Economics 246 — Fall 2009

International Macroeconomics

Problem Set 1

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1 Exponential Period Utility

There are two periods. A country's representative household has the exponential period utility function

$$u(C) = -\gamma \exp(-C/\gamma)$$

with $\gamma \in (0, \infty)$ and maximizes lifetime utility $U_1 = u(C_1) + \beta u(C_2)$ subject to

$$C_1 + RC_2 = Y_1 + RY_2 \equiv W,$$

where $R \equiv 1/(1+r)$ is the price of tomorrow's consumption in terms of today's consumption and W is initial wealth. The value of W depends on R.

1. Derive the Euler equation and solve it for C_2 as a function of C_1, R and β.

2. What is the optimal level of C_1 considering W, R and β as given?

3. Differentiate this consumption function of C_1 with respect to R (differentiate W with respect to R too) and show that

$$\frac{dC_1}{dR} = -\frac{C_1}{1+R} + \frac{Y_2}{1+R} + \frac{\gamma}{1+R}\big(1 - \ln(\beta/R)\big)$$

4. Derive the intertemporal elasticity of substitution of the exponential period utility $(-u'(C)/Cu''(C))$.

5. Use this result to show that the derivative dC_1/dR in part 3 can be expressed as

$$\frac{dC_1}{dR} = \frac{\sigma(C_2)\, C_2}{1+R} - \frac{C_2}{1+R} + \frac{Y_2}{1+R}.$$

   Interpret the three additive terms in this derivative.

2 Stochastic Current Account Model

There are infinitely many periods. A country's representative household has the linear-quadratic period utility function

$$u(C) = C - \frac{a_0}{2}C^2$$

with $a_0 \in (0, \infty)$ and maximizes expected lifetime utility

$$U_t = \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u\left(C_s\right) \right]$$

subject to

$$CA_s = B_{s+1} - B_s = rB_s + \tilde{Z}_s - C_s \quad \forall s \geq t$$

where $R \equiv 1/(1+r) = \beta$ and $\tilde{Z}_s (\equiv \tilde{Y}_s - \tilde{G}_s - \tilde{I}_s)$ is *random* net output.

1. Derive the stochastic Euler equations and show that C_t satisfies

$$C_t = rR \left((1+r)B_t + \sum_{s=t}^{\infty} R^{s-t} \mathbb{E}_t[\tilde{Z}_s] \right) = rB_t + \mathbb{E}_t[\hat{\tilde{Z}}_s]$$

2. Show that $CA_t \equiv B_{t+1} - B_t = \tilde{Z}_t - \mathbb{E}_t[\hat{\tilde{Z}}_t]$, where the hat denotes the permanent level of the variable. The permanent level $\hat{X}$ of a random variable $\tilde{X}$ is defined as $\sum_{s=t}^{\infty} R^{s-t} \hat{\tilde{X}} \equiv \mathbb{E}_t \left[\sum_{s=t}^{\infty} R^{s-t} \tilde{X}_s \right]$.

3. Define $\Delta\tilde{Z}_s \equiv \tilde{Z}_{s+1} - \tilde{Z}_s$ and suppose that $\lim_{T\to\infty} R^T \mathbb{E}_t \left[\tilde{Z}_{t+T} \right] = 0$. Show that the current account follows a martingale. In other words, show that current account innovations (unexpected changes to the current account) are unrelated to any past realizations of state variables.

   Hint: Show that the current account can be rewritten as

   $$CA_t = -R \sum_{s=t}^{\infty} R^{s-t} \mathbb{E}_t \left[\Delta\tilde{Z}_s \right]$$

   for $\lim_{T\to\infty} R^T \mathbb{E}_t \left[\tilde{Z}_{t+T} \right] = 0$ and find $CA_t - \mathbb{E}_{t-1}\left[CA_t\right]$.

4. How is this finding related to Hall's (1978) result that consumption follows a martingale?

3 Current Account and Terms of Trade

In a small open economy, the representative individual maximizes lifetime utility

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \frac{\left(X_s^{\gamma} M_s^{1-\gamma}\right)^{1-1/\sigma} - 1}{1 - 1/\sigma},$$

where X is consumption of an exported good and M consumption of an imported good. The country completely specializes in production of the export good. The endowment of this good is constant at Y. The representative individual faces the fixed world interest rate $r = (1-\beta)/\beta$ in terms of the real consumption index $C = X^{\gamma} M^{1-\gamma}$ (so a loan of 1 real consumption unit today returns $1 + r$ real consumption units tomorrow). There is no investment or government spending.

1. Let p bet the price of the export goods in terms of the import good. So, a rise in p is an improvement in the terms of trade. Show that the welfare-based price index P in terms of imports is

$$P = p^{\gamma}/[\gamma^{\gamma}(1-\gamma)^{1-\gamma}].$$

2. Show that the home country's current account identity is

$$B_{t+1} - B_t = rB_t + \frac{p_t(Y - X_t)}{P_t} - \frac{M_t}{P_t}.$$

   What is the corresponding intertemporal budget constraint for the representative consumer?

3. Show that utility maximization (Marshallian demands for X_t and M_t) and expenditure minimization (Hicksian demands for X_t and M_t) both imply that $P_t C_t = p_t X_t + M_t$.

4. Derive the first-order conditions of the representative agents's intertemporal consumption problem. What are the optimal paths for C_t, X_t and M_t? For this purpose, express C_t in terms of the representative agent's present net wealth using the intertemporal budget constraint and note that the representative agent's present net wealth is time dependent if relative prices vary over time.

5. Suppose initial expectations are that p remains constant over time. There is an unexpected *temporary* fall in the terms of trade from p to $p' < p$. What is the effect on the current account $CA_t = B_{t+1} - B_t$ from part 2? What if p *permanently* drops to p'?

6. Now suppose foreign net wealth B is indexed to the import good M rather than to real consumption. Accordingly, let r denote the own-rate of interest on the import-denominated bond but assume again that $r = (1-\beta)/\beta$. How does a *temporary* drop in the terms of trade from p to $p' < p$ affect the current account now? How do you explain differences, if any, to part 5? [*Hint*: You might find it instructive to consider the effect of a one-percent change in p_t on p_t/P_t and the current account balance under either denomination.]

4 Heterogeneous Firms and the Terms of Trade with an Initially Balanced Current Account (a variant of Ghironi & Melitz, *QJE* 2005)

This question asks you to revisit the Harberger-Laursen-Metzler effect in the context of firm heterogeneity and endogenous entry in a small-open economy.

Consider a representative household who maximizes expected lifetime utility: $\sum_{s=t}^{\infty} \beta^{s-t} \mathbb{E}_t [u(C_s)]$, where period utility $u(C) = (C^{1-1/\sigma} - 1)/(1 - 1/\sigma)$ has a constant intertemporal elasticity of substitution $\sigma > 0$ and $\beta \in (0, 1)$ is the subjective discount factor. The consumption basket contains a continuum of goods

$$C_t = \left(\int_{\omega \in \Omega_t} c_t(\omega)^{1-\frac{1}{\theta}} \, d\omega \right)^{\frac{1}{1-\frac{1}{\theta}}}, \quad \theta > 1,$$

where θ is the elasticity of substitution across goods. At any time t, only a subset of goods $\Omega_t \subset \Omega$ is available. The household inelastically supplies L units of labor.

Firms produce output (their individual variety) from labor with productivity $Z_t \, z$, where Z_t is an economy-wide productivity parameter and common to all domestic firms, whereas z is the firm's individual productivity. So, the unit cost of production at time t is $w_t/(Z_t \, z)$. There is endogenous firm entry and exogenous firm exit. If an entrant chooses to start production, the firm incurs a one-time sunk cost f_E in terms of labor, resulting in the expense $w_t f_E/Z_t$. Firms shut down with an exogenous probability $\delta \in (0, 1)$. If a domestic firm chooses to export in a given period, it incurs a per-period fixed cost of exporting f_X in terms of labor, resulting in the (repeated) per-period expense $w_t f_X/Z_t$, and its product ships at a proportional transportation cost $\tau \geq 1$ ($\tau \equiv 1/(1-\kappa)$ for iceberg transportation costs κ). Every firm is a monopolist in the market for its variety.

A firm's individual productivity z is drawn from a Pareto distribution with minimum productivity $\underline{z}$ and shape parameter k so that $G(z) = 1 - (\underline{z}/z)^k$. Assume that $k > \theta - 1 > 0$.

Define the real exchange rate as $Q_t \equiv P_t^*/P_t$ (setting the nominal exchange rate to unity), where P_t and P_t^* are the welfare-based home and foreign price indices to be derived below. There is a time-invariant worldwide interest rate $r_t = r$ such that $\beta = R \equiv 1/(1 + r)$.

1. Use expenditure minimization to show that the welfare-based price index at time t is
$$P_t = \left(\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} \, d\omega \right)^{\frac{1}{1-\theta}}.$$
2. Show that demand for variety ω is
$$c_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t.$$

3. Write down the profit maximization problem for a firm with productivity z, derive monopoly price $p_{D,t}(z)$ as a function of $Z_t\, z$ and show that the *real* profit flow (dividend) from domestic sales in period t is

$$d_{D,t}(z) = \frac{1}{\theta} \left(\frac{p_{D,t}(z)}{P_t} \right)^{1-\theta} C_t.$$

Show that real dividends are constant fraction $1/\theta$ of real sales. For $\theta > 1$, do more productive firms set higher or lower prices? For $\theta > 1$, do more productive firms have higher or lower profits? [*Hint*: Real variables are defined as the nominal variables divided by the home economy's welfare-based price index.]

4. Using the results from 3, show that the inverse monopoly price $1/p_{D,t}(z)$ and the dividend $d_{D,t}(z)$ from domestic sales are Pareto distributed given that z is Pareto distributed with minimum productivity $\underline{z}$ and shape parameter k. What are the minimum inverse price and shape parameter of the inverse price distribution, what are the minimum dividend and shape parameter of the dividend distribution? [*Hint*: Show that, for a Pareto distributed random variable z with shape parameter k and minimum $\underline{z}$, the transformed random variable $x = A\,(z)^B$ is Pareto distributed with shape k/B and minimum $A\,(\underline{z})^B$. You may find a change in variables useful for the proof.]

5. The destination-market price of an export from home is $p_{X,t}(z) = \tau p_{D,t}(z)$. Why? Using the results from 3, derive the cutoff value $\underline{z}_{X,t}$ at which a firm with productivity $z = \underline{z}_{X,t}$ is indifferent between entering the export market and remaining a domestic seller. How does $\underline{z}_{X,t}$ depend on τ, f_X and Q_t? [*Hint*: For home firms, real foreign variables are also defined as the nominal variables divided by the home economy's welfare-based price index.]

6. Show that the productivity distribution for exporters is Pareto with minimum productivity $\underline{z}_{X,t}$ and shape parameter k. Show that the share of exporters among domestic firms $N_{X,t}/N_{D,t}$ equals the cumulative distribution function evaluated at the cutoff value $\underline{z}_{X,t}$, or $N_{X,t}/N_{D,t} = 1 - G(\underline{z}_{X,t}) = (\underline{z}/\underline{z}_{X,t})^k$.

7. Derive the mean inverse price $\tilde{p}_{D,t}$ and the mean dividend $\tilde{d}_{D,t}$ for all firms with domestic sales. Show that $k > \theta - 1 > 0$ is a necessary condition for the mean dividend (and therefore mean and total sales) to exist. Derive the mean inverse price $\tilde{p}_{X,t}$ and the mean dividend $\tilde{d}_{X,t}$ for all home exporters. Using $N_{X,t}/N_{D,t}$ from 6, derive the total dividend of the mean home firm, including exporters. [*Hint*: The mean of a Pareto distributed random variable z with shape parameter k and minimum $\underline{z}$ is $k\underline{z}/(k-1)$.]

8. Denote with $N_{D,t}$ the mass of firms that continue in operation since $t-1$ and with $N_{E,t}$ the mass of firms that newly enter. Then $N_{D,t+1} = (1 -$

$\delta)(N_{D,t}+N_{E,t})$. Explain the representative household's budget constraint

$$B_{t+1}+\tilde{v}_t(N_{D,t}+N_{E,t})x_{t+1}+C_t=(1+r)B_t+(\tilde{d}_t+\tilde{v}_t)N_{D,t}x_t+w_tL,$$

where $\tilde{d}_t \equiv \tilde{d}_{D,t}+\tilde{d}_{X,t}$ is the dividend of the mean firm and $\tilde{v}_t$ is the mean firm's value. x_t denotes the household's beginning of period holdings of domestic firms.

9. The household maximizes expected lifetime utility given the budget constraint in 8. Derive the Euler equations for B_{t+1} and x_{t+1}. Use forward-iteration of the Euler equation for x_{t+1} to show that the mean firm's ex-dividend value is

$$\tilde{v}_t=\sum_{s=t+1}^{\infty}\left(\frac{1-\delta}{1+r}\right)^{s-t}\mathbb{E}_t\left[\left(\frac{C_s}{C_t}\right)^{-\frac{1}{\sigma}}\tilde{d}_s\right].$$

Argue that firm entry occurs until $\tilde{v}_t=w_tf_E/Z_t$.

10. *Open question.* Suppose that $B_t=0$ and $x_t=1$ and define the terms of trade as

$$ToT_t \equiv \left(\frac{N_{X,t}(\tilde{p}_{X,t})^{1-\theta}}{N^*_{X,t}(\tilde{p}^*_{X,t})^{1-\theta}}\right)^{\frac{1}{1-\theta}},$$

where $N^*_{X,t}$ is the mass of foreign exporters and $\tilde{p}^*_{X,t}$ the price of the mean foreign exporter's shipments to home (mean home imports price). Consider an unanticipated *permanent* terms-of-trade deterioration because of a permanent productivity drop abroad (a permanent reduction in Z^*_s for $s\geq t$). What is the equilibrium path of B_s for $s\geq t+1$? How does it depend on the elasticity of intertemporal substitution σ? [*Hint*: Derive a current account relationship CA_s as a function of the world economy's fundamentals and use the fact that $B_s=\sum_{\varsigma=1}^{s}CA_\varsigma$ because $B_t=0$; then derive the response of firm entry to the permanent productivity shock abroad to find the fundamental current account response to the shock through entry.]