Graduate Topics Course FGV–EESP — July/August 2015

International Economics: Micro- and Macroeconomic Connections

Select Problems

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Instructor: Marc-Andreas Muendler

E-mail: muendler@ucsd.edu

1 *July 27*: Microfoundations of the Price Index

For the following four demand systems, find the minimum expenditure P that just allows for one unit of the consumption basket $C = 1$. In this formulation, the micro-foundation of the price index P is the value function of the expenditure minimization problem.

1. For the CES Consumption Index $C \equiv \left(\int_0^1 \alpha(z)^{1/\theta} c(z)^{\frac{\theta-1}{\theta}} \mathrm{d}z \right)^{\frac{\theta}{\theta-1}}$, minimize expenditure $\int_0^1 p(z)c(z)\mathrm{d}z$ for all $\{c(z)\}_0^1$ such that $C \geq 1$, state the first-order conditions, derive Hicksian demands and show that $P = \left(\int_0^1 \alpha(z) p(z)^{1-\theta} \mathrm{d}z \right)^{\frac{1}{1-\theta}}$.

2. For the Log Consumption Index $C = \left\{ \int_0^1 \ln c(z)\mathrm{d}z \right\}$, minimize expenditure $\int_0^1 p(z)c(z)\mathrm{d}z$ for all $\{c(z)\}_{z\in[0,1]}$ such that $C \geq 0$ and show that $P = \exp\left[\int_0^1 \ln p(z)\mathrm{d}z \right]$.

3. For the CES index in Tradables and Nontradables $C \equiv \left[\gamma^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} C_S^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$, minimize expenditure $C_T + p\, C_S$ for $\{C_S, C_T\}$ such that $C \geq 1$ and show that $P = \left[\gamma + (1-\gamma) p^{1-\theta} \right]^{1/(1-\theta)}$.

4. For Cobb-Douglas utility in Home and Foreign goods $C \equiv \frac{(C_h)^\gamma (C_f)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}$, minimize expenditure $p_h\, C_h + p_f\, C_f$ for $\{C_h, C_f\}$ such that $C \geq 1$ and show that $P = (p_h)^\gamma (p_f)^{1-\gamma}$.

5. Compute the limits of the CES Consumption Index $C \equiv [aC_1^\rho + (1-a)C_2^\rho]^{1/\rho}$ as $\rho \to -\infty$ and $\rho \to 0$. (*Hint:* Apply L'Hôpital's rule to the log CES index.) Compute the elasticity of substitution between C_1 and C_2. What is the elasticity of substitution for $\rho \in \{-\infty, 0, 1\}$?

2 *July 28*: Properties of the Pareto Distribution

Consider a Pareto distributed variable $\phi \sim \mathcal{P}(\phi_0, \theta)$, where ϕ_0 is called the location parameter and θ the shape parameter. The Pareto distribution function is $F(Z \leq \phi) = 1 - (\phi_0/\phi)^\theta$.

1. Show that the Pareto density function is $\mu(\phi|\phi_0, \theta) = \theta(\phi_0)^\theta/(\phi)^{\theta+1}$.

2. For any $\phi^* > \phi_0$, show that the conditional distribution function is: $F(\phi|\phi \geq \phi^*) = 1 - (\phi^*/\phi)^\theta$, also a Pareto distribution function.

3. Consider a transformed random variable $A(\phi)^B$ with $A, B > 0$. Show that the transformed variable is Pareto distributed with location parameter $A\,(\phi_0)^B$ and shape parameter θ/B.

4. Show that the mean of a Pareto distributed variable ϕ is $\mathbb{E}\left[\phi|\phi_0, \theta\right] = \theta\phi_0/(\theta - 1)$ if $\theta > 1$.

3 *July 29*: Diverse Firms and Trade

There is a mass of firms in source country s, each firm with an individual productivity ϕ drawn from a national distribution. A firm with ϕ produces output $q = L/\phi$ using labor L. The firms also incurs a fixed entry cost F to operate at all. Exporters incur an additional per-unit transport cost $\tau_d > 1$ (iceberg trade costs so that $\tau_d \geq 1$ goods needed for one good to arrive abroad) as well as a fixed export market entry cost F_X.

A firm ϕ faces a residual demand curve for its individual variety with constant elasticity in every market. At a price $p_d(\phi)$, the firm's demand is

$$q_d(\phi) = \frac{p_d(\phi)^{-\sigma}}{P_d^{-\sigma}} \frac{y_d L_d}{P_d} \qquad \text{where} \quad P_d \equiv \left(\int_\phi p_d(\phi)^{1-\sigma} \mathrm{d}\phi \right)^{\frac{1}{1-\sigma}}$$

and $y_d L_d$ is national income. Conditional on entry, firm ϕ maximizes profits at every destination d

$$\pi_d(\phi) = p_d(\phi) q_d(\phi) - \tau_d w_s \, q_d(\phi)/\phi - w_s F_0 \quad [-\, w_s F_X],$$

where d can be the home market or the foreign destination and w_s is the economy-wide wage.

1. Under the assumptions made, a firm can treat domestic entry and export-market entry as independent decisions. Why?

2. Use a firm's demand function $q_d(\phi) = p_d(\phi)^{-\sigma} y_d L_d / P_d^{1-\sigma}$ in profits $\pi_d(\phi)$ to derive the optimal price of a firm with productivity ϕ.

3. Show that a firm ϕ's revenues are

$$r_d(\phi) = \left(\frac{P_d}{\eta \, \tau_d \, w_s} \right)^{\sigma-1} y_d L_d \; \phi^{\sigma-1}.$$

4. For a fixed cost F_d ($F_d = F_0$ in the home market and $F_d = F_0 + F_X$ abroad), show that profits are

$$\pi_d(\phi) = \frac{r_d(\phi)}{\sigma} - w_s F_d.$$

5. How do prices, revenues and profits change with ϕ?

6. Use 3 and 4 to derive, for a fixed cost F_d ($d = 0, X$), the productivity threshold ϕ_d at which an entrant just breaks even. Does ϕ_d fall or increase in F_d? Why?

7. Sketch in words how the real wage responds when F_d falls.

8. In the model so far, all workers earn the same wage w_s. Outline, in a few sentences, one set of possible additional assumptions that can generate an inter-firm wage differential. In that extended model, how does real wage inequality respond when F_d falls?

4 *July 30*: Diverse Firms and Horizontal FDI

Consider the following version of the Helpman, Melitz & Yeaple model (AER 2004) of horizontal foreign direct investment (FDI). There are two countries, and there is a continuum of firms in each country. In each country lives a measure of L_d consumers, who inelastically supply one unit of labor and own the shares of domestic firms. The L_d representative consumers have identical CES preferences over a continuum of varieties

$$U_d = \left[\sum_{s=1}^{2} \int_{\omega \in \mathbf{\Omega}_{sd}} q_{sd}(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right]^{\frac{\sigma}{\sigma-1}} \qquad \text{with } \sigma > 1,$$

where s denotes the source country and d the destination country of a variety shipment.

Each firm produces one variety ω. A firm's production technology is constant returns to scale given the firm's productivity ϕ. Firms draw ϕ from a Pareto distribution $F(\phi) = 1 - (b_s/\phi)^{\theta}$. It will be convenient to call all firms ω with a given productivity level the firms ϕ.

Firms choose to enter their respective home market and any foreign destination. There are two modes of entry into the foreign destination: exports from the respective home market, or horizontal foreign direct investment. There are iceberg transportation costs τ_{sd} between countries for exporting. There is a fixed cost F_D to enter the domestic market, a fixed cost F_X for exporting to the foreign market, and a fixed cost F_I to enter the foreign market through horizontal FDI.

1. Show that demand for a variety $q_{sd}(\omega)$ is

$$q_{sd}(\omega) = \frac{(p_{sd})^{-\sigma}}{(P_d)^{1-\sigma}} \, y_d L_d \qquad \text{with} \quad P_d \equiv \left(\int_{\omega \in \mathbf{\Omega}_{sd}} p_{sd}^{1-\sigma} \, \mathrm{d}\omega \right)^{\frac{1}{1-\sigma}}.$$

2. Show that profit maximization of firm with productivity ϕ implies:

$$p_{sd}(\phi) = \eta \, \frac{\tau_{sd} \, w_s}{\phi} \qquad \text{with} \quad \eta = \frac{\sigma}{\sigma - 1}.$$

3. Show that a firm's gross operational profits from producing in source country s and shipping to destination market d are

$$\Pi(\tau_{sd}\, w_s) \equiv \left(\frac{P_d\, \phi}{\eta\, \tau_{sd}\, w_s}\right)^{\sigma-1} \frac{y_d L_d}{\sigma}.$$

4. Show that net profits are $\Pi(\tau_{ss}\, w_s, F_D)$ for national non-exporters, $\Pi(\tau_{sd}\, w_s, F_X)$ for exporters, and $\Pi(\tau_{dd}\, w_d, F_I)$ for horizontal multinationals, where

$$\begin{aligned} \Pi(\tau_{ss}\, w_s, F_D) &= \left(\frac{P_s\, \phi}{\eta\, w_s}\right)^{\sigma-1} \frac{y_s L_s}{\sigma} - F_D \\ \Pi(\tau_{sd}\, w_s, F_X) &= \left(\frac{P_d\, \phi}{\eta\, \tau_{sd}\, w_s}\right)^{\sigma-1} \frac{y_d L_d}{\sigma} - F_X \\ \Pi(\tau_{dd}\, w_d, F_I) &= \left(\frac{P_d\, \phi}{\eta\, w_d}\right)^{\sigma-1} \frac{y_d L_d}{\sigma} - F_I. \end{aligned}$$

5. Derive the following break-even points for a firm as productivity thresholds: ϕ_D (break-even between shutdown and national non-exporting), ϕ_X (break-even between national non-exporting status and exporting), and ϕ_I (break-even between exporting status and horizontal multinational status). What chain of inequalities do (w_d/w_s) and the fixed costs need to satisfy so that $\phi_D < \phi_X < \phi_I$? What is the chain of inequalities for symmetric countries with identical incomes, wages and price indexes?

6. Is it possible to find conditions so that $\phi_D < \phi_I < \phi_X$? Is it possible to find conditions so that $\phi_X < \phi_D < \phi_I$? How would your answer change for symmetric countries with identical incomes, wages and price indexes?

5 *August 3*: Translog Demand Systems

Burgess (REStat 1974) has extended Christensen, Jorgenson & Lau's (REStat 1973) single-product translog (transcendental logarithmic) cost function to the case of multiple products (such as products shipped to N different destination markets or made in N different source countries):

$$\begin{aligned} \ln C_j &= \alpha + \sum_{k=1}^{N} \alpha_k \ln Q_j^k + \sum_{\ell=1}^{N} \tau_\ell \ln w_\ell + \sum_{k=1}^{N}\sum_{\ell=1}^{N} \chi_{k\ell} \ln Q_j^k \ln w_\ell \\ &\quad + \frac{1}{2}\sum_{k=1}^{N}\sum_{\ell=1}^{N} \lambda_{k\ell} \ln Q_j^k \ln Q_j^\ell + \frac{1}{2}\sum_{k=1}^{N}\sum_{\ell=1}^{N} \delta_{k\ell} \ln w_k \ln w_\ell, \end{aligned} \qquad (1)$$

where the subscript j denotes a firm or an industry, depending on application, Q_j^ℓ is output at or for location ℓ, and w_ℓ is a factor price at or for location ℓ. There are N locations that differentiate the product.

1. Is the cost function (1) separable in individual products for product-level cost functions $c_j^\ell(\cdot)$ so that $C_j(\mathbf{Q}_j; \mathbf{w}) = \sum_\ell c_j^\ell(Q_j^\ell; \mathbf{w})$?

2. For (1) to be homogeneous of degree one in factor prices for any given output vector $\mathbf{Q}_j$, parameters must satisfy certain conditions. What condition does $\sum_{\ell=1}^N \tau_\ell$ have to satisfy? What does $\sum_{\ell=1}^N \chi_{k\ell}$ have to satisfy for all k? What condition do the sums $\sum_{k=1}^N \delta_{k\ell}$, $\sum_{\ell=1}^N \delta_{k\ell}$ and $\sum_{k=1}^N \sum_{\ell=1}^N \delta_{k\ell}$ have to satisfy? By symmetry, we must have $\delta_{k\ell} = \delta_{\ell k}$. How many symmetry restrictions are there for N locations?

Now consider capital K^ℓ a quasi-fixed factor in the short run. Following Brown & Christensen (equation 10.21 of chapter 10 in Berndt & Field 1981: *Modeling and measuring natural resource substitution*), one can augment (1) to a short-run translog multiproduct cost function

$$\begin{aligned}
\ln C_j^V \;=\;& \alpha + \sum_{k=1}^N \alpha_k \ln Q_j^k + \sum_{\ell=1}^N \tau_\ell \ln w_\ell + \sum_{k=1}^N \sum_{\ell=1}^N \chi_{k\ell} \ln Q_j^k \ln w_\ell \\
&+ \frac{1}{2} \sum_{k=1}^N \sum_{\ell=1}^N \lambda_{k\ell} \ln Q_j^k \ln Q_j^\ell + \frac{1}{2} \sum_{k=1}^N \sum_{\ell=1}^N \delta_{k\ell} \ln w_k \ln w_\ell \\
&+ \sum_{k=1}^N \kappa_k \ln K_j^k + \sum_{k=1}^N \sum_{\ell=1}^N \mu_{k\ell} \ln K_j^k \ln Q_j^\ell \\
&+ \sum_{k=1}^N \sum_{\ell=1}^N \zeta_{k\ell} \ln K_j^k \ln w_\ell + \frac{1}{2} \sum_{k=1}^N \sum_{\ell=1}^N \psi_{k\ell} \ln K_j^k \ln K_j^\ell .
\end{aligned} \tag{2}$$

3. What additional condition on $\sum_{\ell=1}^N \zeta_{k\ell}$ is now needed for linear homogeneity of (2) in factor prices?

4. Use Shepard's Lemma to derive firm or industry j's demand for factor ℓ from (2).

5. Show that the cost share of factor ℓ in j's total costs C_j^V is

$$\theta_j^\ell \;=\; \tau_\ell + \sum_{k=1}^N \chi_{k\ell} \ln Q_j^k + \sum_{k=1}^N \zeta_{k\ell} \ln K_j^k + \sum_{k=1}^N \delta_{k\ell} \ln w_k .$$

6. The constant-output *cross-price elasticity of substitution* between factors ℓ and k is defined as

$$\varepsilon_{\ell k} \equiv \frac{\partial \ln X_j^\ell}{\partial \ln w_k} = w_k \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \Big/ \left(\frac{\partial C_j}{\partial w_\ell} \right),$$

where X_j^ℓ is factor demand. Show that the second equality follows from Shepard's Lemma. Derive the cross-price elasticity of substitution ($\ell \neq k$ off diagonal) and the own-price elasticity ($\ell = k$ on diagonal) for the translog cost function C_j^V.

7. The partial *Allen-Uzawa elasticity of substitution* between two factors of production ℓ and k is defined as
$$\sigma_{\ell k}^{AU} \equiv C_j \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \bigg/ \left(\frac{\partial C_j}{\partial w_\ell} \frac{\partial C_j}{\partial w_k} \right) = \frac{\varepsilon_{\ell k}}{\theta_j^k},$$
where $\varepsilon_{\ell k}$ is the (constant-output) cross-price elasticity of factor demand and θ_j^k is the share of the kth input in total cost. Show that the second equality follows from Shepard's Lemma. Derive the Allen-Uzawa elasticity on and off the diagonal for the translog cost function C_j^V.

8. *Morishima* elasticities are superior to Allen-Uzawa elasticities. Blackorby & Russel (AER 1989) show that, among other benefits, Morishima elasticities preserve Hicks's notion that the elasticity of substitution between two factors of production should completely characterize the curvature of an isoquant. Allen-Uzawa elasticities fail in this regard when there are more than two inputs. The *Morishima elasticity of substitution* can be derived as a natural generalization of Hicks's two-factor elasticity and is defined as
$$\sigma_{\ell k}^{M} \equiv w_\ell \cdot \frac{\partial^2 C_j}{\partial w_\ell \, \partial w_k} \bigg/ \left(\frac{\partial C_j}{\partial w_k} \right) - w_\ell \cdot \frac{\partial^2 C_j}{(\partial w_\ell)^2} \bigg/ \left(\frac{\partial C_j}{\partial w_\ell} \right) = \varepsilon_{k\ell} - \varepsilon_{\ell\ell},$$
where $\varepsilon_{k\ell}$ is the (constant-output) cross-price elasticity of factor demand. Show that the second equality follows from Shepard's Lemma. Derive the Morishima elasticity on and off the diagonal for the translog cost function C_j^V. [*Note*: Morishima elasticities are inherently asymmetric because Hicks's definition requires that only the price w_ℓ in the ratio w_ℓ/w_k vary.]

6 *August 5*: Current Account and Terms of Trade

In a small open economy, the representative individual maximizes lifetime utility
$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(X_s^\gamma M_s^{1-\gamma}\right)^{1-1/\sigma} - 1}{1 - 1/\sigma},$$
where X is consumption of an exported good and M consumption of an imported good. The country completely specializes in production of the export good. The endowment of this good is constant at Y. The representative individual faces the fixed world interest rate $r = (1-\beta)/\beta$ in terms of the real consumption index $C = X^\gamma M^{1-\gamma}$ (so a loan of 1 real consumption unit today returns $1 + r$ real consumption units tomorrow). There is no investment or government spending.

1. Let p bet the price of the export goods in terms of the import good. So, a rise in p is an improvement in the terms of trade. Show that the welfare-based price index P in terms of imports is
$$P = p^\gamma / [\gamma^\gamma (1-\gamma)^{1-\gamma}].$$

2. Show that the home country's current account identity is
$$B_{t+1} - B_t = rB_t + \frac{p_t(Y - X_t)}{P_t} - \frac{M_t}{P_t}.$$
What is the corresponding intertemporal budget constraint for the representative consumer?

3. Show either that utility maximization (Marshallian demands for X_t and M_t) or that expenditure minimization (Hicksian demands for X_t and M_t) implies $P_t C_t = p_t X_t + M_t$.

4. Derive the first-order conditions of the representative agents's intertemporal consumption problem. What are the optimal paths for C_t, X_t and M_t? For this purpose, express C_t in terms of the representative agent's present net wealth using the intertemporal budget constraint and note that the representative agent's present net wealth is time dependent if relative prices vary over time.

5. Suppose initial expectations are that p remains constant over time. There is an unexpected *temporary* fall in the terms of trade from p to $p' < p$. What is the effect on the current account $CA_t = B_{t+1} - B_t$ from part 2? What if p *permanently* drops to p'?

6. Now suppose foreign net wealth B is indexed to the import good M rather than to real consumption. Accordingly, let r denote the own-rate of interest on the import-denominated bond but assume again that $r = (1-\beta)/\beta$. How does a *temporary* drop in the terms of trade from p to $p' < p$ affect the current account now? How do you explain differences, if any, to part 5? [*Hint*: You might find it instructive to consider the effect of a one-percent change in p_t on p_t/P_t and the current account balance under either denomination.]

7 *August 6*: Stochastic Current Account Model

There are infinitely many periods. A country's representative household has the linear-quadratic period utility function

$$u(C) = C - \frac{a_0}{2}C^2$$

with $a_0 \in (0, \infty)$ and maximizes expected lifetime utility

$$U_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right]$$

subject to

$$CA_s = B_{s+1} - B_s = rB_s + \tilde{Z}_s - C_s \quad \forall s \geq t$$

where $R \equiv 1/(1+r) = \beta$ and $\tilde{Z}_s (\equiv \tilde{Y}_s - \tilde{G}_s - \tilde{I}_s)$ is *random* net output.

1. Derive the stochastic Euler equations and show that C_t satisfies

$$C_t = rR\left((1+r)B_t + \sum_{s=t}^{\infty} R^{s-t}\mathbb{E}_t[\tilde{Z}_s]\right) = rB_t + \mathbb{E}_t[\tilde{Z}_s]$$

2. Show that $CA_t \equiv B_{t+1} - B_t = \tilde{Z}_t - \mathbb{E}_t[\hat{\tilde{Z}}_t]$, where the hat denotes the permanent level of the variable. The permanent level $\hat{X}$ of a random variable $\tilde{X}$ is defined as $\sum_{s=t}^{\infty} R^{s-t}\hat{\tilde{X}} \equiv \mathbb{E}_t\left[\sum_{s=t}^{\infty} R^{s-t}\tilde{X}_s\right]$.

3. Define $\Delta \tilde{Z}_s \equiv \tilde{Z}_{s+1} - \tilde{Z}_s$ and suppose that $\lim_{T\to\infty} R^T \mathbb{E}_t \left[\tilde{Z}_{t+T}\right] = 0$. Show that the current account follows a martingale. In other words, show that current account innovations (unexpected changes to the current account) are unrelated to any past realizations of state variables.

   Hint: Show that the current account can be rewritten as

   $$CA_t = -R \sum_{s=t}^{\infty} R^{s-t} \mathbb{E}_t \left[\Delta \tilde{Z}_s\right]$$

   for $\lim_{T\to\infty} R^T \mathbb{E}_t \left[\tilde{Z}_{t+T}\right] = 0$ and find $CA_t - \mathbb{E}_{t-1}\left[CA_t\right]$.

4. Does consumption follow a martingale?