Risk Neutral Investors Do Not Acquire Information*

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Abstract

Give a risk neutral investor the choice to acquire a costly signal prior to asset market equilibrium. She refuses to pay for the signal under general conditions. The reason is that a risk neutral investor is indifferent between a risky asset or a safe bond in optimum and expects the same return to her portfolio *ex ante*, whether or not she acquires information. Risk neutrality thus implies the absence of costly information from asset price in competitive asset markets.

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Risk neutral investors populate financial market models and are widely assumed to be informed. Benchmark models such as Kyle (1985), Glosten and Milgrom (1985) and Back and Baruch (2004), for instance, consider risk neutral and informed investors. Apart from early financial-information models with risk averse investors by Grossman and Stiglitz (1980) or Verrecchia (1982), risk neutrality is the assumption of choice in models such as Holden and Subrahmanyam (1992), Foster and Viswanathan (1993), Rochet and Vila (1994), Back, Cao and Willard (2000), or Barlevy and Veronesi (2000). Leland (1992) and Repullo (1999) consider insiders who are risk neutral. Risk neutral investors, however, choose not to become informed in competitive asset markets.

Consider a canonical model of portfolio and consumption choice with risk neutral investors. Make one change: Add a choice of information acquisition *ex ante*. The risk neutral investor refuses to pay for information if she anticipates asset markets to clear. Put differently, risk neutrality implies the absence of costly information from asset price. The intuitive reason is that a risk neutral investor is indifferent whether she holds a risky asset or a safe bond in her portfolio. When acquiring information before making her portfolio choice, a risk neutral investor rationally anticipates that asset price will equal expected returns in competitive financial-market equilibrium. Hence, she expects her actions upon signal realizations to yield the same return *ex ante* as uninformed actions do, which makes her indifferent to signals. She will accept signals for free, but refuse to incur any cost of information acquisition.

The no-acquisition result implies that findings on optimal experimentation with risk neutral agents outside a market setting (Bolton and Harris 1999; Moscarini and Smith 2001; Cripps, Keller and Rady 2005) only carry over to competitive markets under additional assumptions such as no price taking or bounded budget sets. Some recent approaches to financial information acquisition make investors risk neutral in competitive markets. Risk neutral investors in Jackson (1991) or Jackson and Peck (1999), for instance, acquire information because equilibrium price fails to aggregate contemporaneous asset demand. Similarly, risk neutral investors in Barlevy and Veronesi (2000) become implicitly risk averse through credit constraints and thus value information.

A theorem clarifies that non-standard assumptions are necessary for risk neutral investors to acquire information. A risk neutral investor with no restriction on short sales acquires information only if asset markets fail to clear or fail to be competitive, or if utility is not intertemporally separable. Otherwise, a risk neutral investor will neither spend money to obtain information nor sacrifice leisure to process information. In competitive asset markets, risk neutral investors remain rationally uninformed.

1 No acquisition of financial information

Consider two periods, today 0 and tomorrow 1, and two assets: one bond b with safe gross return $R = 1 + r$ and one risky asset x with uncertain payoff θ. No assumption is placed on the distribution of θ. So, the model applies to risky securities in general.

Before asset markets open, a signal s has been sent to investor i and informs her about the risky asset return tomorrow. A risk neutral investor maximizes the expected net present value of consumption

$$\mathbb{E}\left[C_0^i + \beta\, C_1^i | \mathcal{F}\right],$$

where β is the discount factor and C_t^i consumption of investor i at time t. The investor's information set $\mathcal{F}$ includes the signal realization s. Today's budget constraint implies $C_0^i = q_0^i - (b_1^i + P_0 x_1^i)$, where b_1^i and x_1^i are investor i's choices of bond and risky asset holdings, P_0 is the price of the risky asset, and $q_0^i = b_0^i + P_0 x_0^i$ is investor i's initial portfolio endowment. Tomorrow, consumption will be $C_1^i = R b_1^i + \theta x_1^i$. For a price-taking risk neutral investor, the conditions for optimal portfolio choice (b_1^{i*}, x_1^{i*}) are

$$R = 1/\beta \qquad \text{and} \qquad P_0 = \beta\, \mathbb{E}\left[\theta | \mathcal{F}\right] \tag{1}$$

in competitive asset markets.

In rational-expectations equilibrium, the investor's information set $\mathcal{F}$ includes both the signal realization s and equilibrium price P_0. Alternative equilibrium definitions, such as those in Jackson (1991) or Jackson and Peck (1999) for instance, exclude concurrent P_0 from $\mathcal{F}$. The following arguments apply to any of these equilibrium concepts. The following arguments only require that investors have expectations consistent with equilibrium and that conditions (1) are satisfied in asset market equilibrium.[1]

The two single-period budget constraints imply the intertemporal budget constraint

$$C_0^i + \tfrac{1}{R} C_1^i = q_0^i + \tfrac{1}{R}\left(\theta_1 - R P_0\right) x_1^i. \tag{2}$$

The net present value of an investor's consumption stream equals the value of the initial endowment q_0^i plus the (discounted) excess return $\theta_1 - R P_0$ of the risky asset holdings x_1^i, beyond the opportunity cost of holding the bond.

[1] Note that existence of any asset market equilibrium implies that every risk neutral investor must hold the same posterior expectation, so either asset price in rational-expectations equilibrium equalizes $\mathbb{E}\left[\theta | s^i, P_0\right]$ to a unique value for all i, or $\mathbb{E}\left[\theta | s^i\right] = \mathbb{E}\left[\theta | s^j\right]$ for all i, j

Having received a signal realization s, investor i assesses the impact that the signal realization has on the expected net present value of her optimal consumption. By (2),

$$C_0^{i*} + \tfrac{1}{R}\mathbb{E}\left[C_1^{i*}|s\right] = \mathbb{E}\left[q_0^i|s\right] + \tfrac{1}{R}\,\mathbb{E}\left[\,\theta_1 - RP_0\,|\,s\,\right]x_1^{i*} \tag{3}$$

depends on s because her optimal asset demands b_1^{i*} and x_1^{i*} respond to the signal realization.

Suppose investor i is asked to pay for her signal s. How much will she pay? The signal realization s is still unknown to her (she would not pay for something known). To evaluate the signal, the investor rationally anticipates her expected asset-demand response to the signal S (if she did not anticipate to act on the signal it would have no value to begin with). By (3) and iterated expectations, she evaluates the net present value of her intertemporal consumption

$$\mathbb{E}_S\left[C_0^{i*}\right] + \tfrac{1}{R}\mathbb{E}_S\left[C_1^{i*}\right] = \mathbb{E}_S\left[q_0^i\right] + \tfrac{1}{R}\mathbb{E}_S[(\theta_1 - RP_0)\,x_1^{i*}] \tag{4}$$

before she learns the signal realization s, and compares it to the net present value of her intertemporal consumption in the absence of a signal.

A rational investor anticipates that unbounded demand cannot be an equilibrium outcome in competitive asset markets. So, the investor rationally infers that her optimality conditions (1) must be satisfied with equality. Equivalently, $\mathbb{E}_S[(\theta_1 - RP_0)\,x_1^{i*}] = 0$ and investor i's expected utility before she receives the signal realization becomes

$$\mathbb{E}_S\left[C_0^i\right] + \beta\,\mathbb{E}_S\left[C_1^i\right] = \mathbb{E}_S\left[q_0^i\right].$$

As a consequence, her only benefit from a signal can come from an endowment revaluation $\mathbb{E}_S[q_0^i] = b_0^i + \mathbb{E}_S[P_0]\,x_0^i$ as her response to the signal changes asset price in market equilibrium. The expected equilibrium price level, however, is the same with and without a signal by iterated expectations: $\mathbb{E}_S[P_0] = \beta\mathbb{E}_S[\mathbb{E}\,[\theta|S]] = \beta\mathbb{E}\,[\theta]$ by (1). Note that this argument applies to any competitive market equilibrium where optimality conditions (1) hold with equality, including rational-expectations equilibrium. So, to a risk neutral investor, there is no expected benefit from a signal.

The no-acquisition result is true more generally.

Theorem 1 *Suppose signals are costly. Then a price taking investor with no limit on short sales and intertemporally separable von-Neumann-Morgenstern utility acquires a signal prior to equilibrium in competitive financial markets only if she is not risk neutral.*

Proof. Investor i has intertemporally separable von-Neumann-Morgenstern utility

$$U_t^i = \mathbb{E}\left[\sum_{\tau=t}^{t+T} \beta^{\tau-t} u(C_\tau^i) \,\middle|\, \mathcal{F}_t^i\right],$$

where instantaneous utility $u(C)$ is linear and strictly increases in C, she lives T periods (possibly $T \to \infty$) and $\mathcal{F}_t^i$ is her information set at t. Denote the ex-dividend price of the risky asset in period τ with P_τ. Then her intertemporal budget constraint is

$$b_{\tau+1}^i + P_\tau x_{\tau+1}^i = R_\tau b_\tau^i + (\theta_\tau + P_\tau) x_\tau^i - C_\tau^i.$$

Forward-iterate the budget constraint to find the net present value of consumption

$$\sum_{\tau=t}^{t+T} R_{t,\tau}^{-1} C_\tau^i = R_t q_t^i + \sum_{\tau=t}^{t+T} R_{t,\tau}^{-1} (\theta_\tau + P_\tau - R_\tau P_{\tau-1}) x_\tau^i - R_{t,T}^{-1} q_{t+T+1}^i, \qquad (5)$$

satisfied with certainty, where $R_{t,\tau}^{-1} \equiv (\Pi_{\tau=t+1}^s R_\tau)^{-1}$ and $q_t^i = b_t^i + P_t x_t^i$. In optimum, $q_{t+T+1}^{i*} = 0$ (for $T \to \infty$ by the transversality condition).

For a risk neutral investor, $u'(C)$ is a constant and the Euler equations for optimal portfolio choice become

$$R_{\tau+1} = 1/\beta \qquad \text{and} \qquad \mathbb{E}\left[P_\tau | \mathcal{F}_t^i\right] = \beta\, \mathbb{E}\left[\theta_{\tau+1} + P_{\tau+1} | \mathcal{F}_t^i\right]. \qquad (6)$$

The expected net present value of consumption, the expectation of (5), is equivalent to von-Neumann-Morgenstern utility, which turns into

$$U_t^i = \sum_{\tau=t}^{t+T} \beta^{\tau-t}\, \mathbb{E}\left[C_\tau^i | \mathcal{F}_t^i\right] = \mathbb{E}\left[q_t^i | \mathcal{F}_t^i\right]/\beta = b_t^i/\beta + \mathbb{E}\left[P_t | \mathcal{F}_t^i\right] x_t^i/\beta \qquad (7)$$

under Euler conditions (6). By iterated expectations, $\mathbb{E}\left[\mathbb{E}\left[P_t | \mathcal{F}_t^i\right]\right] = \mathbb{E}\left[P_t\right]$ so that expected utility $U_t^i = \mathbb{E}_{\mathcal{F}}[U_t^i]$ is identical in the presence and in the absence of the expected receipt of a signal. ■

For a risk neutral investor, financial information has no utility value.

2 Discussion

Theorem 1 assumes about equilibrium only that a risk neutral investor's optimality conditions are satisfied. Theorem 1 does not extend to risk averse investors, however, because a signal reduces the *ex ante* expected variance of future consumption. By variance decomposition, $\mathbb{E}_S\left[\mathbb{V}\left(\theta | S\right)\right] = \mathbb{V}\left(\theta\right) - \mathbb{V}_S\left(\mathbb{E}\left[\theta | S\right]\right)$.[2]

[2] As shown in Muendler (2007) for a portfolio choice model with a similar structure, information acquisition is an equilibrium outcome for countably many risk averse investors even under fully revealing asset price.

Theorem 1 is based on von-Neumann-Morgenstern utility and a set of assumptions on asset market equilibrium. Some assumptions are not necessary. An investor's life time was assumed to be known; but uncertainty about life expectancy does not change the result. The proof invoked the law of iterated expectations; but it's failure under strategic uncertainty does not make information valuable to risk neutral investors. Other assumptions are crucial: utility is intertemporally additive, asset markets are competitive and they clear, there is no excess demand for the risky asset in equilibrium, and *ex ante* utility must exist.[3] These assumptions are common in canonical portfolio choice models. They need not hold, however, and deserve scrutiny. Models such as Jackson and Peck (1999) or Barlevy and Veronesi (2000), where risk neutral investors acquire information, remove at least one of the key assumptions from classic portfolio choice.

The no-acquisition result for financial information is reminiscent of lacking firm entry into competitive markets in general equilibrium under constant returns to scale when entry is costly (McKenzie 1959). One resolution of the free entry problem in general equilibrium uses a sequence of Cournot-competition equilibria (for an accessible exposition see Novshek and Sonnenschein 1987). This suggests that the limit of a Cournot-style model of investor behavior in financial markets might also achieve a reconciliation of risk neutrality with financial-information acquisition.

2.1 Necessary assumptions for the no-acquisition result

Market clearing need not be satisfied. Froot, Scharfstein and Stein (1992) make information valuable to risk neutral investors by not permitting the market to clear. Instead, half of the orders is randomly deferred to a future period. In Jackson and Peck (1999), risk neutral investors simultaneously submit bid and offer functions in a Shapley and Shubik (1977) market game. This disconnects asset price from information on the fundamental because investors submit bids based on their own information and the anticipated bids of others, without being able to condition on equilibrium price. The first order condition $P = \beta \mathbb{E}[\theta|s]$ fails, so market clearing does not result in a price that reflects the expected asset value.

Credit constrained investors value information even if they are risk neutral (Barlevy and Veronesi 2000). The credit constraint removes from the optimality condition its knife-edge property—by which asset price equals the expected

[3]The existence of *ex ante* utility is required for investors to evaluate signals. Undefined *ex ante* utility such as in the bubble economy of Bhattacharyya and Lipman (1995), for instance, is not permissible. Infinite *ex ante* utility such as in the speculative game of Nalebuff (1989) makes information acquisition irrelevant if information costs are finite.

return $P = \beta\mathbb{E}[\theta|s]$, or else demand becomes unbounded. If investor i lacks resources to go long in the asset, she has to accept $P < \beta\mathbb{E}[\theta|s]$ (reflected in a strictly negative Lagrange-multiplier under a Kuhn-Tucker approach). If credit constraints happen to bind all risk neutral investors in equilibrium, a strictly positive excess return prevails. As a result, expected excess return $\mathbb{E}_S[(\theta - RP)x^{i*}]$ is non-zero *ex ante* and signal acquisition becomes worthwhile for a risk neutral investor. The asset price now depends on thes initial wealth distribution and also reflects investors' credit constraints.

Utility can be intertemporally non-separable in many forms. Consider a risk neutral investor i whose discount rate β^i is state dependent (it may depend on her uncertain state of health) and not revealed before the resolution of the asset return. Signals inform her about both her expected utility parameters and the asset return. Then the expected net present value of her optimal consumption exhibits a correlation between her discount rate and asset return,

$$\mathbb{E}\left[q_0^i|s\right] + \mathbb{E}\left[\beta^i(\theta - RP)|s\right] x^{i*},$$

replacing the right-hand side of (4). If a signal reveals joint information on utility parameters and asset returns, it can have a positive utility value for a risk neutral investor. Lacking intertemporal separability is a form of lacking risk neutrality because the correlation between an investor's discount rate and the asset return now matters for portfolio choice.

2.2 Unrelated assumptions and the no-acquisition result

The law of iterated expectations can fail under strategic uncertainty (e.g. Morris and Shin 2002). Then, individual *ex post* expectations of market price $\mathbb{E}^i\left[\bar{\mathbb{E}}[P]\right]$ (where superscript i is a shorthand for investor i's information set) do not simplify to the average of investors' expectations of market price $\bar{\mathbb{E}}[P]$. Strategic uncertainty does not alter a risk averse investor's valuation of a signal, however, because *ex ante* expected equilibrium price with and without the private signal remains identical: $\mathbb{E}_S[\mathbb{E}^i[P_t|S]] = \mathbb{E}^i[P_t]$. For a risk neutral investor, private information does not result in a utility improvement *ex ante* because a private signal only adds precision, for which a risk neutral investor does not care. So, even under strategic uncertainty, financial information has no utility value.[4]

A competitive fringe of risk neutral traders or market makers is part of several microstructure models of financial information (e.g. Kyle 1989; Hirshleifer, Subrahmanyam and Titman 1994; Vives 1995). Market makers observe aggregate demand. One might argue that the costs of information acquisition for market makers are zero because information on aggregate demand is just a byproduct

[4]For a formal proof see the working paper version (Muendler 2005).

of their market making. If so, market makers' risk neutrality would not impede their information acquisition. Market makers' information on aggregate demand, however, is secondary information in that it derives from the primary information behind informed investors' demands. Those informed investors cannot be risk neutral, otherwise they would not acquire information.

3 Concluding remarks

How much income or leisure does a risk neutral investor give up to acquire information? In competitive asset markets, the answer is no income and no leisure at all. A risk neutral investor is indifferent between holding a risky asset or a safe bond in equilibrium. Hence, she expects her actions upon signal realizations to yield the same return *ex ante* as uninformed actions do. This makes signals useless to her. In competitive asset markets, risk neutral investors can only be informed by accident.

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