douglas production function

$$L_i = (L_i^{\text{on}})^{1-\gamma}(L_i^{\text{off}})^{\gamma},$$

where L_i^{off} denotes offshored labor and $\gamma \in (0, 1)$ captures the intensity of onshore labor. When offshored, a foreign unit of labor costs βw^* for the home economy to contract foreign labor at a distance.

To standardize the analysis, consider industry 1 to be relatively intensive in offshorable L-labor and industry 2 intensive in non-offshorable N-labor. Only the offshoring cost parameter β in the model is free to change.

1. Derive optimal inputs L_i^{on} and L_i^{off} from the cost minimization problem for $wL_i^{\text{on}} + \beta w^* L_i^{\text{off}}$ given some $L_i = \bar{L}_i$, and express L_i^{on} as a function of L_i^{off}. What is the elasticity of substitution between L_i^{on} and L_i^{off}: $-\mathrm{d}\ln[L_i^{\text{on}}/L_i^{\text{off}}]/\mathrm{d}\ln[w/(\beta w^*)]$?

2. What is the shadow price $\bar{w}$ of L_i given optimal choices of L_i^{on} and L_i^{off}? [*Hint*: Obtain the Lagrange multiplier from the cost minimization problem.] Express the ratio of shadow prices $\bar{w}/\bar{w}^*$ as a function of the productivity ratio A/A^* given factor-price equalization $w/w^* = A/A^*$. State demand for L_i^{off} as a function of wages, parameters and L_i.

3. Under these functional forms, is there offshoring for $\beta w^* > w$? In other words, is $A^* < A$ a necessary condition for offshoring? Is there two-way offshoring (home contracting from abroad and abroad contracting from home)? Show that your answers hold for any function $L_i = G(L_i^{\text{off}}; L_i^{\text{on}})$ that satisfies the Inada conditions $\lim_{L_i^{\text{off}} \to 0} \partial G(L_i^{\text{off}}; L_i^{\text{on}})/\partial L_i^{\text{off}} = \infty$ and $\lim_{L_i^{\text{off}} \to \infty} \partial G(L_i^{\text{off}}; L_i^{\text{on}})/\partial L_i^{\text{off}} = 0$.

4. Use the first-order conditions for optimal inputs to show that

$$\hat{L}_i^{\text{off}} = \hat{L}_i^{\text{on}} - \hat{\beta} = \hat{L}_i - (1-\gamma)\,\hat{\beta},$$

where hats over a variable x denote relative changes $\hat{x} \equiv \mathrm{d}\ln x$.

5. Show that, in equilibrium, total offshorable labor supply to the home economy is

$$L = L_1 + L_2 = (L^{\text{on}})^{1-\gamma}(L^{\text{off}})^{\gamma}.$$

[*Hint*: Use the fact that $L_i^{\text{on}}/L_i^{\text{off}}$ is a constant across both industries for given β.]

6. For simplicity, suppose from now on that offshoring goes only one way, with domestic production using foreign offshore labor but not the reverse (β for offshoring by the domestic economy is finite, but infinite for offshoring by the foreign economy). Factor market clearing then is equivalent to

$$\begin{aligned} a_{L1}Q_1 + a_{L2}Q_2 &= (L^{\text{on}})^{1-\gamma}(L^{\text{off}})^{\gamma} \\ a_{N1}Q_1 + a_{N2}Q_2 &= N \end{aligned}$$

for the unit labor requirements $a_{Lj} = L_j/Q_j$ and $a_{Nj} = N_j/Q_j$. Show that

$$\begin{aligned} \alpha_{L1}\hat{Q}_1 + (1-\alpha_{L1})\hat{Q}_2 &= (1-\gamma)\hat{L}^{\text{on}} + \gamma\hat{L}^{\text{off}} \\ \alpha_{N1}\hat{Q}_1 + (1-\alpha_{N1})\hat{Q}_2 &= \hat{N} \end{aligned}$$

for adequately defined α_{L1} and α_{N1}. State α_{L1} and α_{N1}.

7. What does inelastic labor supply and the absence of cross-border migration imply for $\hat{L}^{\text{on}}$ and $\hat{N}$?

8. Use inelastic labor supply and the results from 4 and 5 to show that

$$\begin{pmatrix} \hat{Q}_1 \\ \hat{Q}_2 \end{pmatrix} = -\frac{\gamma}{\alpha_{L1} - \alpha_{N1}} \begin{pmatrix} 1 - \alpha_{N1} \\ -\alpha_{N1} \end{pmatrix} \cdot \hat{\beta}.$$

Under the assumptions made in the beginning, what are the signs of $\hat{Q}_1$ and $\hat{Q}_2$?

9. Much of the empirical literature on wage inequality and trade uses wage-bill shares in estimation. Define the onshore wage-bill share of non-offshorable labor in industry i as

$$\theta_{Ni}^{\text{on}} = \frac{sN_i}{wL_i^{\text{on}} + sN_i}.$$

Show that the relative change in the wage-bill share of non-offshorable labor is

$$\hat{\theta}_{Ni}^{\text{on}} = (1 - \theta_{Ni}^{\text{on}})(\hat{s} - \hat{w} + \hat{N}_i - \hat{L}_i^{\text{on}}).$$

10. Suppose factor-price equalization holds. Use the results from 8 and 9 to derive $\hat{\theta}_{N1}^{\text{on}}$ and $\hat{\theta}_{N2}^{\text{on}}$ as functions of θ_{Ni}^{on}, parameters and $\hat{\beta}$. Do the $\hat{\theta}_{Ni}^{\text{on}}$ responses to $\hat{\beta}$ differ in sign? How do their responses to $\hat{\beta}$ differ in magnitude?

2 Helpman, Melitz & Yeaple (AER 2004) and Horizontal FDI

There are two countries, and there is a continuum of firms in each country. In each country lives a measure of L_d consumers, who inelastically supply one unit of labor and own the shares of domestic firms. The L_d representative consumers have identical CES preferences over a continuum of varieties

$$U_d = \left[\sum_{s=1}^{2} \int_{\omega \in \mathbf{\Omega}_{sd}} q_{sd}(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \qquad \text{with } \sigma > 1,$$

where s denotes the source country and d the destination country of a variety shipment.

Each firm produces one variety ω. A firm's production technology is constant returns to scale given the firm's productivity ϕ. Firms draw ϕ from a Pareto distribution $F(\phi) = 1 - (b_s/\phi)^{\theta}$. It will be convenient to call all firms ω with a given productivity level the firms ϕ.

Firms choose to enter their respective home market and any foreign destination. There are two modes of entry into the foreign destination: exports from the respective home market, or horizontal foreign direct investment. There are iceberg transportation costs τ_{sd} between countries for exporting. There is a fixed cost F_D to enter the domestic market, a fixed cost F_X for exporting to the foreign market, and a fixed cost F_I to enter the foreign market through horizontal FDI.

1. Show that demand for a variety $q_{sd}(\omega)$ is

$$q_{sd}(\omega) = \frac{(p_{sd})^{-\sigma}}{(P_d)^{1-\sigma}} \, y_d L_d \qquad \text{with} \quad P_d \equiv \left(\int_{\omega \in \mathbf{\Omega}_{sd}} p_{sd}^{1-\sigma} \, \mathrm{d}\omega \right)^{\frac{1}{1-\sigma}}.$$

2. Show that profit maximization of firm with productivity ϕ implies:

$$p_{sd}(\phi) = \eta \frac{\tau_{sd}\, w_s}{\phi} \qquad \text{with} \quad \eta = \frac{\sigma}{\sigma - 1}.$$

3. Show that a firm's gross operational profits from producing in source country s and shipping to destination market d are

$$\Pi(\tau_{sd}\, w_s) \equiv \left(\frac{P_d\, \phi}{\eta\, \tau_{sd}\, w_s} \right)^{\sigma-1} \frac{y_d L_d}{\sigma}.$$

4. Show that net profits are $\Pi(\tau_{ss}\, w_s, F_D)$ for national non-exporters, $\Pi(\tau_{sd}\, w_s, F_X)$ for exporters, and $\Pi(\tau_{dd}\, w_d, F_I)$ for horizontal multinationals, where

$$\begin{aligned}
\Pi(\tau_{ss}\, w_s, F_D) &= \left(\frac{P_s\, \phi}{\eta\, w_s} \right)^{\sigma-1} \frac{y_s L_s}{\sigma} - F_D \\
\Pi(\tau_{sd}\, w_s, F_X) &= \left(\frac{P_d\, \phi}{\eta\, \tau_{sd}\, w_s} \right)^{\sigma-1} \frac{y_d L_d}{\sigma} - F_X \\
\Pi(\tau_{dd}\, w_d, F_I) &= \left(\frac{P_d\, \phi}{\eta\, w_d} \right)^{\sigma-1} \frac{y_d L_d}{\sigma} - F_I.
\end{aligned}$$

5. Derive the following break-even points for a firm as productivity thresholds: ϕ_D (break-even between shutdown and national non-exporting), ϕ_X (break-even between national non-exporting status and exporting), and ϕ_I (break-even between exporting status and horizontal multinational status). What chain of inequalities do (w_d/w_s) and the fixed costs need to satisfy so that $\phi_D < \phi_X < \phi_I$? What is the chain of inequalities for symmetric countries with identical incomes, wages and price indexes?

6. Is it possible to find conditions so that $\phi_D < \phi_I < \phi_X$? Is it possible to find conditions so that $\phi_X < \phi_D < \phi_I$? How would your answer change for symmetric countries with identical incomes, wages and price indexes?

3 Translog Cost Functions

Burgess (REStat 1974) has extended Christensen, Jorgenson & Lau's (REStat 1973) single-product translog (transcendental logarithmic) cost function to the case of multiple products (such as products shipped to N different destination markets or made in N different source countries):

$$
\begin{aligned}
\ln C_j &= \alpha + \sum_{k=1}^{N} \alpha_k \ln Q_j^k + \sum_{\ell=1}^{N} \tau_\ell \ln w_\ell + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \chi_{k\ell} \ln Q_j^k \ln w_\ell \\
&\quad + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \lambda_{k\ell} \ln Q_j^k \ln Q_j^\ell + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \delta_{k\ell} \ln w_k \ln w_\ell,
\end{aligned} \tag{1}
$$

where the subscript j denotes a firm or an industry, depending on application, Q_j^ℓ is output at or for location ℓ, and w_ℓ is a factor price at or for location ℓ. There are N locations that differentiate the product.

1. Is the cost function (1) separable in individual products for product-level cost functions $c_j^\ell(\cdot)$ so that $C_j(\mathbf{Q}_j; \mathbf{w}) = \sum_\ell c_j^\ell(Q_j^\ell; \mathbf{w})$?

2. For (1) to be homogeneous of degree one in factor prices for any given output vector $\mathbf{Q}_j$, parameters must satisfy certain conditions. What condition does $\sum_{\ell=1}^{N} \tau_\ell$ have to satisfy? What does $\sum_{\ell=1}^{N} \chi_{k\ell}$ have to satisfy for all k? What condition do the sums $\sum_{k=1}^{N} \delta_{k\ell}$, $\sum_{\ell=1}^{N} \delta_{k\ell}$ and $\sum_{k=1}^{N} \sum_{\ell=1}^{N} \delta_{k\ell}$ have to satisfy? By symmetry, we must have $\delta_{k\ell} = \delta_{\ell k}$. How many symmetry restrictions are there for N locations?

Now consider capital K^ℓ a quasi-fixed factor in the short run. Following Brown & Christensen (equation 10.21 of chapter 10 in Berndt & Field 1981: *Modeling and measuring natural resource substitution*), one can augment (1) to a short-run translog multiproduct cost function

$$
\begin{aligned}
\ln C_j^V &= \alpha + \sum_{k=1}^{N} \alpha_k \ln Q_j^k + \sum_{\ell=1}^{N} \tau_\ell \ln w_\ell + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \chi_{k\ell} \ln Q_j^k \ln w_\ell \\
&\quad + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \lambda_{k\ell} \ln Q_j^k \ln Q_j^\ell + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \delta_{k\ell} \ln w_k \ln w_\ell \\
&\quad + \sum_{k=1}^{N} \kappa_k \ln K_j^k + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \mu_{k\ell} \ln K_j^k \ln Q_j^\ell \\
&\quad + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \zeta_{k\ell} \ln K_j^k \ln w_\ell + \frac{1}{2} \sum_{k=1}^{N} \sum_{\ell=1}^{N} \psi_{k\ell} \ln K_j^k \ln K_j^\ell .
\end{aligned} \tag{2}
$$

3. What additional condition on $\sum_{\ell=1}^{N} \zeta_{k\ell}$ is now needed for linear homogeneity of (2) in factor prices?

4. Use Shepard's Lemma to derive firm or industry j's demand for factor ℓ from (2).

5. Show that the cost share of factor ℓ in j's total costs C_j^V is

$$\theta_j^\ell \quad = \quad \tau_\ell + \sum_{k=1}^{N} \chi_{k\ell} \ln Q_j^k + \sum_{k=1}^{N} \zeta_{k\ell} \ln K_j^k + \sum_{k=1}^{N} \delta_{k\ell} \ln w_k.$$

6. The constant-output *cross-price elasticity of substitution* between factors ℓ and k is defined as

$$\varepsilon_{\ell k} \equiv \frac{\partial \ln X_j^\ell}{\partial \ln w_k} = w_k \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \right),$$

where X_j^ℓ is factor demand. Show that the second equality follows from Shepard's Lemma. Derive the cross-price elasticity of substitution ($\ell \neq k$ off diagonal) and the own-price elasticity ($\ell = k$ on diagonal) for the translog cost function C_j^V.

7. The partial *Allen-Uzawa elasticity of substitution* between two factors of production ℓ and k is defined as

$$\sigma_{\ell k}^{AU} \equiv C_j \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \frac{\partial C_j}{\partial w_k} \right) = \frac{\varepsilon_{\ell k}}{\theta_j^k},$$

where $\varepsilon_{\ell k}$ is the (constant-output) cross-price elasticity of factor demand and θ_j^k is the share of the kth input in total cost. Show that the second equality follows from Shepard's Lemma. Derive the Allen-Uzawa elasticity on and off the diagonal for the translog cost function C_j^V.

8. *Morishima* elasticities are superior to Allen-Uzawa elasticities. Blackorby & Russel (AER 1989) show that, among other benefits, Morishima elasticities preserve Hicks's notion that the elasticity of substitution between two factors of production should completely characterize the curvature of an isoquant. Allen-Uzawa elasticities fail in this regard when there are more than two inputs. The *Morishima elasticity of substitution* can be derived as a natural generalization of Hicks's two-factor elasticity and is defined as

$$\sigma_{\ell k}^{M} \equiv w_\ell \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_k} \right) - w_\ell \cdot \frac{\partial^2 C_j}{(\partial w_\ell)^2} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \right) = \varepsilon_{k\ell} - \varepsilon_{\ell\ell},$$

where $\varepsilon_{k\ell}$ is the (constant-output) cross-price elasticity of factor demand. Show that the second equality follows from Shepard's Lemma. Derive the Morishima elasticity on and off the diagonal for the translog cost function C_j^V. [*Note*: Morishima elasticities are inherently asymmetric because Hicks's definition requires that only the price w_ℓ in the ratio w_ℓ/w_k vary.]