The Action Value of Information
and the Natural Transparency Limit*

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Abstract

Add an opening stage of signal acquisition to a canonical portfolio choice model and let investors have rational expectations about the ensuing Walrasian equilibrium. The expected marginal utility of a signal (its action value) falls in the number of signals and turns strictly negative at a finite number because signals diminish the asset's excess return. There is a natural transparency limit at which rational investors pay to inhibit information disclosure. Prior to the limit, financial information is a public good and justifies intervention. To instill more transparency, cutting costs of information acquisition is superior to disclosure because disclosure crowds out private information acquisition and risks a violation of the transparency limit.

Keywords: information acquisition; portfolio choice; rational expectations equilibrium; informational efficiency; transparency

JEL classiciation: D81, D83, G14

Recent financial crises and corporate scandals have prompted calls for more transparency. Morris and Shin (2002) show in a stylized model of the financial market that more public information may reduce welfare. Their result has been widely interpreted as counsel against transparency and been challenged within the original setting (Svensson 2005) as well as in a series of extensions (Angeletos and Pavan 2004, Heinemann and Cornand 2004, Hellwig 2004). The model of this paper removes the role of higher-order expectations from the game and instead precedes portfolio choice with an experimentation game of signal acquisition. The results provide common ground for both proponents and sceptics of financial transparency: the underprovision of information in private markets calls for public intervention, but public disclosure is harmful beyond a natural transparency limit.

When rational investors anticipate the effect of information and their actions on equilibrium price, the resulting *action value of information* rises with investors' risk aversion, rises with asset market volume, and rises with the variance of the asset return—as it should be. Most important, however, the action value of information is not bound to be positive. At the *natural transparency limit*, investors pay to prevent further disclosure.

Through revealing asset price, one investor's action permits other investors to update beliefs so that information becomes a public commodity. Private investors do not internalize the benefit to others and acquire too little information when information is a public good.[1] This basic mechanism—cast aside in stylized game-theoretic models—provides a rationale for governmental intervention. But information turns into a public bad at the natural transparency limit. The reason is that, by removing risk, information diminishes the excess return. Investors are compensated for the risk of an asset's lacking information with a payoff beyond the safe return. The utility loss from diminished excess returns—omitted from abstract experimentation models—outweighs information benefits if the variance of the asset return is low, if the market volume is small, or if a large amount of information is already available. To take an example, risk averse investors are compensated for Argentina's default risk with returns that exceed the expected losses under default. That is why they lend. More information strictly diminishes this excess return. At the transparency limit, rational lenders reject further information to keep the excess return.

Information and transparency are matters of degree. Beyond binary information acquisition as in Grossman and Stiglitz (1980) or Hellwig (1980), the present paper gives investors a choice of a number of signals in the spirit of

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[1]Asset price is fully revealing in this paper. Under partially revealing price, information has public-goods character nonetheless because price continues to permit belief updating (Admati 1985).

experimentation. To make public and private information comparable in price impact, the present paper considers a finite number of investors whose individual actions affect price. Verrecchia (1982) and Kim and Verrecchia (1991) consider the complementary case of infinitely many investors with no price impact.

Recent theoretical models address the value of information and welfare effects of transparency in two different ways. A game-theoretic and a general-equilibrium literature, on the one hand, model the real side of the economy and investigate conditions for welfare improving public disclosure. This research often stops short of giving investors a rational choice of information (e.g. Morris and Shin 2002, Angeletos and Pavan 2004; Krebs 2005). The literature on experimentation, on the other hand, shows that there are decreasing but strictly positive marginal benefits to information and that the public-goods character of information induces free-riding (e.g. Moscarini and Smith 2001, Cripps, Keller and Rady 2005). The abstract experimentation framework does not tie the value of information to economic primitives.[2] The present paper attempts to bridge these strands of literature and gives investors a rational information choice through costly experimentation in a canonical model of portfolio choice.

In practice, evidence on the social value of information is scarce and offers little guidance to judge how strongly the market outcome differs from the transparency optimum. This uncertainty notwithstanding, the present analysis of rational information choice provides indications of suitable policies.

Better abstain from public provision of private information. It is because hedge funds and startup companies are not transparent that they generate excess returns. Public disclosure of private information can disrupt the market and push it beyond the transparency limit, where rational investors pay to prevent disclosure. With the enforcement of New Zealand's compulsory disclosure rules in 1996, for instance, the country's banks no longer receive on-sight visits under prudential supervision but undergo frequent external audits and credit-rating disclosures (Vishwanath and Kaufmann 2001). New Zealand's extensive disclosure rules may have resulted in a welfare improvement for investors, or not.

Reduce, instead, the costs of information acquisition to achieve more transparency. The analysis of rational information choice suggests that lower access costs remedy the need for financial information but cannot result in a violation of the transparency limit. Private investors will gather no more information than up to the desirable limit. Examples of such information cost reducing policies are: the timely dissemination of disclosed information, the equitable dissemination of information beyond financial intermediaries to ultimate investors, the assessment of rating agencies' and auditors' standards to facilitate interpretation of

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[2]The literature on information transmission in oligopolistic product markets shows that the value of information is closely linked to market conditions (Raith 1996).

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disclosed information, the use of lucid language instead of subdued suggestions in disclosed official reports, and the promotion of internationally accepted accounting standards.[3]

Detrimental effects of disclosure are not confined to financial markets. Hirshleifer (1971) demonstrates how public information can erode risk-sharing opportunities and reduce *ex ante* welfare. Within financial markets, Marín and Rahi (2000) show that market completeness and the ensuing information revelation may reduce welfare because of a Hirshleifer effect. The diminishing-excess-return effect, more generally, inflicts utility losses even on investors with a safe endowment and no risk-sharing appetite, beyond a Hirshleifer effect.

Insufficient public information and a high degree of asymmetric information, on the other hand, can also hamper financial markets—similar to markets for lemons (Akerlof 1970). In the extreme, less informed traders refuse to trade and cause a financial market breakdown (Bhattacharya, Reny and Spiegel 1995). This adverse selection problem prompts issuers to design securities with payoffs that are independent of private information (Rahi 1996, DeMarzo and Duffie 1999). The concern with financial crises and corporate scandals, however, is not that investors refuse to hold the risky asset before the crisis or scandal; it is that investors strongly demand the asset in the absence of transparency. Fully revealing price is a device to check, in individual utility terms, the value of public information. For these reasons, the present paper removes asymmetric information and adopts a fully revealing financial market equilibrium. Fully revealing asset price heightens the diminishing-excess-return effect, a key source for informational welfare losses, but information under partially revealing price also exhibits the effect (Admati 1985).

Public disclosure crowds out private information gathering in the present model. Tong (2005) finds this in a coordination game too, whereas Kim and Verrecchia (1991) present conditions when announced public disclosure induces investors to buy more precise private information. Tong provides empirical evidence from panel data of analysts' forecasts on stocks in thirty countries: disclosure standards promote accuracy but crowd out the number of analysts per stock.

Easley and O'Hara (2004) show that less asymmetric information about an asset diminishes its excess return. In the present model with no asymmetric information, it is publicly inferrable information that diminishes the excess return. Easley, Hvidkjaer and O'Hara (2002) provide evidence for a diminishing-excess-return effect when information becomes more uniform across investors: as the probability that trades are based on insider information drops by ten percentage

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[3]The International Monetary Fund (1999) champions such cost reducing policies but also proposes to publicly provide so-far undisclosed information.

points, the expected return of a NYSE stock diminishes by 2.5 percentage points a year. This diminishing-return effect is consistent with both less private and more public information.

Drawing on Raiffa and Schlaifer's (1961) decision theory, this paper adds an experimentation stage to standard portfolio choice. The paper considers Poisson distributed signals and gamma distributed asset returns. With the Poisson-gamma pair, Wall Street equilibrium has a closed form so that the information market equilibrium is tractable. The existence of a transparency limit is not specific to the Poisson-gamma pair. In fact, for the common normal-normal distribution pair, the natural transparency limit under fully revealing price is *no* information (Muendler 2002, ch. 4).

The Poisson signal distribution reflects the discreteness of financial information such as Standard & Poor's or Moody's investment grades, and is a statistical approximation to many binomial thumbs-up, thumbs-down signals. Public information, if superior to private information, arrives in bundles of multiple Poisson signals rather than in single signals. To be informative, public and private signals must depend on the same financial fundamentals. The success of Nelson's (1991) exponential ARCH model in empirical finance suggests that the gamma distribution represents a particularly relevant family of return distributions.[4] Realistically, gamma distributed payoffs cannot be negative so that investors can never lose more than their principal.

The action value of information is the expected utility benefit of a signal under the condition that no strategic action can be taken at Wall Street. An extension of the action value to strategic interaction in the asset market remains for future research. The natural transparency limit, I conjecture, will shift but not disappear.

The following section 1 presents the rational information choice model and discusses its assumptions. There is a unique and fully revealing equilibrium at Wall Street, derived in section 2. Moving backward in time, section 3 establishes equilibrium in the preceding information market and shows how financial primitives determine the value of information. Section 4 investigates the welfare properties of the information market equilibrium and discusses transparency policies. Section 5 concludes with final remarks.
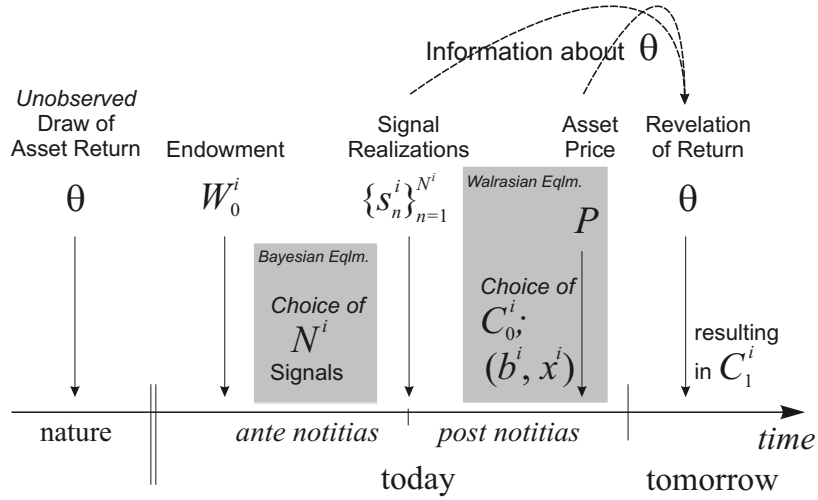
Figure 1: **Timing of decisions and information revelation**

1 Portfolio and Information Choice

Consider a canonical expected utility model of portfolio choice. There are two periods, today and tomorrow, and there are two assets: One riskless bond b and one risky stock x. Assets are perfectly divisible. The riskless bond sells at a price of unity today and pays a real interest rate $r \in (-1, \infty)$ tomorrow so that the gross interest factor is $R \equiv 1 + r \in (0, \infty)$. The risky asset sells at a price P today and pays a gross return θ tomorrow.

Add to this model an opening stage of signal acquisition. Investors hold prior beliefs about the distribution of the risky asset return and can hire spy robots (acquire signals) to receive reports (get signal realizations and update beliefs). Markets for spy robots (signals) S_n^i open at 9am today. Robot n, hired by investor i, reports back exclusively to investor i with a signal realization s_n^i before 10am. How many different spy robots N^i should investor i hire?

Each investor knows that she will base her portfolio decision, to be taken at 10am today, on the information that she is about to receive from her N^i spy robots. Signals are independent, conditional on the asset return. Moreover, the asset price at 10am will also contain information. The reason is that each investor chooses her portfolio given her observations of signal realizations ($\{s_n^i\}_{n=1}^{N^i}$), and the Walrasian auctioneer at Wall Street clears the market by calling an equilibrium price. In the benchmark case of a *fully revealing* equilibrium, the asset

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[4]Special cases of the gamma distribution are the exponential, the chi-squared, and the Erlang distribution, for instance. Davis (1993) presents an earlier model in finance that employs the gamma distribution.

price is invertible in a sufficient statistic of all investors' *posterior* beliefs and hence permits the rational extraction of all relevant market information. This is the case in the present paper.

The timing is illustrated in Figure 1. Every investor i is endowed with certain initial wealth W_0^i. At 9am, investors choose the number of signals (spy robots) N^i. To do so, they maximize *ante notitias* expected utility based on their prior beliefs before signal realizations become known (*ante notitias*). Investors then receive the realizations $\{s_1^i, ..., s_{N^i}^i\}$ of these N^i signals (they get to know the content of the spy robots' reports) and update their beliefs. When Wall Street opens at 10am today, investors choose consumption today and tomorrow, C_0^i and C_1^i, and decide how much of the risky asset to hold. At this stage, they maximize *post notitias* expected utility based on their posterior beliefs.[5] The Walrasian auctioneer in the financial market sets the price P for the risky asset such that the stock market clears. The bond market clears given the interest factor R so that RP is the opportunity cost of holding a risky asset rather than a bond.

The acquisition of signals changes the expected asset price P *ante notitias*. So, investors can affect the expected value of their endowments $W_0^i = b_0^i + P x_0^i$ by buying signals. This purely distributional incentive to acquire information complicates the analysis but does not change basic insights about the action value of information or the transparency limit. Appendix E provides a general proof of the key proposition of this paper and accounts for the wealth effect of information. To focus the analysis, consider homogeneous investors with $x_0^i = 0$. There is a sole (foreign) issuer of the risky asset, and this initial asset owner is considered irrelevant for information acquisition. Appendix E shows that the sole owner of the risky asset may indeed not want to acquire any signal.

1.1 Conjugate updating

Financial information often comes in discrete levels such as Standard & Poor's or Moody's investment grades, or on a three-level buy-hold-sell scale. It therefore appears not only convenient but realistic to consider discrete signals. Poisson distributed signals in particular exhibit several useful statistical properties. For a large number of repetitions, Poisson probabilities approximate binomial (good news, bad news) signal distributions (Casella and Berger 1990, Example 2.3.6). The sum of N^i conditionally independent Poisson signals is itself Poisson distributed with mean and variance $N^i \theta$ (appendix A).

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[5]To clarify the timing of signal realizations, I distinguish between *ante notitias* and *post notitias* expected utility. *Ante notitias* expected utility is different from *prior* expected utility in that the arrival of N^i signals is rationally incorporated in *ante notitias* expected utility. Raiffa and Schlaifer (1961) favored the terms "prior analysis," "pre-posterior analysis" and "posterior analysis." Laffont (1985) used *ex ante*, *interim*, and *ex post*.

Assumption 1 (Poisson distributed signals and conjugate updating). *Signals are Poisson distributed and update the prior distribution of the asset return θ to a posterior distribution from the same family.*

A gamma distribution of the asset return, $\theta \sim \mathcal{G}(\alpha^i, \beta^i)$, uniquely satisfies assumption 1 (Robert 1994, Proposition 3.3). The parameters $\{\alpha^i, \beta^i\}$ are part of investor i's information set. The parameter α^i is sometimes referred to as the shape parameter and $1/\beta^i$ as the scale parameter.

A gamma distributed asset return has the advantage that its support is strictly positive so that, realistically, negative returns cannot occur. In contrast, a normal asset return would imply that stock holders must cover losses beyond the principal ($\theta < -P$) with a strictly positive probability. Moreover, the Poisson-gamma pair makes sure that signals have positive value even for investors with no risky asset endowment. A normal-normal pair of signal-return distributions would result in zero information acquisition in the present model (Muendler 2002, ch. 4).

The mean of a gamma distributed return θ is α^i/β^i, and its variance $\alpha^i/(\beta^i)^2$. Thus, the mean-variance ratio is β^i. It will play a key role. An important property of the Poisson-gamma pair relates to the updating of beliefs (Robert 1994, Proposition 3.3):

Fact 1 (Conjugate updating). *Suppose the prior distribution of θ is a gamma distribution with parameters $\bar{\alpha} > 0$ and $\bar{\beta} > 0$. Signals $S_1^i, ..., S_{N^i}^i$ are independently drawn from a Poisson distribution with the realization of θ as parameter. Then the* post notitias *distribution of θ, given realizations $s_1^i, ..., s_{N^i}^i$ of the signals, is a gamma distribution with parameters $\alpha^i = \bar{\alpha} + \sum_{n=1}^{N^i} s_n^i$ and $\beta^i = \bar{\beta} + N^i$.*

Derivations in this paper draw on some further useful properties of the Poisson-gamma distribution pair. Those are reported in appendix A.

It is instructive to consider investors who are identical in beliefs and risk aversion. This homogeneity is necessary for price to become fully revealing. If investors also know market size, asset price will become fully revealing.

Assumption 2 (Common priors and risk aversion). *Investors have common prior beliefs about the joint signal-return distribution, and the same risk aversion.*

Assumption 3 (Known market size). *The average supply of the risky asset $\bar{x}$ and the total number of investors I are certain.*

So, investor i's information set is $\{\alpha^i, \beta^i; \bar{x}, I; RP\}$.

1.2 Portfolio choice

A signal costs c. The intertemporal budget constraint of investor i is

$$b^i + Px^i = b_0^i - C_0^i - cN^i \tag{1}$$

today, and

$$C_1^i = Rb^i + \theta x^i \tag{2}$$

will be available for consumption tomorrow.

Assumption 4 (Expected utility). *Investors $i = 1, \ldots, I$ evaluate consumption with additively separable CARA utility U^i at individual discount rates ρ^i:*

$$U^i = \mathbb{E}\left[u(C_0^i) + \rho^i u(C_1^i) \mid \alpha^i, \beta^i; \bar{x}, I; RP\right], \tag{3}$$

where $u(C) = -\exp\{-AC\} < 0$.

After having received the realizations of her N^i signals $\{s_j^i\}_{j=1}^{N^i}$ and updated beliefs to *post notitias* beliefs, every investor i maximizes expected utility (3) with respect to consumption C_0^i today and C_1^i tomorrow given (1) and (2). For ease of notation, abbreviate investor i's conditional expectations with $\mathbb{E}^i[\cdot] \equiv \mathbb{E}[\cdot \mid \alpha^i, \beta^i; \bar{x}, I; RP]$ when they are based on *post notitias* beliefs, and with $\mathbb{E}_{ante}[\cdot] \equiv \mathbb{E}[\cdot \mid \bar{\alpha}, \bar{\beta}; \bar{x}, I; RP]$ for *ante notitias* beliefs in anticipation of N^i signal receipts. *Post notitias* expectations coincide for all investors under fully revealing price.

For a gamma distributed asset return, demand for the risky asset becomes (see appendix B)

$$x^{i*} = \frac{\beta^i}{A} \frac{\mathbb{E}^i[\theta] - RP}{RP} \equiv \frac{\beta^i}{A} \cdot \xi^i. \tag{4}$$

Demand for the risky asset decreases in price and the riskless asset's return; demand is the higher the less risk averse investors become (lower A) or the higher the expected mean-variance ratio β^i of the asset is. Investors go short in the risky asset whenever their return expectations fall short of opportunity cost, $\mathbb{E}^i[\theta] < RP$, and go long otherwise. Under CARA, demand for the risky asset is independent of wealth W_0^i. Bond demand b^i varies to satisfy the wealth constraint.

The factor $\mathbb{E}^i[\theta - RP]/RP$ is an individual investor i's *expected relative excess return* over opportunity cost. Risk averse investors demand this premium. For later reference, define

$$\xi^i \equiv \frac{\mathbb{E}^i[\theta] - RP}{RP}. \tag{5}$$

The expected relative excess return ξ^i has important informational properties that crucially affect incentives for information acquisition.

1.3   Rational information choice equilibrium

At the outset, every investor chooses the number of signals she wants to receive. She does this by maximizing *ante notitias* utility given her beliefs before the realizations of the signals arrive. At this time she does not know more than the prior parameters of the respective distributions, but she builds her *ante notitias* beliefs by taking into account how signals will likely change beliefs at 10am. *Ante notitias* utility is $\mathbb{E}_{ante}\left[U^{i*}\right] = \mathbb{E}_{ante}\left[u(C_0^{i*})\right] + \rho^i \mathbb{E}_{ante}\left[u(C_1^{i*})\right]$ by iterated expectations. The optimal number of signals $N^{i*} \geq 0$ maximizes *ante notitias* utility $\mathbb{E}_{ante}\left[U^{i*}\right]$.

In the spirit of competitive equilibrium, a rational expectations equilibrium (REE) that clears both the asset market and the market for signals can be defined as a Walrasian equilibrium at Wall Street preceded by a Bayesian public-goods equilibrium in the market for detective services. I call this REE extension a Rational Information Choice Equilibrium, or RICE.

Rational Bayesian investors choose signals given the expected asset market REE at Wall Street under anticipated information revelation. The equilibrium in the market for signals is a benchmark public-goods equilibrium similar to Samuelson's (1954) definition. Agents mutually respond to other agents' rational demand for signals in the Nash equilibrium. Most important, investors know all other investors' signal choices $\sum_{k \neq i} N^{k,*}$ when taking their portfolio decision at Wall Street. In practice, international lenders know the frequency of a sovereign debtor's reports and the number of rating agencies who classify the country's solvency. Stock market investors know the annual number of corporate statements and can obtain published head counts of financial analysts who track a given stock.

Definition 1 (RICE). *A rational information choice equilibrium (RICE) is an allocation of x^{i*} risky assets, b^{i*} riskless bonds, and N^{i*} signals to investors $i = 1, ..., I$ and an asset price P along with consistent beliefs such that*

- *the portfolio (x^{i*}, b^{i*}) is optimal given RP and investors' post notitias beliefs for $i = 1, ..., I$,*

- *the market for the risky asset clears, $\sum_{i=1}^{I} x^{i*} = I\bar{x}$, and*

- *the choice of signals N^{i*} is optimal for investors $i = 1, ..., I$ given the sum of all other investors' signal choices $\sum_{k \neq i} N^{k,*}$ and marginal signal cost c.*

Investors evaluate *ante notitias* expected utility for their signal choice. *Ante notitias* expected utility has no closed form, however, unless R is constant. Assumption 5 assures this.

Assumption 5 (Single-price responses to signal realizations). *The equilibrium price of an asset only responds to signal realizations on its own return.*

The assumption is equivalent to the limiting case where markets for individual risky assets are small relative to the overall market for riskless bonds so that single signal realizations alter R negligibly little (see appendix C for a formal derivation). Economies with large safe forms of debt such as government debt and small open economies are examples.

2 Financial Market Equilibrium

To solve for RICE, we can work backward. For now, restrict attention to the partial REE at Wall Street, given any market equilibrium for spy robots. Investors $i = 1, ..., I$ have received the spy robots' reports (realizations of their conditionally independent N^i signals). It is 10am, and investors choose portfolios (x^{i*}, b^{i*}) given their *post notitias* information sets.

In REE, rational investors consider their own signal realizations and also extract information from price. RP and $\sum_{n=1}^{N^i} s_n^i$ are correlated in equilibrium. So, the *post notitias* distribution of the asset return, based on this information set, can be complicated. If price P is fully revealing, however, the information sets of all investors coincide. This gives the rational beliefs in REE a closed and linear form analogous to fact 1.

Proposition 1 (Unique asset market REE). *Under assumptions 1 through 4, the asset market REE in RICE is unique and symmetric, conditional on the number of signals N^k and their total realizations $\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k$, with*

$$\alpha^i = \bar{\alpha} + \sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k \equiv \alpha, \tag{6}$$

$$\beta^i = \bar{\beta} + \sum_{k=1}^{I} N^k \equiv \beta, \tag{7}$$

$$RP = \frac{\alpha}{\beta} \frac{1}{1 + \xi}, \tag{8}$$

where $x^{i*} = \bar{x}$ and $\xi^i = \xi \equiv A\bar{x}/\beta$.

Proof. By (4) and for beliefs (6) and (7), $x^{i*} = \alpha/(A\,RP) - \beta/A$ for all i. So, market clearing $x^{i*} = \bar{x}$ under definition 1 of RICE implies (8).

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Uniqueness of beliefs (6) and (7) follows by construction. By (4) and market clearing, RP can always be written as an affine function of all signal realizations $\sum_{k=1}^{I} \sum_{n=1}^{N^k} s_n^k$ so that every investor i can infer everyone else's signal realizations from her knowledge of own signal realizations. Everyone else's signals $\sum_{k \neq i} \sum_{n=1}^{N^k} s_n^k$ and own signals $\sum_{n=1}^{N^i} s_n^i$ are Poisson distributed by fact 3 but conditionally independent. So, a rational investor applies Bayesian updating following fact 1. Hence, $\alpha^i = \bar{\alpha} + \sum_{n=1}^{N^i} s_n^i + \sum_{k \neq i}^{I} \sum_{n=1}^{N^k} s_n^k$ and $\beta^i = \bar{\beta} + N^i + \sum_{k \neq i}^{I} N^k$. $\sum_{k \neq i}^{I} N^k$ is known by definition 1 of RICE.

No less than all signals can get revealed in REE. Otherwise at least one investor would not base demand on her signal, which is ruled out in an REE. ∎

The equilibrium price P fully reveals aggregate information of all market participants. Formally, aggregate information is the total of all signals received: $\sum_{i=1}^{I} \sum_{n=1}^{N^i} s_n^i$. This is a sufficient statistic for every moment of θ given $\sum_{i=1}^{I} N^i$ (which is known by definition 1 of RICE). It can be shown that the equilibrium price is fully revealing if and only if assumptions 1 through 4 are satisfied.

In fully revealing REE, investors' information sets coincide by (6) and (7). Consequently, the expected relative excess return $\xi^i = \xi$ (5) coincides. It becomes

$$\xi = \frac{\mathbb{E}[\theta] - RP}{RP} = \frac{A\bar{x}}{\beta} = \frac{A\bar{x}}{\bar{\beta} + \sum_{k=1}^{I} N^k} \in (0, \bar{\xi}] \quad \text{where} \quad \bar{\xi} \equiv \frac{A\bar{x}}{\bar{\beta}}. \quad (9)$$

The limit $\bar{\xi}$ is the *elementary excess return*: the maximal expected excess return absent information acquisition.

The expected relative excess return $\mathbb{E}^i[\theta - RP]/RP$ will be crucial for individual incentives to buy information: information diminishes the expected excess return. Equilibrium price P reveals signal realizations. So, private information becomes publicly known to investors through informative price and risk averse investors value the risky asset more, thus bidding up price. Therefore, investors expect higher opportunity cost of the risky asset $\mathbb{E}_{ante}[RP]$ in the face of reduced uncertainty.

Proposition 2 (Diminishing expected excess return). *Under assumptions 1 through 4, the expected relative excess return ξ in asset market REE strictly falls in the number of signals, while the expected opportunity cost of the risky asset $\mathbb{E}_{ante}[RP]$ strictly increases in the number of signals* ante notitiam.

Proof. Note that $\xi = \mathbb{E}_{ante}[\xi]$ by (9). The number of signals $\bar{N} = \sum_{k=1}^{I} N^k$ strictly diminishes ξ by (9). The number of signals strictly raises $\mathbb{E}_{ante}[RP] = (\bar{\alpha} + \bar{\alpha}\bar{N}/\bar{\beta})/(A\bar{x} + \beta)$ since $\partial \mathbb{E}_{ante}[RP]/\partial \bar{N} = \bar{\xi}/\beta^2(1 + \xi)^2 > 0$. ∎

To choose the number of spy robots, investors evaluate the utility benefit in expected asset market equilibrium. *Post notitias* expected utility $\mathbb{E}^i[U^i] = \mathbb{E}[U^i|\alpha^i, \beta^i; \bar{x}, I; RP]$, given signal realizations, is

$$\begin{aligned}
\mathbb{E}^i[U^i] &= -\delta^i \exp\{-AR(b_0^i - cN^i)\}^{\frac{1}{1+R}} \\
&\quad \times \exp\left\{-ARP(x_0^i - x^{i*})\right\}^{\frac{1}{1+R}} \left[\beta^i/(\beta^i + Ax^{i*})\right]^{\frac{\alpha^i}{1+R}},
\end{aligned}$$

using (4) in utility (3), where $\delta^i \equiv \frac{1+R}{R}(\rho^i R)^{\frac{1}{1+R}}$. In asset market equilibrium with (6), (7) and (8), *post notitias* expected utility becomes

$$\mathbb{E}^i[U^i] = -\delta^i \exp\{-AR(b_0^i - cN^i)\}^{\frac{1}{1+R}} \exp\{\xi/(1+\xi)\}^{\frac{\alpha}{1+R}} (1+\xi)^{-\frac{\alpha}{1+R}}, \quad (10)$$

where $\xi = A\bar{x}/\beta$ is the expected excess return.
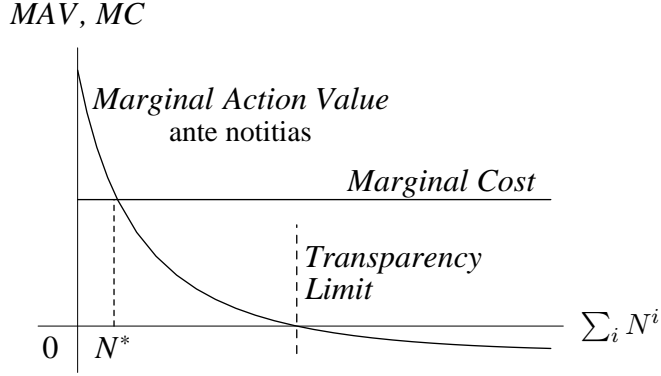
3 The Action Value of Information

Given the expected financial market equilibrium, how many spy robots do investors hire in RICE? Investors dislike the diminishing effect of information on the expected relative excess return ξ but anticipate a more educated portfolio choice if they can receive spy robots' reports. In their *ante notitias* choice of the optimal number signals, risk averse investors weigh the diminishing excess return and the marginal cost of a signal against the benefit of a more informed intertemporal consumption allocation. The net marginal utility benefit is the net action value of information.

Taking prior expectations of (10) yields *ante notitias* expected utility

$$\begin{aligned}
\mathbb{E}_{ante}\left[\mathbb{E}^i[U^i]\right] &= -\delta^i \exp\left\{-A\frac{R}{1+R}(W_0^i - cN^i)\right\} \quad (11) \\
&\quad \times \left[1 + \left(\left[(1+\xi)\exp\left\{-\frac{\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\frac{\bar{\xi}}{\xi}\right]^{-\bar{\alpha}}
\end{aligned}$$

(see appendix D). The cost of signals cN^i enters (11) in the form of an initial wealth reduction. The last factor in (11) captures the effect of the relative excess return $\xi \in (0, \bar{\xi}]$ on utility. The term $(1+\xi)\exp(-\xi/(1+\xi))$ strictly exceeds unity since $\xi > 0$ for finitely many spy robots by (9). Hence, the last factor in (11) is well defined.

Although the number of signals is discrete, one can take the derivative of *ante notitias* utility with respect to N^i to describe optimal signal choice. Strict monotonicity of the first-order condition in the relevant range will prove this to be admissible.

MAV, MC

Marginal Action Value
ante notitias

Marginal Cost

Transparency
Limit

$0 \quad N^*$

$\sum_i N^i$

Financial primitives: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 2: **Action value of information and the transparency limit**

Differentiate (11) with respect to N^i, and divide by $-\mathbb{E}_{ante}[U^{i*}] > 0$ for clarity, to find the marginal net action value of information

$$-\frac{1}{\mathbb{E}_{ante}[U^{i*}]}\frac{\partial \mathbb{E}_{ante}[U^{i*}]}{\partial N^i} = -A\frac{R}{1+R}c \tag{12}$$

$$+\frac{\bar{\alpha}}{\bar{\beta}}\frac{\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}}\left(1 - \frac{1}{1+R}\frac{\xi^2}{(1+\xi)^2}\right) - 1}{1 + \left(\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\frac{\bar{\xi}}{\xi}}.$$

The first term on the right hand side of (12) is negative and represents the *marginal cost* of a signal (MC) in expected utility terms. The second term expresses the *marginal action value* of a signal (MAV) and can be positive or negative. The incentive for information acquisition does not depend on an investor's patience.

Figure 2 depicts the marginal action value MAV of a signal and its marginal cost, and shows the optimal information choice N^* if spy robots were divisible. The optimal discrete number of spy robots lies in an open neighborhood around N^*. An additional spy robot can diminish the expected excess return ξ so strongly that this negative effect more than outweighs the benefits. Then the marginal action value of information turns negative. Figure 2 depicts this turnaround point: the *natural transparency limit*. Investors in a market close to the transparency limit suffer a strict utility loss if public disclosure pushes information beyond the transparency limit.

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How can the action value of information turn negative? Recall that the action value answers the investor's question how an additional signal changes her utility *ex ante* if she takes into consideration everyone else's response at Wall Street to her action upon the signal realization. A negative action value means that an investor strictly prefers not to receive information because she expects her and, through revealing price, everyone else's signal-induced actions to reduce her welfare. An investor rationally stops acquiring signals before the action value turns negative. At zero action value, the investor accepts a signal for free. At negative action value, the investor pays to prevent public disclosure because an additional signal updates everyone's beliefs and diminishes excess return below the minimally desirable level.

The marginal action value of information (12) is not invertible in the number of signals in closed form. Buying signals N^k, however, is just the converse of choosing the expected relative excess return ξ because ξ strictly falls in N^k (proposition 2). So, information acquisition is equivalent to choosing the remaining excess return.
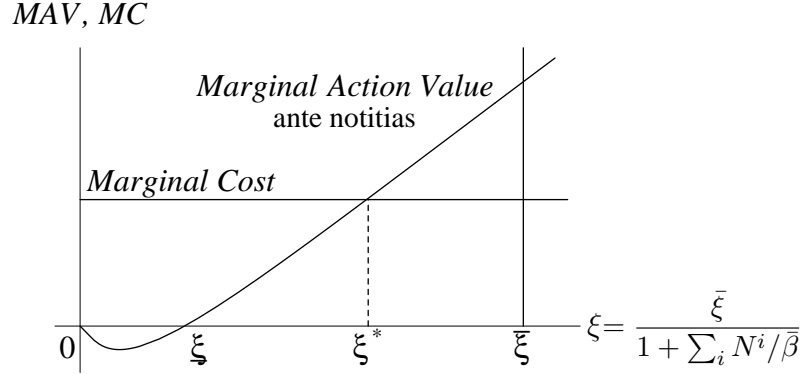
Proposition 3 (Action value of information). *Under assumptions 1 through 5, the following is true for the marginal action value* $\mathrm{MAV}(\xi)$ *of a signal.*

- *The marginal action value* $\mathrm{MAV}(\xi)$ *is strictly positive iff* $\xi > \underline{\xi}$, *where the expected excess return at the transparency limit* $\underline{\xi} \in (0, \infty)$ *uniquely solves* $\mathrm{MAV}(\underline{\xi}) = 0$.

- *The marginal action value of information* $\mathrm{MAV}(\xi)$ *strictly increases in* ξ *in the range* $\xi \in [\underline{\xi}, \bar{\bar{\xi}}]$, *provided the elementary excess return* $\bar{\bar{\xi}}$ *exceeds the expected excess return* $\underline{\xi}$ *at the transparency limit.*

Proof. See appendix E for the general case and set $x_0^i = 0$. ■

Figures 3 through 5 depict the marginal signal cost (MC) and marginal action value of a signal (MAV) under varying financial primitives. (By assumption 5 we can consider R a primitive instead of average initial wealth $\bar{b}$.)

For a strictly positive interest factor R, MAV turns positive at one unique point $\underline{\xi} > 0$ and subsequently increases unboundedly in ξ. The unique zero point $\underline{\xi}$ solves $MAV(\underline{\xi}) = 0$ and is independent of the elementary excess return $\bar{\bar{\xi}}$ (and $\bar{\alpha}, \bar{\beta}$). In contrast to cases of non-convexities (e.g. Chade and Schlee 2002) but in line with findings from the experimentation literature (e.g. Moscarini and Smith 2001), the value of information is well behaved in our rational Bayesian model of portfolio choice. In the range where signals have positive utility value, the marginal action value MAV of an additional signal strictly monotonically

MAV, MC

$$\xi = \frac{\bar{\xi}}{1 + \sum_i N^i / \bar{\beta}}$$

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Financial primitives: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 3: **Action value of information and expected excess return**
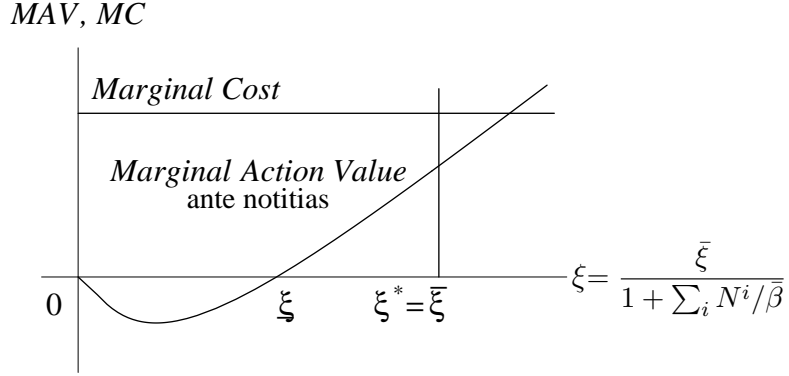
falls. *Ante notitias* expected utility is thus strictly concave in signals in the relevant range.

Under what conditions do investors buy information? Figure 3 shows a case (for the same parameters as in Figure 2). When investors acquire signals, they move ξ away from the elementary excess return $\bar{\xi}$ and to the west. The *MAV* curve has a long arm in the positive range that slopes strictly upward by proposition 3. So, as long as the elementary excess return $\bar{\xi}$ is large enough, there is a strictly positive expected excess return ξ^* at which the marginal action value *MAV* of a signal equals marginal cost *MC*. Although the relative excess return could attain any real value in principle, signals are not perfectly divisible. As a consequence, the precise optimal number of signals yields an expected relative excess return in an open interval around ξ^*.

An interior equilibrium can only occur if the elementary excess return $\bar{\xi}$ is large. So, investors will acquire a strictly positive amount of information only if the financial primitives meet the following two conditions. First, supply of the risky assets needs to be strong so that $\bar{x}$ is high. Then investors anticipate that they will hold a relatively large portion of their savings in the risky asset, and information about the risky asset return becomes important to them. Second, investors need to be sufficiently risk averse relative to prior beliefs about the mean-variance ratio $\bar{\beta}$ so that $A/\bar{\beta}$ is high. Because the benefit of information stems from lowering the prior variance of the portfolio, information matters more for investors who are more risk averse.

In short, financial primitives determine whether information is valuable. In-

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MAV, MC

Marginal Cost

Marginal Action Value
ante notitias

$$\xi = \frac{\bar{\xi}}{1 + \sum_i N^i / \bar{\beta}}$$

$0 \qquad\qquad \xi \qquad\qquad \xi^* = \overline{\xi}$

The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Financial primitives: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 3$, $c = .1$.

Figure 4: **No information acquisition at high cost**

formation is not a good in itself. When the elementary excess return $\overline{\xi}$ drops too low, the marginal action value MAV of a signal cannot reach or exceed marginal cost, and nobody will acquire a signal so that $\xi^* = \overline{\xi}$. This case is depicted in Figure 4 (risky asset supply is reduced by more than half compared to Figure 3). The elementary excess return $\overline{\xi}$ is low if relatively few risky assets are in the market (low $\bar{x}$), or if investors are little risk averse (low A), or when the prior mean-variance ratio of the asset return is relatively high (high $\bar{\beta}$) so that risk matters little compared to payoff. Then investors do not value information enough to buy it.

What if signal cost drops to zero? Even then, there are market conditions where information has no or negative value. Figure 5 depicts a case in which the price of a signal c is zero but information will not be acquired (risky asset supply is reduced to a seventh of the level in Figure 3). When the amount of available information is large already, the price externality that diminishes expected excess return ξ weighs more heavily than the benefits of resolving uncertainty. The marginal action value MAV is strictly negative and investors find additional disclosure undesirable even at zero signal cost.

The marginal action value MAV vanishes as the excess return ξ goes to zero. In this limit, no investor wants to purchase a costly signal. But every investor would accept signals for free. The economy tends to $\xi \to 0$ when no risky assets are supplied to the market ($\bar{x} \to 0$). Similarly, when investors become risk neutral ($A \to 0$), or when the prior variance tends to zero ($\bar{\beta} \to \infty$), then there is no benefit of holding information but also no harm done. Finally, if investors were
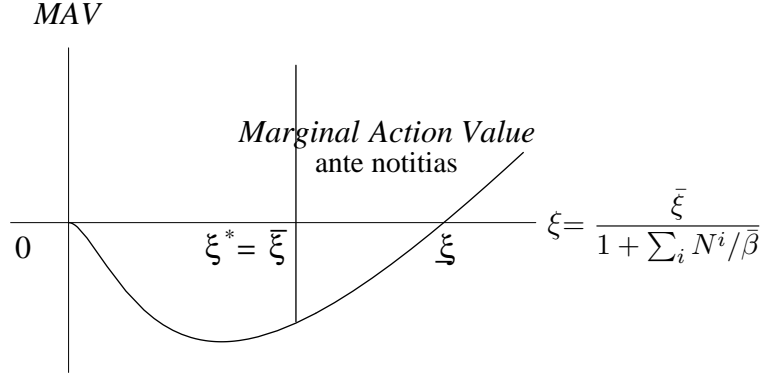
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The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Financial primitives: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 1$.

Figure 5: **Rejection of information beyond transparency limit**

given infinitely many signals for free, all uncertainty would be resolved. Then ξ would also reach zero but the return realization θ would become known with certainty and the previously risky asset would turn into a perfect substitute to the bond. The common cause for information to lose its value in all these cases is that the relative excess return ξ is driven down to zero so that no investor chooses to hold any risky asset. In this limit, information does not have a negative value either. Investors are simply unaffected. If investors don't think at 9am that they will be holding a risky asset at 10am, they know they will never need to act upon information. An infinite amount of information makes investors indifferent to it in the presence of a safe bond.

The following proposition summarizes these insights.

Proposition 4 (RICE). *Under assumptions 1 through 5, information acquisition occurs in RICE under the following conditions.*

- *Investors acquire a strictly positive and finite number of signals in signal market equilibrium if and only if the elementary excess return $\bar{\bar{\xi}}$ strictly exceeds the excess return $\underline{\xi}$ at the transparency limit, where $\underline{\xi}$ solves $\mathrm{MAV}(\underline{\xi}) = 0$.*

- *If the cost of a signal is zero but $R > 0$, then there are two signal market equilibria, one of which involves an infinite amount of freely received signals.*
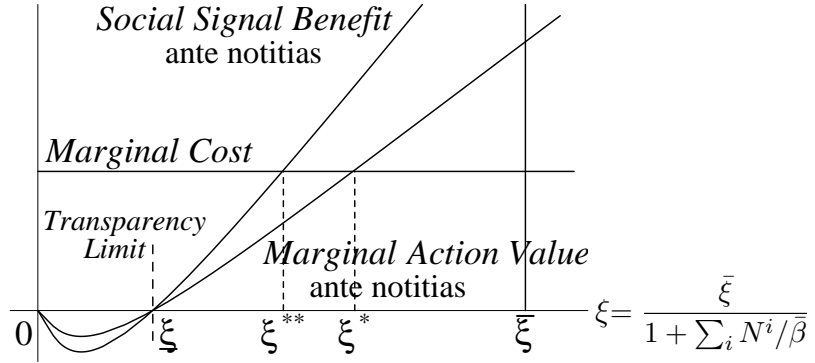
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The expected relative excess return ξ strictly decreases in the number of signals $\sum_i N^i$.
Financial primitives: $A = 2$, $\bar{\alpha} = 1.3$, $\bar{\beta} = 1$, $R = 1.1$, $\bar{x} = 7$, $c = .1$.

Figure 6: **Socially desirable information choice**

RICE does not determine how many signals a single investor holds. In equilibrium, one investor may acquire all $\sum_i N^i$ signals while nobody else buys any signal, or all investors may hold the same number of signals. Signals are public commodities and therefore perfect strategic substitutes under fully revealing price because any fellow investors' signal is as useful (or detrimental) as one's own signal.

4 Welfare and the Transparency Limit

Consider an omniscient social planner who uses a Pareto criterion to judge information allocation in the private market.

Definition 2 (Informational Pareto efficiency) *An allocation of x^{i**} risky assets, b^{i**} riskless bonds, and N^{i**} signals to investors $i = 1, ..., I$ is called informationally Pareto efficient under given financial primitives $(\overline{\xi}, R)$ if there is no other allocation such that all investors are at least as well off and at least one investor is strictly better off.*

It does not matter for this Pareto criterion that information can change from a public good into a public bad. The criterion is conditional on financial primitives. To investigate whether the RICE in section 3 is Pareto efficient, imagine a benevolent social planner who can order every consumer j to buy exactly N^{j**} signals.

The social planner maximizes $\sum_{j=1}^{I} \mathbb{E}_{ante}[U^j]$ with respect to $\{N^1, ..., N^I\}$. Thus, similar to Samuelson's (1954) condition for public good provision, a benevolent social planner's first-order conditions for information allocation are not (12) but instead

$$
-\frac{1}{\mathbb{E}_{ante}[U^{j**}]} \frac{\partial \sum_{k=1}^{I} \mathbb{E}_{ante}[U^{k**}]}{\partial N^k} = -A\frac{R}{1+R}\, c \tag{13}
$$

$$
+\frac{\bar{\alpha}}{\bar{\beta}} \frac{\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}}\left(1 - \frac{1}{1+R}\frac{\xi^2}{(1+\xi)^2}\right) - 1}{1 + \left(\left[(1+\xi)\exp\left\{\frac{-\xi}{1+\xi}\right\}\right]^{\frac{1}{1+R}} - 1\right)\frac{\bar{\xi}}{\xi}} \left(1 + \sum_{k \neq j}^{I} \frac{\mathbb{E}_{ante}[U^{k**}]}{\mathbb{E}_{ante}[U^{j**}]}\right)
$$

for any $j \in 1, ..., I$, written in terms of that investor j's utility. Thus, compared to the privately perceived benefits, the potential social benefits SB that a social planner considers boost the private action value of information MAV by a factor of $1 + (1/\mathbb{E}_{ante}[U^{j**}]) \cdot \sum_{k \neq j}^{I} \mathbb{E}_{ante}[U^{k**}] > 1$. Therefore, if information is a public bad, a benevolent social planner wants to implement an even smaller amount of information than the private market. Because private investors buy no information when information is a public bad, the market equilibrium provides maximal welfare.

On the other hand, if information is a public good under given market conditions, a social planner wants more information to be allocated than markets provide. So, there is a cause for governmental intervention to achieve more transparency. Individual investors do not take into account that their signal acquisition also benefits other investors through revealing price. When information is valuable under given financial primitives, markets allocate (weakly) less information than desirable. In Figure 6, a social planner wants to allocate information so that relative excess return is brought down from around ξ^* to around ξ^{**}. Signals are not divisible, however, and one cannot infer from condition (13) that a social planner wants to implement strictly more information.[6]

Two equilibria exist when information is free. They are Pareto ranked: the equilibrium with finite information yields strictly higher utility for all investors than the equilibrium with infinite information since investors incur utility losses as the excess return ξ moves west from the transparency limit $\underline{\xi}$. Even if signal costs are zero, only the market outcome with finite information is efficient but

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[6]An additional signal can diminish relative excess return ξ so strongly that all investors are worse and not better off. So, discreteness of the number of signals only permits a conditional efficiency statement up to discrete tolerance. In Figure 6, a social planner wants to allocate information so that relative excess return is brought down from around ξ^* to around ξ^{**}. However, if an additional signal makes the implementable level of ξ drop far below ξ^{**}, investors are better off if relative excess return ξ remains at the market equilibrium level around ξ^*.

not the one with infinite information. So, the *natural transparency limit* stands on firm Pareto efficiency grounds even in the extreme. As long as the bond is valuable ($R > 0$), neither markets nor the social planner want to completely resolve uncertainty. Risk-averse investors want to enjoy a positive excess return ξ over opportunity cost. Investors in incomplete markets want a second asset that is not a perfect substitute to the safe bond.

Proposition 5 summarizes the arguments.

Proposition 5 *Under assumptions 1 through 5, the following is true in a RICE.*

- *If $\overline{\xi} \leq \underline{\xi}$, then the equilibrium is informationally Pareto efficient.*

- *If $c > 0$ and at least one signal is acquired in equilibrium, then the equilibrium is not informationally Pareto efficient up to discrete tolerance and results in too little signal provision.*

- *If $c = 0$, then the equilibrium with infinitely much information is not Pareto efficient for $R > 0$, whereas the equilibrium with finite information is informationally Pareto efficient.*

Government cannot dictate information gathering. Aware of the social planner's objective, rational investors will reduce their information acquisition in response to anticipated public disclosure. In the present framework, public disclosure fully crowds out private activity. No matter how many signals private investors would acquire, they know a benevolent social planner will fill in the difference up to x^{i**} (Figure 6). Anticipating this public intervention, rational investors acquire no information.

Government is not omniscient. The Pareto criterion is at best suggestive to judge optimal transparency and public disclosure of information. There is simply no evidence how far the social optimum differs from private market outcomes and how far, or whether at all, the market outcome differs from the transparency limit. Proposition 5 implies, however, that when investors seek information, financial primitives must be such that information is a public good and underprovided. This justifies governmental intervention to improve transparency. Public disclosure of private information—about a project's state, a company's prospects, or a sovereign debtor's internal revenues—may nevertheless be harmful. Public disclosure can push the asset market beyond the transparency limit, where rational investors pay to prevent further information. Reducing the costs of information acquisition, on the other hand, will also result in more transparency but no violation of the transparency limit. So, not only practicability but welfare concerns too suggest that reducing costs of information acquisition is a superior strategy to achieve adequate financial transparency.

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5   Concluding Remarks

Information is a tertiary commodity in financial markets. Investors care for the primary good, consumption. Assets are means to the end of consumption and therefore secondary. Information, in turn, is valuable only if it helps investors improve their asset holdings. This makes information a tertiary commodity. Not surprisingly, the action value of information changes with financial primitives. Giving rational investors a choice of information before they decide on their portfolios shows that there is a natural transparency limit. Public disclosure of information may violate the transparency limit. Therefore, to instill transparency, cutting down costs of information acquisition appears more adequate than disclosure.

Financial information is a public commodity because investors use revealing asset price to update beliefs. This constitutes a rationale for intervention to instill transparency. The paper considered the benchmark of a fully revealing asset price. A partially revealing price does not uproot the basic transmission mechanism of private information through price; partial revelation just makes price less informative. Under partially revealing price, the transparency limit conceivably moves to a higher number of signals. In fact, partially revealing price makes it rational for investors to acquire duplicates of identical signals. This paper considered the benchmark of Walrasian equilibrium and did not admit strategic investors to Wall Street. Strategic investors could manipulate asset demands to prevent perfect revelation of their information. The natural transparency limit, I conjecture, will shift even farther but not fade away.

Appendix

This appendix lists properties of Poisson and gamma distributions, and uses them for proofs to propositions and statements in the text. Derivations are based on moment generating functions (MGF).

A Properties of the Poisson-gamma distribution

Fact 1 in the text states how Poisson signals update beliefs about gamma distributed returns. Several further properties are useful.

A.1 Poisson signals

Poisson distributed signals $S_n^i|\theta \overset{i.i.d.}{\sim} \mathcal{P}(\theta)$ have a density

$$f\left(s_n^i\,|\theta\right) = \begin{cases} \exp\{-\theta\}\,\theta^{s_n^i}/s_n^i! & \text{for } s_n^i > 0 \\ 0 & \text{for } s_n^i \leq 0 \end{cases}$$

Fact 2 (Poisson MGF). *The MGF of a Poisson signal is*

$$M_{S|\theta}(t) = \exp\{\theta(\exp\{t\} - 1)\}.$$

Proof. Casella and Berger (1990). ∎

Fact 3 (Sum of Poisson signals). *The sum of N independently Poisson distributed signals with a common mean and variance θ, $S_1 + \ldots + S_N$, has a Poisson distribution with parameter $N\theta$.*

Proof. The distribution of the sum of N independent Poisson variables is the product $\Pi_{n=1}^N f\left(s_n^i\,|\theta\right) = \exp\{-N\theta\}\,\theta^{\sum_{n=1}^N s_n^i}/\sum_{n=1}^N s_n^!$, a Poisson distribution with parameter $N\theta$. ∎

A.2 Gamma returns

Given an individual investor i's information set $\{\alpha^i, \beta^i\}$, the risky asset return is distributed $\theta \sim \mathcal{G}(\alpha^i, \beta^i)$ so that its density is

$$\pi\left(\theta\,|\alpha^i, \beta^i\right) = \begin{cases} (\beta^i)^{\alpha^i}\,\theta^{\alpha^i-1}\,\exp\{-\beta^i\theta\}/\Gamma(\alpha^i) & \text{for } \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

where the gamma function is given by $\Gamma(\alpha^i) \equiv \int_0^\infty z^{\alpha^i-1}e^{-z}\,\mathrm{d}z$. The two parameters α^i and β^i must be positive.

Fact 4 (Gamma MGF). *The MGF of a gamma distributed return is*

$$M_{\theta|\alpha^i,\beta^i}(t) = \left(\frac{\beta^i}{\beta^i - t}\right)^{\alpha^i}.$$

Proof. Casella and Berger (1990). ∎

A.3 Poisson-gamma signal-return distribution

The following property of the Poisson-gamma distribution pair proves useful for the derivation of *ante notitias* expected indirect utility.

Fact 5 (Expected signal effect on utility). *For two arbitrary constants B and ξ, $\bar{N}$ Poisson distributed signals $S_1, ..., S_{\bar{N}}$ and a conjugate prior gamma distribution of their common mean θ, the following is true:*

$$\mathbb{E}_{ante}\left[(1+\xi)^{-B \cdot \sum\limits_{n=1}^{\bar{N}} s_n} \cdot \exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B \cdot \sum_{n=1}^{\bar{N}} s_n\right\}\right]$$

$$= (1+\xi)^{\bar{\alpha}B} \exp\left\{\bar{\alpha}\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\left(1 + \left[(1+\xi)^B \exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}B\right\} - 1\right]\frac{\beta}{\bar{\beta}}\right)^{-\bar{\alpha}},$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the parameters of the prior gamma distribution of θ, and $\beta = \bar{\beta} + \bar{N}$ is the according parameter of the post notitias *distribution.*

Proof. By iterated expectations $\mathbb{E}_{ante}[\cdot] = \mathbb{E}_\theta[\mathbb{E}[\cdot|\theta]]$. The 'inner' expectation $\mathbb{E}[\cdot|\theta]$ is equal to

$$\mathbb{E}[\cdot|\theta] = \sum_{(\sum_{n=1}^{\bar{N}} s_n)=0}^{\infty} (1+\xi)^{-B\sum\limits_{n=1}^{\bar{N}} s_n} \exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\sum_{n=1}^{\bar{N}} s_n\right\} f\left(\sum_{n=1}^{\bar{N}} s_n\right)$$

$$= \exp\left\{-\bar{N}\theta\left(1 - (1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)\right\},$$

because the sum $\sum_{n=1}^{\bar{N}} s_n$ is Poisson distributed with mean $\bar{N}\theta$ (fact 3). Thus, by the MGF of a gamma distribution (fact 4),

$$\mathbb{E}_{ante}[\cdot] = \mathbb{E}_\theta\left[\exp\left\{-\theta\left(1 - (1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)(\beta - \bar{\beta})\right\}\right]$$

$$= (\bar{\beta})^{\bar{\alpha}}\left(\bar{\beta} + \left(1 - (1+\xi)^{-B}\exp\left\{-\frac{\xi(\omega^i-1)}{1+\xi}B\right\}\right)(\beta - \bar{\beta})\right)^{-\bar{\alpha}}.$$

since $\bar{N} = \beta - \bar{\beta}$ (fact 1). Simplifying the last term and factoring out $(1+\xi)^B \exp\{\frac{\xi(\omega^i-1)}{1+\xi}B\}$ proves fact 5. ∎

B Optimality conditions and portfolio value

Define $t \equiv -Ax^i \in (-\infty, 0)$ for the moment generating function (MGF) $M_{\theta|\mathcal{F}^i}(t)$, where the information set $\mathcal{F}^i \equiv \{\alpha^i, \beta^i; \bar{x}, I; RP\}$. Maximizing (3) over x^i and b^i for CARA (assumption 4 and 2) yields the first-order conditions

$$\frac{P}{\rho^i} = H^i \, M'_{\theta|\mathcal{F}^i}(t) \quad \text{and} \quad \frac{1}{\rho^i R} = H^i \, M_{\theta|\mathcal{F}^i}(t), \tag{B.1}$$

where $H^i \equiv \exp\{-A[(1+R)b^i + Px^i - W_0^i - cN^i]\}$. Note that H^i, W_0^i, C_1^i and C_0^i are functions of $\mathcal{F}^i$ since RP depends on $\mathcal{F}^i$.

With the definition of H^i, the optimal portfolio value can be written

$$
\begin{aligned}
b^i + Px^i &= \frac{1}{1+R}\left(W_0^i - cN^i + RP\,x^i - \frac{1}{A}\ln H^i\right) \\
&= \frac{1}{1+R}\left[b_0^i + RP(x_0^i/R + x^i) + \frac{1}{A}\ln \rho^i RM_{\theta|\mathcal{F}^i}(-Ax^i) - cN^i\right],
\end{aligned}
\tag{B.2}
$$

where the second line follows from the bond first-order condition in (B.1).

The matrix of cross-derivatives for the two assets b^i and x^i reflects the second-order conditions:

$$\mathbf{B} = -A^2 \rho^i \exp\{-ARb^i\} \begin{vmatrix} R(1+R)M_{\theta|\mathcal{F}^i}(t) & \cdot \\ (1+R)M'_{\theta|\mathcal{F}^i}(t) & PM'_{\theta|\mathcal{F}^i}(t) + M''_{\theta|\mathcal{F}^i}(t) \end{vmatrix} \tag{B.3}$$

by (B.1). It is negative definite for the gamma distribution (fact 4).

Using the MGF of the gamma distribution (fact 4) in first-order condition (B.1) yields risky asset demand (4) in the text.

C Bond return response to stock information

Taking logs of both sides of the bond first-order condition in (B.1) yields

$$A(1+R)b^i - Ab_0^i + AP(x^i - x_0^i) = \ln[\rho^i RM_{\theta|\mathcal{F}^i}(-A\,x^i)] + AcN^i,$$

a permissible operation since $\rho^i, R, M_{\theta|\mathcal{F}^i}(\cdot) > 0$ by their definitions. Summing up both sides over investors i and dividing by their total number yields

$$AR\bar{b} - \ln \rho^i R - \ln M_{\theta|\mathcal{F}^i}(t) - Ac\sum_{k=1}^{I} N^k/I = 0 \tag{C.1}$$

where $\bar{b} \equiv \sum_{i=1}^{I} b_0^i/I$ is the average initial bond endowment per investor and $t \equiv -A\bar{x}$. Equation (C.1) implicitly determines the gross bond return R. *Post notitias*, $M_{\theta|\mathcal{F}^i}(t)$ and R respond to the signal realization. Define $\bar{s} \equiv \sum_{k=1}^{I}\sum_{n=1}^{N^k} s_n^k$.

Applying the implicit function theorem to (C.1) for the MGF of the gamma distribution $M_{\theta|\alpha,\beta}(t) = [\beta/(\beta - t)]^\alpha$ yields

$$\frac{dR}{d\bar{s}} = -\frac{\ln(1+\xi)}{A\bar{b} - 1/R}$$

for $\alpha = \bar{\alpha} + \bar{s}, \beta = \bar{\beta} + \sum_{k=1}^{I} N^k$ by (6) and $\xi = A\bar{x}/\beta$ given $\sum_{k=1}^{I} N^k$. The bond return falls in response to a favorable signal realization $\bar{s}$ iff $\bar{b} > 1/(AR)$. So, in principle, R too is a function of the signal realization $\bar{s}$. For large bond endowments $\bar{b}$, however,

$$\lim_{\bar{b}\to\infty} dR/d\bar{s} = 0.$$

Similarly, $dR/d\bar{s} = 0$ for $\xi = \bar{x} = 0$.

D *Ante notitias* expected indirect utility

For a gamma distributed asset return, *post notitias* expected indirect utility (10) becomes

$$\mathbb{E}^i\left[U^{i*}\right] = -\delta^i \exp\left\{-A\frac{R}{1+R}(W_0^i - cN^i)\right\} \exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}\right\}^{-\frac{\alpha^i}{1+R}}(1+\xi)^{-\frac{\alpha^i}{1+R}} \quad \text{(D.1)}$$

where $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$ is the relative endowment of investors with the risky asset, and $\xi \equiv A\bar{x}/\beta$. With fact 5 at hand, one can set $B \equiv 1/(1+R)$ (by assumption 5) and obtains *ante notitias* expected utility (11) for $\omega^i = 0$.

E Monotonic action value of information

Define the relative endowment of investors with the risky asset as $\omega^i \equiv x_0^i/\bar{x} \in [0, I]$. The expected relative excess return ξ is bounded by $\xi \in (0, \bar{\xi}]$. Under assumptions 1 through 5, the marginal action value $MAV(\xi, \omega^i)$ of a signal is $MAV(\xi, \omega^i) = g(\xi, \omega^i)/h(\xi, \omega^i)$ with

$$m(\xi, \omega^i) \equiv \left[(1+\xi)\exp\left\{\frac{\xi(\omega^i-1)}{1+\xi}\right\}\right]^{\frac{1}{1+R}}, \quad \text{(E.1)}$$

$$h(\xi, \omega^i) \equiv 1 + \left[m(\xi, \omega^i) - 1\right]\frac{\bar{\xi}}{\xi}, \quad \text{(E.2)}$$

$$g(\xi, \omega^i) \equiv -\frac{\xi^2}{\bar{\xi}}\frac{\partial h(\xi, \omega^i)}{\partial \xi} = m(\xi, \omega^i)\left(1 - \frac{1}{1+R}\frac{\xi(\xi+\omega^i)}{(1+\xi)^2}\right) - 1. \quad \text{(E.3)}$$

Proposition 3 is a special case for $\omega^i = 0$. The general proof for $\omega^i \in \mathbb{R}_+$ proceeds in four steps.

First, claim 1 sates useful properties of $m(\xi, \omega^i)$ for the discussion of $g(\xi, \omega^i)$ and $h(\xi, \omega^i)$. Second, claim 2 establishes that the numerator $g(\xi, \omega^i)$ strictly increases in ξ for $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that it is not bounded above. So, the numerator boosts the marginal action value of information $MAV(\xi, \omega^i)$ higher and higher as ξ rises. Third, claim 3 establishes that the denominator $h(\xi, \omega^i)$ is bounded below and above in the positive range, and that it strictly decreases in ξ iff the numerator is strictly positive. So, the denominator cannot explode and boosts the marginal action value $MAV(\xi, \omega^i)$ higher where the marginal action value $MAV(\xi, \omega^i)$ is positive. The latter two claims imply that $MAV(\xi, \omega^i)$ strictly increases in ξ for $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that $MAV(\xi, \omega^i)$ is unbounded for arbitrarily large ξ. So, fourth and last, $MAV(\xi, \omega^i)$ ultimately attains strictly positive values and continues to strictly increase in that positive range.

Claim 1 $m(\xi, \omega^i)$ *strictly increases in* ω^i; $m(0, \omega^i) = 1$; *and* $m(\xi, \omega^i) > 1$ *for any* $\xi > 0$, $\omega^i \geq 0$ *and* $R \in (0, \infty)$.

Proof. By (E.1), $\partial m(\xi, \omega^i)/\partial \xi = m(\xi, \omega^i)\xi/(1 + \xi) > 0$, which establishes the first part of the claim.

Taking natural logs of both sides of (E.1) is permissible since $m(\xi, \omega^i) > 0$ and shows that $m(\xi, \omega^i) \geq 1$ iff $\ln(1 + \xi) \geq -\xi(\omega^i - 1)/(1 + \xi)$. Since $m(\xi, \omega^i)$ strictly increases in ω^i, consider $\omega^i = 0$. So, $m(\xi, 0) \geq 1$ iff $\ln(1 + \xi) \geq \xi/(1 + \xi)$. Note that equality holds at $\xi = 0$ but $\ln(1 + \xi)$ increases strictly faster in ξ than $\xi/(1 + \xi)$ increases in ξ for any $\xi > 0$. So, $m(\xi, 0) \geq 1$. Since $m(\xi, \omega^i)$ strictly increases in ω^i, $m(\xi, \omega^i) \geq 1$. ∎

Claim 2 $g(\xi, \omega^i)$ *strictly increases in* ξ *iff* $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$. *In addition,* $\lim_{\xi \to 0} g(\xi, \omega^i) = 0$ *and* $\lim_{\xi \to \infty} g(\xi, \omega^i) = +\infty$.

Proof. The first derivative of $g(\xi, \omega^i)$ with respect to ξ is

$$\frac{\partial g(\xi, \omega^i)}{\partial \xi} = \frac{\xi}{(1+R)^2(1+\xi)^4} m(\xi, \omega^i) \left[R(\xi + \omega^i)^2 - (1+R)(\omega^i - 1)^2 \right].$$

So, $\partial g(\xi, \omega^i)/\partial \xi = 0$ at $\xi = 0$ and at $\xi = |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ (the negative root is ruled out by $\xi \geq 0$). Evaluating $\partial g(\xi, \omega^i)/\partial \xi = 0$ around the zero points shows that $g(\xi, \omega^i)$ strictly decreases in ξ if $\xi \in (0, |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i)$ and strictly increases if $\xi \in (|\omega^i - 1|\sqrt{1 + 1/R} - \omega^i, \infty)$.

$\lim_{\xi \to 0} g(\xi, \omega^i) = m(0, \omega^i) - 1 = 0$ by claim 1. $\lim_{\xi \to \infty} g(\xi, \omega^i) = -1 + \lim_{\xi \to \infty} \exp\{\xi/(1 + R)\} = +\infty$ since $R \in (0, \infty)$. ∎

Claim 2 implies that there must be a $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ that uniquely solves $g(\underline{\xi}^i, \omega^i) = 0$ because $g(\xi, \omega^i)$ strictly decreases as long as $\xi < |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$) but subsequently strictly increases in ξ.

Claim 3 $h(\xi, \omega^i)$ *strictly decreases in ξ iff* $g(\xi, \omega^i) > 0$. $h(\xi, \omega^i)$ *is bounded in* $h(\xi, \omega^i) \in (1, h(\underline{\xi}^i, \omega^i)]$ *for* $\xi \in (0, \overline{\xi}]$ *and* $R \in (0, \infty)$, *where* $h(\underline{\xi}^i, \omega^i) > 1$, $\overline{\xi}$ *is given by (9) and* $\underline{\xi}^i$ *solves* $g(\underline{\xi}^i, \omega^i) = 0$.

Proof. By (E.3), $\partial h(\xi, \omega^i)/\partial \xi < 0$ iff $g(\xi, \omega^i) > 0$. So, $h(\xi, \omega^i)$ attains its global maximum at $\underline{\xi}^i$, which solves $g(\underline{\xi}^i, \omega^i) = 0$, and $h(\xi, \omega^i)$ attains its global minimum either for $\xi \to 0$ or for $\xi \to \infty$. By L'Hôpital's rule, $\lim_{\xi \to 0} m(\xi, \omega^i)/\xi - 1/\xi = 0$ so $\lim_{\xi \to 0} h(\xi, \omega^i) = 1$. Similarly, for $R \in (0, \infty)$, $\lim_{\xi \to \infty} h(\xi, \omega^i) = 1$ while $\lim_{\xi \to \infty} h(\xi, \omega^i) = 1 + \overline{\xi} \exp\{\omega^i - 1\}$ for $R \to 0$. This establishes that $h(\xi, \omega^i) \in (1, h(\underline{\xi}^i, \omega^i)]$ for $\xi \in (0, \overline{\xi}]$. ∎

Claims 2 and 3 imply that $MAV(\xi, \omega^i)$ strictly increases in ξ for $\xi > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ and that $MAV(\xi, \omega^i)$ is unbounded for arbitrarily large ξ. So, $MAV(\xi, \omega^i)$ attains strictly positive values if and only if $\xi > \underline{\xi}^i$, where $\underline{\xi}^i > |\omega^i - 1|\sqrt{1 + 1/R} - \omega^i$ solves $g(\underline{\xi}^i, \omega^i) = 0$, and $\underline{\xi}^i \in (0, \infty)$ is independent of $\overline{\xi}$ and unique given $R \in (0, \infty)$.

The paper considered a sole owner j of the risky project with $\omega^j = I$ and argued that her incentives for information acquisition can be ignored. Claim 4 confirms that a sole owner may not value signals: the marginal action value of information approaches negative infinity as the relative risky asset endowment ω^j (the project size $I\bar{x}$) increases for a given average endowment $\bar{x}$.

Claim 4 $g(\xi, \omega^i)$ *strictly decreases in ω^i iff* $\omega^i > 1 + R(1 + \xi)$. *In addition,* $\lim_{\xi \to \infty} g(\xi, \omega^i) = -\infty$.

Proof. The first derivative of $g(\xi, \omega^i)$ with respect to ω^i is

$$\frac{\partial g(\xi, \omega^i)}{\partial \omega^i} = \frac{1}{1 + R} \frac{\xi^2}{(1 + \xi)^2} m(\xi, \omega^i) \left[R(1 + \xi) - (\omega^i - 1) \right],$$

where $m(\xi, \omega^i)$ is given by (E.1). So, $\partial g(\xi, \omega^i)/\partial \omega^i = 0$ at $\omega^i = 1 + R(1 + \xi)$. Evaluating $\partial g(\xi, \omega^i)/\partial \xi = 0$ around this unique zero point shows that $g(\xi, \omega^i)$ strictly increases in ω^i if $\omega^i \in [0, 1 + R(1 + \xi))$ and strictly increases if $\omega^i \in (1 + R(1 + \xi), I]$. So, $\lim_{\omega^i \to \infty} g(\xi, \omega^i) = -\infty$ for $R \in (0, \infty)$ and $\xi \in (, \overline{\xi}]$. ∎

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