Economics 232c — Spring 2003

International Macroeconomics

Problem Set 3

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Due:	**Thu, June 5, 2003**
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1 Trending Fundamentals in a Target Zone Model (Froot and Obstfeld 1991)

Suppose the exchange rate follows the fundamental equation

$$e_t = k_t + \frac{\eta}{h}\mathbb{E}_t\left[\mathrm{d}e_{t+h}\right],$$

where k_t is the fundamental value. As opposed to the model in class, suppose the change in the fundamental value follows the trending process

$$\mathrm{d}k_{t+h} = \mu h + h^{1/2}\nu \mathrm{d}z_{t+h},$$

where $\mathrm{d}z_{t+h}$ is drawn from a mean-zero i.i.d. normal distribution with unit variance. Under a free float $e_t = k_t$. Suppose the Home and Foreign central bank announce a target zone for the exchange rate: $e_t \in (\underline{e}, \overline{e})$.

This problem asks you to show step by step that the equilibrium exchange rate process must satisfy

$$e = G(k) + \eta\mu G'(k) + \frac{\eta\nu^2}{2}G''(k), \tag{1-1}$$

which has a solution of the form

$$G(k) = k + \eta\mu + +\eta\mu G'(k) + \frac{\eta\nu^2}{2}G''(k), \tag{1-2}$$

1. Suppose that $e = G(k)$ is a solution, where $G(\cdot)$ is twice continuously differentiable. Show that

$$G(k_t) = k_t + \frac{\eta}{h}\mathbb{E}_t\left[G(k_{t+h}) - G(k_t)\right]$$

2. Use a second-order Taylor approximation of $\mathbb{E}_t\left[G(k_{t+h}) - G(k_t)\right]$ at $k_{t+1} = k_t$ to show that

$$\mathbb{E}_t\left[\mathrm{d}G'(k_{t+h})\right] = \mu h G'(k_t) + \frac{hv^2}{2}G''(k_t).$$

[*Hint*: Show that $\mathbb{E}_t\left[G'(k_t)\mathrm{d}k_{t+h}\right] = \mu h G'(k_t)$.]

3. Using these results, show that

$$G(k) = k + \eta\mu + +\eta\mu G'(k) + \frac{\eta\nu^2}{2} G''(k).$$

is the solution.

4. A general solution to a second-order differential equation of this form is $G(k) = k + \alpha + b_1 \exp(\lambda_1 k) + b_2 \exp(\lambda_2 k)$ for some constants b_1, b_2. Show that the previous results imply

$$G(k) = k + \eta\mu + +\eta\mu G'(k) + \frac{\eta\nu^2}{2} G''(k).$$

What quadratic equation determines the values λ_1, λ_2.

5. A particular solution to the general solution (1-2) can be found by imposing restrictions that e must satisfy at the limits of the band. Describe how the particular solution can be derived. [You do not need to derive it.]

2 Open-economy Bank Runs

The Diamond-Dybvig model of bank runs is extended to an open-economy setting, in which international borrowing and lending at a gross interest rate 1 is possible (net real interest rate 0). There are three periods and domestic agents have measure 1. Agents do not discount future consumption in their decision but face uncertainty as to what their preferences will be in the intermediate period 1. Patient agents do not derive any utility from consumption after the intermediate period 1, whereas impatient agents are indifferent between consumption in period 1 or 2.

1. Assume there are no banks, and no international borrowing or lending. In period 0, domestic agents invest their endowment z in the domestic technology, which yields a gross return r after one period and a gross return R after two periods. In period 1, each agent learns if he or she is "impatient", which occurs with probability π, or "patient," which happens with probability $1 - \pi$. Impatient agents liquidate their investments immediately for a gross return of $r < 1$ (a net return $r - 1 < 0$) and consume. Patient types let their investments mature until period 2 and consume zR units then, where $R > 1$. An agent derives utility $u(c)$ from consumption of c at any period and there is no discounting. What is the expected utility of a representative agent? What is expected output in periods 1 and 2?

2. Introduce a perfectly competitive banking sector, which earns zero profits in equilibrium. Banks have the ability to borrow and lend abroad at the net interest rate of 0 but their debt to foreign lenders can never exceed the limit $\overline{f}$. Assume that nonbank domestic residents have no direct access to the international capital market, only through banks. In period 0,

domestic agents deposit their endowments in banks. Banks can borrow and lend abroad, and invest in domestic technology. Suppose the banks simply lend πz abroad in period 0, drawing on these funds to pay off impatient depositors in period 1. Show that the expected utility of a representative agent is higher than in part 1. Why?

3. Banks can do better than the arrangement of part 2. Let x denote the (period 1) consumption of an impatient type and y the (period 2) consumption of a patient type. x^* and y^* denote first-best consumption levels. Suppose banks guarantee each impatient depositor x^* in period 1 and each patient depositor y^* in period 2. To do this, they borrow (or lend) the amount $\overline{f} - \pi x^*$ in period 0 and invest $k^* = z + \overline{f} - \pi x^*$ in the domestic technology. They draw on foreign balances to disburse πx^* in period 1, thereby paying off impatient depositors. In period 2 they repay any net borrowing from periods 0 and 1 and pay patient depositors a total of $(1-\pi)y^*$. Show that, under this banking arrangement, the economy faces the intertemporal budget constraint

$$R\pi x + (1-\pi)y = \overline{f}(R-1) + zR \equiv Rw.$$

4. Consider iso-elastic utility $u(c) = c^{1-\rho} - 1/1 - \rho$. By maximizing expected utility in period 0, show that a representative domestic agent consumes the first-best levels x^* and y^*

$$x^* = \frac{\theta w}{\pi} \qquad \text{and} \qquad y^* = \frac{(1-\theta)Rw}{1-\pi},$$

where

$$\theta = \frac{\pi R^{\frac{\rho-1}{\rho}}}{\pi R^{\frac{\rho-1}{\rho}} + (1-\pi)}.$$

5. Verify that $y^* \geq x^*$ is always satisfies so that patient depositors have no incentive to pretend they are impatient.

6. Show that, by the beginning of period 2, banks will always have borrowed up to their foreign credit limit $\overline{f}$.

7. Now consider bank runs in period 1. Let l be the amount of the investment k^* that banks must liquidate to pay off depositors who withdraw their funds in period 1. Suppose that banks can pre-commit to suspending domestic convertibility of their deposits once period 1 withdrawals force them to liquidate sufficient amounts of their investment that $R(k^* - l) = \overline{f}$. So, foreign creditors can be certain of being repaid in full, even if there is a run by domestic depositors and the banks close their doors to them. Let $\hat{l}$ satisfy $R(k^* - \hat{l}) = \overline{f}$. Show that a run on the first-best banking arrangement is possible if

$$x^* > \pi x^* + r\hat{l}.$$

Here, x^* represents the bank's total potential short-term obligations, which it would be liable to satisfy if all investors claimed to be impatient. Calculate x^* and k^* and show that a sufficient condition for a run to be possible is that $\rho \geq 1$.

3 Exchange Rate Overshooting

Suppose money demand takes the Cagan-like form

$$m_t^d - p_t = \phi y_t^d - \eta\, i_{t+1} \qquad \text{with } \phi, \eta > 0$$

where m_t^d denotes the natural logarithm of nominal money demand, p_t the log price level, y_t^d log aggregate demand and i_{t+1} the nominal interest rate ($\ln(1+i_{t+1})$). Money supply is constant $m_t^s = \overline{m}$.

Suppose the uncovered interest parity condition holds and $i_{t+1} = i_{t+1}^* + e_{t+1} - e_t$, where e_t is the log nominal exchange rate. By definition, the real exchange rate is $q_t = e_t + p_t^* - p_t$.

Suppose aggregate demand increases when the real exchange rate depreciates so that

$$y_t^d = \delta q_t \qquad \text{with } \delta \in (0, \tfrac{1}{\phi})$$

The full-employment level of aggregate supply is

$$y_t^s = \bar{y}.$$

Finally, suppose in Keynesian style, that prices are not immediately set to the expected equilibrium level, but adjusted slowly. In particular, prices obey the response function

$$p_{t+1} - p_t = \pi(y_t^d - y_t^s).$$

Standardize all foreign variables to constants $p_t^* = i_{t+1}^* = 0$, and suppose that money markets clear instantaneously: $m_t^d = m_t^s = \bar{m}$.

1. Show that, under these assumption, a Dornbusch model can be built from three equations:

$$\bar{m} - p_t = \phi y_t^d - \eta(e_{t+1} - e_t), \tag{3-1}$$

$$y_t^d = \delta(e_t - p_t) \qquad \delta \in (0, \tfrac{1}{\phi}), \tag{3-2}$$

$$p_{t+1} - p_t = \pi(y_t^d - \bar{y}). \tag{3-3}$$

2. Find the steady-state values of the exchange rate and the price level ($e_{t+1} = e_t = \bar{e}$, $p_{t+1} = p_t = \bar{p}$).

3. Express both $(e_{t+1} - e_t)$ and $(p_{t+1} - p_t)$ as functions of e_t, p_t and exogenous variables. Find the two functional relationships between p_t and e_t that satisfy $e_{t+1} - e_t = 0$ and $p_{t+1} - p_t = 0$. Draw them in a phase diagram with p_t on the y-axis and e_t on the x-axis. Complete the phase diagram

indicating the motion of the system (using the conditions for $e_{t+1} - e_t \geq 0$ and $p_{t+1} - p_t \geq 0$).

Finally, add a line to the diagram that obeys a 'no-arbitrage condition' as mandated by UIP: $p_t - \bar{p} = -\hat{\theta}\,(e_t - \bar{e})$ for some $\hat{\theta} > 0$. (The $\hat{\theta}$ is not quite the same as in the original Dornbusch model since there is no uncertainty in the present setup.)

4. Is the steady-state stable? If not, what is the unique stable ("saddle") path given a steady-state of $\bar{p}$ and $\bar{e}$?

5. Suppose all variables except for p_t immediately respond to a monetary shock.

   What is the new steady-state? Draw a 'no-arbitrage' line through it.

   What happens to e_t right after an unanticipated reduction in the monetary base from $\bar{m}$ to $\bar{m}' < \bar{m}$? How do e_{t+s} and p_{t+s} evolve over time?

6. Now consider the dynamics of the model around its steady-state. Using (3-2) and the steady-states of $\bar{e}$ and $\bar{p}$ that you found in part 1, express $y_t^d - \bar{y}$ as a function of $e_t - \bar{e}$ and $p_t - \bar{p}$.

7. Using (3-1) along with the results in parts 1 and 6, express $e_{t+1} - \bar{e}$ as a function of $\frac{1}{\eta}(e_t - \bar{e})$ and $\frac{1}{\eta}(p_t - \bar{p})$.

8. Using (3-2) and (3-3) along with the results in part 1, express $p_{t+1} - \bar{p}$ as a weighted sum of $e_t - \bar{e}$ and $p_t - \bar{p}$.

From now on, assume that $\pi = \frac{1}{\eta}$ and $\phi = 3$ for simplicity.

9. Write your findings from 7 and 8 into a system of two difference equations that takes the form

$$\begin{pmatrix} e_{t+1} - \bar{e} \\ p_{t+1} - \bar{p} \end{pmatrix} = \mathbf{A} \cdot \begin{pmatrix} e_t - \bar{e} \\ p_t - \bar{p} \end{pmatrix}. \tag{3-4}$$

   Find the eigenvalues and eigenvectors of the system.

   [*Hint*: An intermediate result is $\mathrm{tr}(\mathbf{A}) = \frac{2(\delta+\eta)}{\eta}$ and $\det(\mathbf{A}) = 1 - \frac{(1-2\eta)\delta}{\eta^2}$.]

10. Is the system stable? That is, do the exchange rate and price levels converge to the steady-state?

    If not, set the coefficient of the unstable root to zero. Then, using the simplified system in part 9 and the results from 6, express p_{t+1} as a function of e_{t+1} and exogenous variables. Draw this function into the phase diagram from part 5 so that it passes through the steady-state. (You can safely pretend that $\bar{y} \neq 0$. Also, remember that $\delta < \frac{1}{\phi}$.)

    What did you just find? What is the intuition for the fact that the economy obeys this relationship?

4 Small-country Redux

The small-country case of a standard open-economy redux model can be derived without solving the intricacies of the full two-country model.

Assume that the small country consumes only a single imported good and a single exported good, over which it has some monopoly power. In particular, the demand curve that the small country faces is

$$y^d = p^{-\theta} C^W,$$

where p is the relative world price of the domestic good and C^W is the exogenous level of world demand. This price p has a negligible (no) effect on the foreign commodity price index P^*.

The small country faces an exogenous own-rate of interest r on the imported good. The small country has no effect on world prices or world aggregate variables.

Assume that absolute Purchasing Power Parity $P = EP^*$ and the Fisher Parity $1 + i_{t+1} = (1 + r)\frac{P_{t+1}}{P_t}$ hold.

1. Let p_y be the home-currency price of the single exported good and P the home-currency price of the imported good. Verify that world-demand can then be rewritten as
$$y^d = \left(\frac{p_y}{P}\right)^{-\theta} C^W.$$

2. Suppose the representative agent maximizes
$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\ln C_s + \chi \ln \frac{M_s}{P_s} - \frac{\kappa}{2} y_s^2\right],$$
where C is consumption of the single imported good. The period budget constraint is
$$P_t B_{t+1} + M_t = (1+r) P_t B_t + M_{t-1} + p_{y,t} y_t - P_t C_t - P_t \tau.$$
Assume that $(1+r)\beta = 1$, and suppose the steady-state levels of the following variables satisfy $\overline{p}_y = \overline{EP}^*$, $\overline{C} = \overline{C}^W$ so that $\overline{p}_y = \overline{P}$. Standardize log variables to be $c^w = p^* = 0$ so that $e_t = p_t$. Derive the agent's first-order conditions, log-linearize around the steady state, and show that
$$\begin{aligned} c_{t+1} &= c_t && \text{(4-1)} \\ m_t - e_t &= c_t - \tfrac{1}{r}(e_{t+1} - e_t) && \text{(4-2)} \\ (\theta + 1) y_t &= -\theta c_t. && \text{(4-3)} \end{aligned}$$

3. Show that $y_t = \theta(e_t - p_{y,t})$ under the assumptions made so that
$$b_{t+1} = (1+r) b_t + (\theta - 1)(e_t - p_{y,t}) - c_t.$$

4. Assume that the small-country-currency price $p_{y,t}$ of the export good is set one period in advance, and reverts to its flexible-price level after a single period absent new shocks. Show that the equilibrium exchange rate must satisfy

$$e = \frac{r(1+\theta) + 2\theta}{\theta r(1+\theta) + 2\theta} m.$$

[*Hint*: Show that $e = \overline{e}$, $\overline{b} = (\theta - 1)e - c$ and $e = \frac{r(1+\theta)+2\theta}{r(\theta^2-1)} c$.]