Choosing the Quality of the Fit Between Complementary Products

Maxim Sinitsyn, UCSD

Abstract

In this paper, I examine how firms should position their complementary products. I assume that there are two competing firms, each producing two complementary products. Each firm decides whether to employ strategies that enhance the quality of the fit (the degree of complementarity) between its pair of complementary products before competing in prices. The consumers have heterogeneous tastes for the four possible bundles. They are willing to pay a price premium in order to purchase a bundle from the same firm if this firm chose to make such bundle more attractive. I find that increasing the degree of complementarity between a firm's complementary products intensifies price competition and often leads to smaller profits. Only when complementarity-enhancing strategies significantly increase the demand for a firm's matching bundle, does the firm benefit from employing them. The highest profits for both firms are obtained when both firms do not employ complementarity-enhancing strategies. Deteriorating the quality of the fit between one's own and a rival's complementary products is never profitable.

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1 Introduction

One of the major decisions faced by firms producing products in complementary categories is how to position these products. There are various strategies that can influence consumers' perception of the quality of the fit (the degree of complementarity) between the firm's complementary products. The firm can give its complementary products the same brand name (umbrella branding).[1] It can adjust the design of its complementary products so that they look more uniform and generate the appearance of a better match.[2] A firm can allow for the same accessory to be used with its various complementary electronic devices[3] or make the connection between these devices more convenient.[4] The manufacturer can negotiate the shelf space with the retailers so that its complementary products are located next to each other, reducing the consumers' search costs (Dawar and Stornelli 2013). Another option is to run advertisements promoting the joint consumption of its products[5] or make the warranty on one product be conditional on the consumer using the same firm's complementary product.[6] In this paper, I examine how this choice of the degree of complementarity affects the pricing strategies of the competing firms and show that increasing the degree of complementarity intensifies price competition and, under certain conditions, decreases

[1] It has been established experimentally and empirically that consumers who purchase a pair of complementary products receive a larger utility if these products share the same brand name (Simonin and Ruth 1995, Ma et al. 2012, Sinitsyn 2012, Rahinel and Redden 2013).

[2] This assumption is a reflection of the Diderot effect—"a force that encourages the individual to maintain a cultural consistency in his/her complement of consumer goods" (McCracken 1988, p. 123). This effect is named after the French philosopher Denis Diderot (1713-1784). In his essay "Regrets on Parting with My Old Dressing Gown" he describes receiving a new dressing gown as a present and consequently replacing all the elements of his study to match the elegance of the gown.

[3] For example, one charger can be used for both iPad and iPhone, while a tv and a dvd player produced by the same firm can share a remote.

[4] For example, AirDrop allows for convenient file sharing between various Apple devices.

[5] For example, Betty Crocker ran a TV commercial showing a cake made with Betty Crocker's complementary products: Devil's Food Cake Mix and Chocolate Frosting (2009BettyC 2009).

[6] For example, a lifetime warranty on BEHR paint requires applying the product according to the label directions which, in turn, suggest using BEHR primer beforehand (Premium Plus ® Interior Ceiling Paint n.d.).

the firms' profits.

In my model, there are two firms, each selling products in two complementary categories. The consumers can mix and match between the firms' products and have heterogeneous tastes for the four possible bundles, selecting the one that offers them the highest utility. Following the set-up of network models of Matutes and Regibeau (1988, 1992), I assume that consumers are uniformly distributed on the unit square with one firm located at the bottom-left corner and another firm located at the upper-right corner. The firms interact over two periods. In the first period, each firm chooses the degree of complementarity between its products, and in the second period, they compete in prices. The degree of complementarity measures the additional utility consumers receive when they purchase two complementary products that are produced by the same firm.

The framework of my paper also contributes to the large body of literature on compatibility decisions in networks. It is well established that, even in the absence of network externalities, when facing a choice between full compatibility and incompatibility, the competing firms set higher prices and obtain higher profits under full compatibility (Matutes and Regibeau 1988, Economides 1989). Consistent with these findings, I employ the model in which the firms operate under full compatibility, i.e., the consumers can assemble their own system by combining the components produced by the rival firms.

The novel feature of my model is that I allow for the matching system composed of the components produced by the same firm to be valued higher than the mixed system. The firm achieves this by either improving the quality of the fit between its components or deteriorating the quality of the fit between its own and the rival's components. The examples of the former strategy are given in the beginning of this introduction. One example of the latter practice was the effort by Microsoft during the first browser war to discourage the users of its Windows platform from using the competitor's Netscape Navigator instead of Microsoft's Internet Explorer. Problems experienced by Windows consumers during the installation of the Netscape Navigator fulfilled the promise of Microsoft's vice president "to make the use of any browser other than Internet Explorer on Windows "a jolting experience" (Finding of Facts, *United States v. Microsoft Corp.*, No. 172).

I solve for the subgame perfect Nash equilibrium and find that a firm's decision to increase the degree of complementarity between its products rests on the relative size of two effects—a direct effect resulting from an increase in consumer demand and a strategic effect resulting from the competitive response of the rival firm, which decreases its prices. If the increase in consumer utility for a matching bundle is significant (the degree of complementarity is high), the direct effect dominates, and in equilibrium both firms employ complementarity-increasing strategies. If, on the other hand, the consumers do not gain much extra utility from purchasing the matching bundle (the degree of complementarity is small), then the strategic effect is stronger, and in equilibrium both firms choose not to use strategies that increase the degree of complementarity.

The intuition for my findings stems from the fact that when consumers do not place a large premium on matching bundles, a lot of them will end up buying mixed bundles. The firms then have less incentive to undercut each other since a decrease in price of one product made by a firm will increase demand for all bundles containing this product, including the bundle with the rival's product. Thus, some of the benefits of a price cut go to the rival, leading the firms to be more reluctant to decrease prices. On the other hand, a large degree of complementarity between the products leads to most of the consumers buying matching bundles. Correspondingly, more of the benefits from a price cut go to the firm that makes it. The incentives to decrease the prices are higher, so the firms' profits decrease. If the firms do not gain much additional demand by increasing the degree of complementarity, they should refrain from adopting this strategy since intensified price competition will decrease their profits. Only when the demand-increasing benefits of complementarity-enhancing strategies are significant do the firms benefit from employing them.[7]

I also find that the related strategy of decreasing the degree of complementarity between one's own and the rival's components is never profitable. Such strategy does not increase demand since a firm which implements it does not capture all the consumers who shift away from mixed bundles. Instead, some of these consumers choose to buy the rival's matching bundle. With more consumers buying matching bundles, price competition intensifies. In the absence of demand-increasing benefits, these lower prices decrease the firm's profit.

2 Model

I consider two firms, A and B, each producing two complementary products. The set of consumers has measure one, and each consumer purchases both complementary products, choosing among four possible bundles: AA, AB, BA, and BB. I model consumer heterogeneity as in Matutes and Regibeau (1988, 1992). Consumers are uniformly distributed on the unit square. Firm A is located at the bottom-left corner, whereas firm B is located at the upper-right corner. A consumer with preferences depicted by a point with the coordinates (x, y) is $x_A = x$ units away from firm A's first component and $y_A = y$ units away from firm A's second component. The same consumer

[7] A similar intuition is found in a price discrimination model of Liu and Serfes (2004) who show that when firms can not separate well the consumers into different groups, the benefits from increased consumer segmentation are outweighed by the costs of increased competition, and the firms choose not to price discriminate. Only with a significant level of consumer segmentation do the firms adopt price discrimination.

is $x_B = 1 - x$ and $y_B = 1 - y$ units away from firm B's first and second components respectively.

The game proceeds in two stages. In stage one, both firms simultaneously choose the degree of complementarity of their products, d_A and d_B.[8] This degree of complementarity is added to the utility of consumers who choose to buy a matching bundle, i.e., a consumer who chooses to purchase bundle ii gets an additional utility of d_i.[9] In Section 6, I analyze a variant of this model in which instead of improving the quality of the fit between own components, the firms deteriorate the quality of the fit between its own and the rival's components.

In the second stage, both firms simultaneously choose prices $p_{k,i}$ for their products, where $k \in \{1, 2\}$ denotes the product category and $i \in \{A, B\}$ denotes the firm.

A consumer who buys bundle ij gets a utility of

$$U_{ij} = S - t(x_i + y_j) - p_{1,i} - p_{2,j} + d_i \cdot I(i = j), \tag{1}$$

where S is the base utility level, x_i and y_j are the distances from the consumer to the first product of firm i and the second product of firm j, t is the transportation cost per unit of distance, and I is the indicator function that is equal to 1 if the consumer buys a matching bundle. The consumers buy the bundle which gives them the highest utility. I assume that S is large enough so that the market is covered (all consumers make a purchase).

While d_i serves a similar role to the bundle discount in the consumer's utility function (both

[8] An alternative setup that does not require the firms to be able to modify the degree of complementary of their existing products moves this decision to the production stage. For example, a firm can have blueprints or patents for two sets of products, X_i and Y_i. Within each set, the products have identical characteristics except for the degree of complementarity with the products from the complementary set. Choosing two products from these sets is then equivalent to choosing d_i in my model. I thank an anonymous referee for raising this issue.

[9] I assume that the choice of d_i does not affect the location of the products on the unit square, i.e., the degree of complementarity is independent of the other characteristics that impact consumer preferences. As the examples in the introduction show, this is often a realistic assumption.

increase the utility if the consumer purchases a matching bundle), this model is not equivalent to the mixed bundling model of Matutes and Regibeau (1992). First, d_i and a bundle discount have different effects on the firm's profit—offering a bundle discount decreases the firm's revenue from matching bundle purchases, while setting a positive degree of complementarity does not decrease the firm's revenue.[10] Second, the two models differ in the timing of the decisions. Sequential decision making is a natural choice for the current model—first, a firm commits to a degree of complementarity between its products and then chooses prices. However, it is unlikely that a firm would commit to a bundle discount before setting the prices for the individual products. Thus, a simultaneous choice of a bundle discount and individual prices is a more appropriate modeling choice for the mixed bundling models.

I will use backward induction to find the subgame perfect Nash equilibrium for this game. In the next section, I proceed by solving for the equilibrium prices that the firms set in the second stage of the game given d_A and d_B chosen in the first stage.

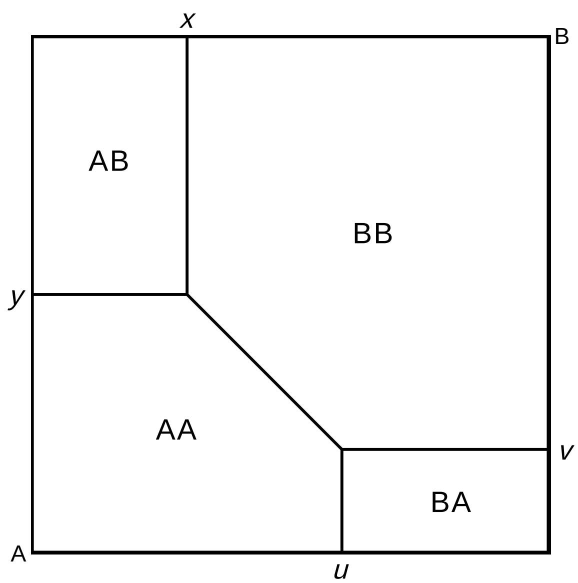
3 Second Stage Competition in Prices

Given the degrees of complementarity d_A and d_B chosen in the first stage, the firms choose prices $p_{1,A}$, $p_{1,B}$, $p_{2,A}$, and $p_{2,B}$ in the second stage. In the main body of the paper, I will consider the case of relatively small values of d_A and d_B with both of them less than or equal to t, while the case of larger values of d_A and d_B is discussed in Section 5. The reason for this focus is that smaller values of d_A and d_B lead to a richer, more realistic pattern of consumer demands with positive demand for all four bundles, which is also what is observed in reality. The larger values

[10] In Section 4, I discuss the issue of a firm having to pay in order to increase the degree of complementarity between its products.

of d_A and d_B lead to the purchase of only matching bundles.[11] Figure 1 shows the division of the consumers between the four different bundles.

Figure 1: Demand for Different Bundles



The following proposition summarizes the firms' optimal prices and profits in the second stage.[12]

Proposition 1 *If both d_A and d_B are smaller than t, then in the second stage, the firms charge prices $p_A = p_{1,A} = p_{2,A} = \frac{d_A - d_B}{6} + \frac{2t^2}{2t+d_A+d_B}$ and $p_B = p_{B,1} = p_{B,2} = \frac{d_B - d_A}{6} + \frac{2t^2}{2t+d_A+d_B}$. The firms earn profits $\pi_A = \frac{\left(d_A^2 - d_B^2 + 2td_A - 2td_B + 12t^2\right)^2}{72t^2(2t+d_A+d_B)}$ and $\pi_B = \frac{\left(d_B^2 - d_A^2 + 2td_B - 2td_A + 12t^2\right)^2}{72t^2(2t+d_A+d_B)}$.*

We can now examine the change in these profits if one of the firms, say firm A, increases the degree of complementarity between its products, d_A, whereas the second firm's degree of

[11] In addition, it is likely that at least for some categories in the real world, the degree of complementarity d is smaller than the measure of consumer heterogeneity t. For example, rough computations (available from the author) based on the estimation of demands for cake mix and cake frosting from Ma et al. (2012) (Table 6), reveal the estimate of t to be about 2.5 times larger than the estimate of d.

[12] All proofs are in the Appendix.

complementarity, d_B, is held constant. An increase in d_A has two opposing effects on the profit function of firm A: a direct effect resulting from the expansion of demand and a strategic effect resulting from the competitive response of firm B, which decreases its prices. We will examine each of these effects in turn starting with the direct effect.

When firm A increases d_A, its demand expands as some consumers who bought a bundle different from AA, but who are located close to the boundary with region AA, switch to buying bundle AA. The demand for one of firm A's products, say in the first category, increases due to the consumers who switch from BA to AA and the consumers who switch from BB to AA. Consumers who originally purchased bundle AB already bought firm A's product in the first category; therefore, they do not contribute to the demand expansion in the first category. This means that the effect of d_A on the expansion of demand in the first category is larger when the boundary between region AA and regions BA and BB is larger.[13] Similarly, the effect of d_A on the expansion of demand in the second category is larger when the boundary between region AA and regions AB and BB is larger. Taken together, an increase in d_A is more effective when the boundary between regions AA and BB is large since the switch from BB to AA contributes to an increase in demand in both categories. The boundary between regions AA and BB is large when regions AA and BB themselves are large, which happens when the degrees of complementarity, d_A and/or d_B, are large. Therefore, when d_A increases, a direct effect resulting from the expansion of demand always increases the profit of firm A, and this effect is larger for larger values of d_A or d_B.

The second, strategic effect, stems from firm B's attempt to regain some of the lost market

[13] The boundary between AA and BB is a line. By "a larger boundary," I mean that this line is longer.

share by cutting its prices in response to an increase in d_A. The aggressiveness of firm B's response depends on the values of d_A and d_B. From the formula for p_B in Proposition 1, we note that the first term, $\frac{d_B - d_A}{6}$, always decreases by the same amount (one-sixth of a change in d_A) and does not depend on the initial levels of d_A and d_B. However, an increase in d_A has a larger effect on the second term, $\frac{2t^2}{2t+d_A+d_B}$, when both d_A and d_B are small. Intuitively, when firm B decreases the price of its product, say in the first category, it gains three different types of consumers: those who switch from AB to BB, those who switch from AA to BA, and those who switch from AA to BB. Of these three types, the latter is the most valuable to firm B since there is a positive spillover of demand into the second category. Therefore, the price cut by firm B is more effective the larger the group of consumers who switch from AA to BB is, i.e., the larger regions AA and BB and the boundary between them are. When d_A and d_B are small, regions AA and BB are also relatively small, and the price cut by firm B is less effective. Therefore, firm B has to cut prices more deeply in response to an increase in d_A. When d_A and d_B are large, the price cut by firm B is more effective; therefore, firm B is less aggressive in its response to an increase in d_A.

In summary, an increase in d_A leads to two opposing effects on firm A's profit. A direct effect resulting from demand expansion increases π_A. This effect increases in magnitude as d_A and d_B increase. A strategic effect resulting from firm B decreasing its prices decreases π_A. This effect decreases in magnitude as d_A and d_B increase. In the next section, I will examine the overall impact of these effects on the profit function and find the firms' optimal choices of the degrees of complementarity.

4 First Stage Choice of the Degree of Complementarity

In the first stage of the game, both firms simultaneously choose their degrees of complementarity, d_A and d_B. I will consider first the case of a binary choice of d_i and then examine the case of d_i being a continuous variable. A binary choice of d_i describes well any marketing strategy that discretely raises the degree of complementarity, such as the use of umbrella branding in the firm's brand architecture.

I assume that, in the absence of such a strategy, the degree of complementarity perceived by the consumers is equal to zero. Then, firm i can either increase the degree of complementarity between its products, $d_i = d$, or keep it at zero, $d_i = 0$, I assume that increasing the degree of complementarity is costless for the firm. This assumption is made to keep the focus of the exposition on the pricing strategies of the firms. In addition, marketing strategies that increase the degree of complementarity, such as the choice of umbrella branding, often do not impose additional costs on a firm. In fact, if a firm utilizes economies of scope in advertising or in production, it might actually decrease its costs by using umbrella branding. Finally, introducing the cost of d into the model is straightforward and has intuitive consequences—an increase in this cost decreases the likelihood of using complementarity-enhancing strategies.

Using the profit functions given in Proposition 1, it is possible to calculate the profits for any choice of d_A and d_B. With each of the firms having a binary choice of the degree of complementarity, the game in the first stage can be represented by a 2×2 payoff matrix shown in Table 1.

The Nash equilibria in this game are summarized in the following proposition.

Proposition 2 *In the Nash equilibrium of the first stage of the game, for the small values of d*

Table 1: Payoff Matrix for the First Stage Choice of Degrees of Complementarity

	$d_B = 0$	$d_B = d$
$d_A = 0$	$t; t$	$\frac{(-d^2-2td+12t^2)^2}{72t^2(2t+d)}; \frac{(d^2+2td+12t^2)^2}{72t^2(2t+d)}$
$d_A = d$	$\frac{(d^2+2td+12t^2)^2}{72t^2(2t+d)}; \frac{(-d^2-2td+12t^2)^2}{72t^2(2t+d)}$	$\frac{t^2}{t+d}; \frac{t^2}{t+d}$

($d \leq \underline{d}$, where $\underline{d} \approx 0.33t$), both firms choose zero degree of complementarity. For the large values of d ($d \geq \overline{d}$, where $\overline{d} \approx 0.76t$), both firms choose the positive degree of complementarity. For the intermediate values of d ($d \in [\underline{d}, \overline{d}]$), either both firms choose zero degree of complementarity or both firms choose the positive degree of complementarity.

The result in Proposition 2 can be understood with the help of the direct and strategic effects of an increase in d_i that were described in the previous section. When d is small ($d \leq \underline{d}$), the direct, demand-increasing effect of choosing $d_i = d$ is dominated by the strategic effect of the rival's more aggressive pricing. Therefore, regardless of the rival's strategy, it is better for the focal firm to choose zero degree of complementarity. Thus, in a Nash equilibrium, both firms choose zero degree of complementarity. As d increases, the direct effect becomes larger, while the strategic effect decreases. The comparison of these effects depends now on the rival's choice of d_j. When $d_j = 0$, the direct effect is smaller than the strategic effect, whereas when $d_j = d$, the direct effect is larger than the strategic effect. Thus, for the intermediate values of d ($d \in [\underline{d}, \overline{d}]$), there are two Nash equilibria—if the rival chooses zero degree of complementarity, the focal firm also chooses it, and when the rival chooses the positive degree of complementarity, the focal firm chooses the positive degree of complementarity as well. The firms' profits are larger in the equilibrium with zero degree of complementarity. Finally, when d is large ($d \geq \overline{d}$), the direct effect dominates the strategic effect regardless of the rival's strategy. Therefore, both firms choose the positive degree of complementarity. This situation is equivalent to the prisoner's dilemma since choosing the positive degree of complementarity is the firms' dominant strategy, but the resulting outcome, $\left(t^2/(t+d); t^2/(t+d)\right)$, is Pareto-inferior to the outcome under zero degree of complementarity, $(t;t)$.

The results of Proposition 2 imply that the firms should consider implementing umbrella branding or other strategies that increase the degree of complementarity for their products with caution. Only when d is sufficiently large (at the level of $\frac{3}{4}t$), the choice of the positive degree of complementarity is dominant regardless of the rival's strategy. Even for the intermediate values of d (between $\frac{1}{3}t$ and $\frac{3}{4}t$), while both equilibria with symmetric choice of strategies are possible, the one with both firms using the positive degree of complementarity results in smaller profits.

The intuition for the finding that zero degree of complementarity results in higher profits for the firms is consistent with and expands on the argument for why compatible networks increase firms' profits. Matutes and Regibeau (1988) argue that when firms sell incompatible components, if one firm decreases its price, it receives all the benefits of a price cut at the expense of a rival firm. When firms sell compatible components, a decrease in the price of one good will increase the demand for all bundles that contain this product, including the bundles containing the rival's product. Therefore, some of the benefits of a price cut will go to the rival, leading the firms to price less aggressively. Although the products are fully compatible in my model, the fact that consumers are willing to pay a premium for matching bundles leads to the similar effect. If the firms choose zero degree of complementarity, a lot of consumers will buy mixed bundles AB and BA. When a large portion of the market is captured by the mixed bundles, the price competition is softer. The firms are reluctant to cut prices as some of the benefits are captured by the rival firm. If the firms choose the positive degree of complementarity, more consumers will buy matching bundles AA and BB, whereas relatively few consumers buy mixed bundles AB and BA. This intensifies the price competition between the firms, leading to smaller profits.

In some situations, it is more appropriate to think of d_i as a continuous variable. There are various methods by which a firm can "fine-tune" the consumers' perception of the degree of complementarity between its products. Choosing a more uniform design for its complementary products, placing them closer to each other on the supermarket shelf, and running promotions that show them used together are some of the possible strategies. While none of these strategies is likely to change the degree of complementarity as drastically as the change from individual to umbrella branding, they, nevertheless, can achieve a marginal change in d_i. Therefore, it is useful to examine the behavior of the profit function while treating d_i as a continuous variable. This will show whether a firm can increase its profits by using strategies that incrementally increase d_i for any possible strategy of the rival d_j. Proposition 3 summarizes the results.

Proposition 3 *For the relatively small values of d_B ($d_B < (\sqrt{13}-3)t \approx 0.61t$), π_A is decreasing for $d_A \leq \frac{-5t-2d_B+\sqrt{d_B^2+2td_B+37t^2}}{3}$ and is increasing for $d_A > \frac{-5t-2d_B+\sqrt{d_B^2+2td_B+37t^2}}{3}$. For the large values of d_B ($d_B \geq (\sqrt{13}-3)t$), π_A always increases.*

The findings in Proposition 3 are consistent with the previous results from Proposition 2. If the rival's degree of complementarity is small, it is unprofitable for the firm to adopt the strategies that marginally increase its own degree of complementarity. On the other hand, if the consumers perceive that the rival's products have a high degree of complementarity then the firm should use all available mechanisms to increase its own degree of complementarity.

5 Large Levels of Degree of Complementarity

The analysis so far focused on the case when the degrees of complementarity of both firms were smaller than t. The results from the previous section show that both firms have an incentive to increase their degrees of complementarity if the rival's degree of complementarity is high. This conclusion is conserved for the situation when the degree of complementarity of both firms is larger than t. In this case, regions AB and BA disappear as both firms sell only their matching bundles AA and BB. The following proposition summarizes the firms' optimal prices.

Proposition 4 *If both d_A and d_B are larger than t, and, without loss of generality, $d_B \geq d_A$, then in the second stage the firms charge prices $p_A = p_{1,A} = p_{2,A} = \frac{2t+d_A-d_B+\sqrt{32t^2+(2t+d_A-d_B)^2}}{16}$ and $p_B = p_{B,1} = p_{B,2} = \frac{-5(2t+d_A-d_B)+3\sqrt{32t^2+(2t+d_A-d_B)^2}}{16}$. Both of these prices increase with their respective degrees of complementarity ($\frac{\partial p_A}{\partial d_A} > 0$ and $\frac{\partial p_B}{\partial d_B} > 0$), and the demands of both firms increase with their respective degrees of complementarity ($\frac{\partial AA}{\partial d_A} > 0$ and $\frac{\partial BB}{\partial d_B} > 0$).*

Since for each firm both price and demand increase with their own degree of complementarity, the firms have an incentive to set the maximum feasible degree of complementarity. In essence, for the large values of d_A and d_B, the competition between the firms is similar to the one-dimensional Hotelling model of competition between the two products—bundles AA and BB. It is not surprising, then, that each firm benefits from increasing its degree of complementarity since it increases the consumers' valuation of its product, which leads to higher profits.

From Table 1, we know that when $d_A = d_B = t$, the profits of both firms are equal to $t/2$. From Proposition 4, when $d_A = d_B > t$, the prices of both firms are equal to $t/2$. Since each firm sells two products and the market is split equally, each firm's profit is also $t/2$. In summary, when both firms have a high degree of complementarity, it is always in the interest of each firm to try to increase its degree of complementarity further. However, in the symmetric equilibrium ($d_A = d_B$), both prices and profits remain constant and do not depend on the firms' degrees of complementarity.

6 Negative Degree of Complementarity with Competition

In this section, I examine a modified version of the basic model in which, instead of increasing the degree of complementarity between its own products, a firm deteriorates the quality of the fit between its own and the rival's complementary products. An example of such scenario is Microsoft degrading the fit between its Windows platform and the competitor's Netscape Navigator. One way of modeling this strategy is to assume that a firm imposes an extra cost on consumers who purchase a mixed bundle. I consider the symmetric case in which firm i deteriorates the value of both bundles ij and ji by the same amount c_i. Then, a consumer's utility function becomes

$$U_{ij} = S - t(x_i + y_j) - p_{1,i} - p_{2,j} - (c_i + c_j) \cdot I(i \neq j), \tag{2}$$

where all the variables are the same as in (1) and c_i is the extra "inconvenience" cost that firm i imposed on the consumer who buys a mixed bundle.

The following proposition summarizes the firms' optimal prices and profits given the choice of c_A and c_B in the first stage.

Proposition 5 *Given c_A and c_B set in the first stage, in the second stage the firms charge prices $p_A = p_{1,A} = p_{2,A} = p_B = p_{B,1} = p_{B,2} = \frac{t^2}{t+c_A+c_B}$. The firms earn profits $\pi_A = \pi_B = \frac{t^2}{t+c_A+c_B}$.*

We note that the profit of each firm decreases in the cost this firm imposes on the consumers buying mixed bundles, regardless of the cost set by the rival. The intuition behind this result can be understood by examining the direct and the strategic effects of an increase in c_i.

When one of the firms, say firm A, increases c_A, mixed bundles AB and BA become less attractive to the consumers. Thus, the consumers who are located close to the boundary with the region containing a matching bundle (either AA or BB) will switch to that matching bundle. We note that regions AB and BA are squares since the boundary between AA and BB is not affected by choices of c_A and c_B and lies on a main diagonal of the unit square. Then, the length of the boundary between AB and AA is the same as the length of the boundary between AB and BB. Thus, the number of consumers switching from AB to AA is the same as the number of consumers switching from AB to BB. This means that the direct effect is absent—a firm gains as many consumers switching from the mixed bundles to its matching bundle as it loses in those who switch from the mixed bundles to the matching bundle of the rival.

An increase in c_A diminishes regions AB and BA while expanding regions AA and BB as well as the boundary between them. Recall from the discussion in Section 4 that when the boundary between regions AA and BB increases, the price competition between the firms intensifies. Therefore, the strategic effect of an increase in c_A leads to smaller prices. Taken together, an increase in c_A does not increase the firm's demand and only leads to lower prices. Therefore, the firm's profit decreases.[14]

In summary, we conclude that the strategy of deteriorating the quality of the fit between one's own and the rival's components is unprofitable. A firm does not gain additional demand as some

[14] Note that the same intuition holds for the situation when a firm could deteriorate the quality of the fit with only one of its components.

consumers who switch away from the mixed bundles buy the rival's matching bundle. On the other hand, the resulting market structure, with a larger portion of consumers buying matching bundles, leads to intensified price competition and lower profits. This analysis is consistent with the relative dearth of real-world examples of firms attempting to deteriorate the quality of mixed bundles. Even the Microsoft-Netscape example given in the Introduction likely represents a predatory strategy by Microsoft trying to drive Netscape out of the market rather than an attempt to increase contemporaneous profits.

7 Concluding Remarks

This paper studies the profit implications of firms being able to choose the quality of the fit between their complementary products. An increase in the quality of the fit shifts consumers' preferences toward matching bundles, which, in turn, affects the price competition between the rival firms.

I find that the firms' profits are the highest when they do not invest in making their own bundles more attractive to the consumers. This leads to many consumers purchasing mixed bundles, which softens price competition. When the firms use strategies that increase the valuation of their matching bundles, fewer consumers buy mixed bundles. With the competition mainly between the two matching bundles, the incentives to undercut the rival increase, leading to smaller profits.

While it would be profitable for the firms to agree not to improve the quality of the fit between their own complementary products, in the absence of the means to enforce such an agreement, it is possible that each firm has an incentive to increase the degree of complementarity. In making this decision, the firm must compare the positive demand-expanding effect versus the negative strategic effect resulting from intensified price competition. If the demand-expanding effect is substantial, the firm should use the strategies that improve the quality of the fit between its complementary products. Having a rival who invests in increasing the degree of complementarity makes adopting similar strategies for the focal firm more likely.

This provides one of the testable implications of my model. If the firms use umbrella branding, which is one of the strategies that is likely to significantly increase the degree of complementarity, then the firms should try to use other strategies, such as joint promotions, in an attempt to further increase the perceived complementarity of their products. This implication is consistent with the findings in Sinitsyn (2012), who reports that in cake mix and cake frosting categories where the firms use umbrella branding, the promotions of complementary products are highly correlated. On the other hand, in detergent and fabric softener categories, where the firms use individual branding (P&G produces detergent Tide, fabric softeners Bounce and Downy; Unilever produces detergents Surf and Wisk, fabric softener Snuggle), price promotions were not coordinated.

References

[1] BettyC2009 (2009, October 26). *BETTY CROCKER UK - NEW TV ADVERT 2009*. Retrieved March 25, 2014 from http://www.youtube.com/watch?v=g5igcSkoDTA

[2] Dawar, N. and J. Stornelli. 2013. Rebuilding the Relationship Between Manufacturers and Retailers. *MIT Sloan Management Review*, 54(2), 83-90.

[3] Economides, N. 1989. Desirability of Compatibility in the Absence of Network Externalities. *The American Economic Review*, 79(5), 1165-1181.

[4] Liu, Q. and K. Serfes. 2004. Quality of Information and Oligopolistic Price Discrimination. *Journal of Economics & Management Strategy*, 13(4), 671-702.

[5] Ma, Y., P. B. Seetharaman, and C. Narasimhan. 2012. Modeling Dependencies in Brand Choice Outcomes Across Complementary Categories. *Journal of Retailing*, 88(1), 47-62.

[6] Matutes, C. and P. Regibeau. 1988. "Mix and Match": Product Compatibility without Network Externalities. *The RAND Journal of Economics*, 19(2), 221-234.

[7] Matutes, C. and P. Regibeau. 1992. Compatibility and Bundling of Complementary Goods in a Duopoly. *The Journal of Industrial Economics*, 40(1), 37-54.

[8] McCracken, G. 1988. *Culture and Consumption: New Approaches to the Symbolic Character of Consumer Goods and Activities*. Indiana University Press, Bloomington and Indianapolis.

[9] Premium Plus ® Interior Ceiling Paint. n.d. Retrieved March 25, 2014, from www.behr.com/consumer/products/interior-paint-and-primer/behn-premium-plus-ceiling-paint

[10] Rahinel, R., and J. P. Redden. 2013. Brands as Product Coordinators: Matching Brands Make Joint Consumption Experiences More Enjoyable. *Journal of Consumer Research*, 39(6), 1290-1299.

[11] Simonin, B. L. and J. A. Ruth. 1995. Bundling as a Strategy for New Product Introduction: Effects on Consumers' Reservation Prices for the Bundle, the New Product, and Its Tie-in. *Journal of Business Research*, 33(3), 219-230.

[12] Sinitsyn, M. 2012. Coordination of Price Promotions in Complementary Categories. *Management Science*, 58(11), 2076-2094.

Appendix

Proof of Proposition 1. The proof will proceed in several steps. In step 1, I compute the consumers' demands for various bundles by calculating the areas AA, AB, and BA from Figure 1. In step 2, I calculate the firms' profit functions and set up the first order conditions. In step 3, I find the firms' prices by solving the system of first order conditions and confirm that these prices result in the market structure depicted in Figure 1. In step 4, I compute the resulting profits. Finally, in step 5, I check that the firms do not have any profitable deviations to the prices that result in a market structure different from the one depicted in Figure 1.

Step 1: In Figure 1, consider the consumer located at the point $(x;1)$ who is indifferent between bundles AB and BB. Using the utility functions from (1), we get $S - t(x+0) - p_{1,A} - p_{2,B} = S - t(1-x+0) - p_{1,B} - p_{2,B} + d_B$. From here, $x = \frac{p_{1,B}-p_{1,A}+t-d_B}{2t}$. Similarly, we find that $y = \frac{p_{2,B}-p_{2,A}+t+d_A}{2t}$, $u = \frac{p_{1,B}-p_{1,A}+t+d_A}{2t}$, and $v = \frac{p_{2,B}-p_{2,A}+t-d_B}{2t}$. The area AB is then $x(1-y)$, the area BA is $(1-u)v$, and the area AA is $xy + \frac{(u-x)(y+v)}{2}$.

Step 2: The profit function of firm A is $\pi_A = p_{1,A}(AA+AB)+p_{2,A}(AA+BA)$. The derivative of π_A with respect to $p_{1,A}$ is

$$\frac{\partial \pi_A}{\partial p_{1,A}} = AA + AB + p_{1,A}\left(\frac{\partial AA}{\partial p_{1,A}} + \frac{\partial AB}{\partial p_{1,A}}\right) + p_{2,A}\left(\frac{\partial AA}{\partial p_{1,A}} + \frac{\partial BA}{\partial p_{1,A}}\right). \tag{3}$$

Noting that y, v, and $u-x$ do not depend on $p_{1,A}$, we can compute the derivatives of the different areas with respect to $p_{1,A}$: $\frac{\partial AA}{\partial p_{1,A}} = y\frac{\partial x}{\partial p_{1,A}} = \frac{p_{2,B}-p_{2,A}+t+d_A}{2t}\frac{-1}{2t} = \frac{p_{2,A}-p_{2,B}-t-d_A}{4t^2}$;

$\frac{\partial AB}{\partial p_{1,A}} = (1-y)\frac{\partial x}{\partial p_{1,A}} = \frac{p_{2,A}-p_{2,B}+t-d_A}{2t}\frac{-1}{2t} = \frac{p_{2,B}-p_{2,A}-t+d_A}{4t^2}$; and

$\frac{\partial BA}{\partial p_{1,A}} = -v\frac{\partial u}{\partial p_{1,A}} = -\frac{p_{2,B}-p_{2,A}+t-d_B}{2t}\frac{-1}{2t} = \frac{p_{2,B}-p_{2,A}+t-d_B}{4t^2}$.

From here, we get that $\frac{\partial AA}{\partial p_{1,A}} + \frac{\partial AB}{\partial p_{1,A}} = \frac{-2t}{4t^2} = -\frac{1}{2t}$ and $\frac{\partial AA}{\partial p_{1,A}} + \frac{\partial BA}{\partial p_{1,A}} = -\frac{d_A+d_B}{4t^2}$. Substituting these expressions into (3) and setting the derivative equal to zero, we get

$xy + \frac{(u-x)(y+v)}{2} + x(1-y) - \frac{p_{1,A}}{2t} - \frac{p_{2,A}(d_A+d_B)}{4t^2} = 0$, which simplifies to

$x + \frac{(u-x)(y+v)}{2} - \frac{p_{1,A}}{2t} - \frac{p_{2,A}(d_A+d_B)}{4t^2} = 0.$

After substituting for x, $u-x$, and $y+v$, we obtain

$\frac{p_{1,B}-p_{1,A}+t-d_B}{2t} + \frac{(d_A+d_B)(2p_{2,B}-2p_{2,A}+2t+d_A-d_B)}{8t^2} - \frac{p_{1,A}}{2t} - \frac{p_{2,A}(d_A+d_B)}{4t^2} = 0$, from where

$2p_{1,A} = p_{1,B} + t - d_B + \frac{(d_A+d_B)(2p_{2,B}-4p_{2,A}+2t+d_A-d_B)}{4t}$, and

$p_{1,A} = \frac{p_{1,B}}{2} + \frac{4t^2-4td_B-(d_A+d_B)(4p_{2,A}-2p_{2,B})+(d_A+d_B)(2t+d_A-d_B)}{8t}$.

Noting that $(d_A+d_B)(2t+d_A-d_B)-4td_B = (d_A-d_B)(2t+d_A+d_B)$, the above equation simplifies to $p_{1,A} = \frac{p_{1,B}}{2} + \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{8t} - \frac{(d_A+d_B)(2p_{2,A}-p_{2,B})}{4t}$. Solving in a similar way for the other best reply functions, we obtain the system of equations, the solution to which characterizes the firms' optimal prices:

$$
\begin{aligned}
p_{1,A} &= \frac{p_{1,B}}{2} + \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{8t} - \frac{(d_A+d_B)(2p_{2,A}-p_{2,B})}{4t} \\
p_{1,B} &= \frac{p_{1,A}}{2} + \frac{4t^2+(d_B-d_A)(d_A+d_B+2t)}{8t} - \frac{(d_A+d_B)(2p_{2,B}-p_{2,A})}{4t} \\
p_{2,A} &= \frac{p_{2,B}}{2} + \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{8t} - \frac{(d_A+d_B)(2p_{1,A}-p_{1,B})}{4t} \\
p_{2,B} &= \frac{p_{2,A}}{2} + \frac{4t^2+(d_B-d_A)(d_A+d_B+2t)}{8t} - \frac{(d_A+d_B)(2p_{1,B}-p_{1,A})}{4t}
\end{aligned}
\tag{4}
$$

Step 3: The first and the third equalities in (4) are equivalent to

$2p_{1,A} - p_{1,B} = \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{4t} - \frac{(d_A+d_B)(2p_{2,A}-p_{2,B})}{2t}$ and

$2p_{2,A} - p_{2,B} = \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{4t} - \frac{(d_A+d_B)(2p_{1,A}-p_{1,B})}{2t}$.

These two equalities can be rewritten as $g = k + mf$ and $f = k + mg$, where $g = 2p_{1,A} - p_{1,B}$, $f = 2p_{2,A} - p_{2,B}$, $k = \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{4t}$, and $m = -\frac{(d_A+d_B)}{2t}$. These simplified equations can be solved by substituting f from the second equation into the first to obtain $g = k+m(k+mg) =$

$k(m+1)+m^2g$, from where $g = \frac{k(m+1)}{1-m^2} = \frac{k}{1-m}$. Noting that $1-m = 1+\frac{(d_A+d_B)}{2t} = \frac{d_A+d_B+2t}{2t}$, we conclude that $2p_{1,A} - p_{1,B} = \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{4t}\frac{2t}{d_A+d_B+2t} = \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{2(d_A+d_B+2t)}$. From here,

$$p_{1,B} = 2p_{1,A} - \frac{4t^2+(d_A-d_B)(d_A+d_B+2t)}{2(d_A+d_B+2t)}. \tag{5}$$

Similarly, the second and the fourth equations from (4) give $2p_{1,B} - p_{1,A} = \frac{4t^2+(d_B-d_A)(d_A+d_B+2t)}{2(d_A+d_B+2t)}$. Substituting $p_{1,B}$ from (5) into this equation, we get $4p_{1,A} - \frac{8t^2+2(d_A-d_B)(d_A+d_B+2t)}{2(d_A+d_B+2t)} - p_{1,A} = \frac{4t^2+(d_B-d_A)(d_A+d_B+2t)}{2(d_A+d_B+2t)}$. From here, $3p_{1,A} = \frac{12t^2+(d_A+d_B+2t)(2d_A-2d_B-d_A+d_B)}{2(d_A+d_B+2t)} = \frac{12t^2+(d_A+d_B+2t)(d_A-d_B)}{2(d_A+d_B+2t)}$ and $p_{1,A} = \frac{d_A-d_B}{6} + \frac{2t^2}{d_A+d_B+2t}$. Similarly, we find the other prices:

$$\begin{aligned} p_A &= p_{1,A} = p_{2,A} = \tfrac{d_A-d_B}{6} + \tfrac{2t^2}{d_A+d_B+2t} \\ p_B &= p_{1,B} = p_{2,B} = \tfrac{d_B-d_A}{6} + \tfrac{2t^2}{d_A+d_B+2t} \end{aligned} \tag{6}$$

It is necessary to confirm that these prices are consistent with the market structure depicted in Figure 1, for which it is enough to check that x and y are between 0 and 1.[15] Substituting the prices given in (6) into the formulas for x and y, we get $x = \frac{p_{1,B}-p_{1,A}+t-d_B}{2t} = \frac{\frac{d_B-d_A}{3}+t-d_B}{2t} = \frac{3t-d_A-2d_B}{6t}$ and $y = \frac{p_{2,B}-p_{2,A}+t+d_A}{2t} = \frac{\frac{d_B-d_A}{3}+t+d_A}{2t} = \frac{3t+2d_A+d_B}{6t}$. Whereas x is always smaller than 1, the inequality $x \geq 0$ holds only when $d_A + 2d_B \leq 3t$. Similarly, y is always greater than 0, but $y \leq 1$ holds only when $2d_A + d_B \leq 3t$. Both of these inequalities hold when both d_A and d_B are less than or equal to t.

Step 4: Using the formulas for x and y obtained in the previous step and using the fact that $x = v$ and $y = u$, we find $AB = BA = x(1-y) = \frac{3t-d_A-2d_B}{6t}\frac{3t-2d_A-d_B}{6t}$ and $AA = xy + \frac{(u-x)(y+v)}{2} = \frac{3t-d_A-2d_B}{6t}\frac{3t+2d_A+d_B}{6t} + \frac{1}{2}\frac{3d_A+3d_B}{6t}\frac{6t+d_A-d_B}{6t} = \frac{3t-d_A-2d_B}{6t}\frac{3t+2d_A+d_B}{6t} + \frac{d_A+d_B}{4t}\frac{6t+d_A-d_B}{6t}$. The profit of

[15] Since each firm charges identical prices in both categories, we note that $x = v$ and $y = u$.

firm A is $\pi_A = p_{1,A}(AA + AB) + p_{2,A}(AA + BA) = 2p_A(AA + AB) =$

$$\frac{(d_A-d_B)(d_A+d_B+2t)+12t^2}{3(d_A+d_B+2t)}\left(\frac{3t-d_A-2d_B}{6t}+\frac{d_A+d_B}{4t}\frac{6t+d_A-d_B}{6t}\right)=$$

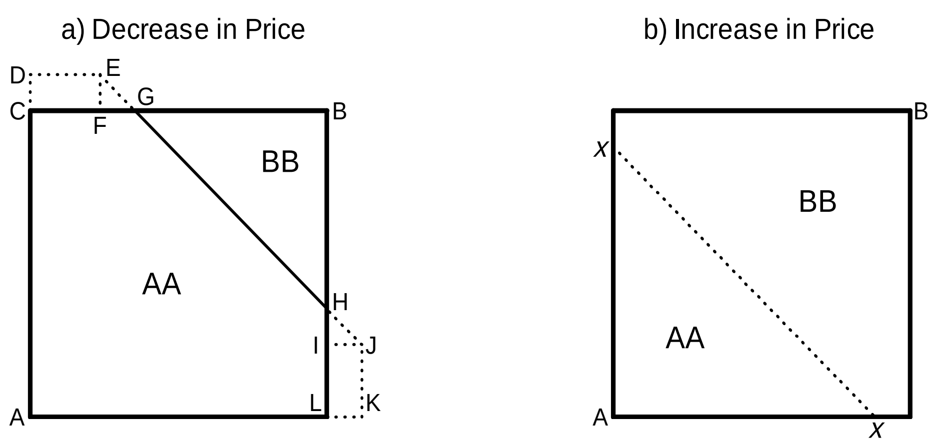
$$\frac{(d_A-d_B)(d_A+d_B+2t)+12t^2}{3(d_A+d_B+2t)}\left(\frac{12t^2-4td_A-8td_B+6td_A+6td_B+d_A^2-d_B^2}{24t^2}\right)=$$

$$\frac{(d_A-d_B)(d_A+d_B+2t)+12t^2}{3(d_A+d_B+2t)}\left(\frac{12t^2+2td_A-2td_B+d_A^2-d_B^2}{24t^2}\right)=\frac{\left(d_A^2-d_B^2+2td_A-2td_B+12t^2\right)^2}{72t^2(d_A+d_B+2t)}.$$

Similarly, the profit of firm B is $\pi_B = \frac{\left(d_B^2-d_A^2+2td_B-2td_A+12t^2\right)^2}{72t^2(d_A+d_B+2t)}$

Step 5: Since the profit function $\pi_A = p_{1,A}(AA+AB)+p_{2,A}(AA+BA)$ is a parabola opening down, as long as the market structure is the same as depicted in Figure 1, firms do not have a profitable deviation to another price. However, it is necessary to check that the firms do not have profitable deviations to the prices that will result in another market structure. Without loss of generality, we will examine possible deviations by firm A. Figure 2 illustrates possible market configurations following a decrease or an increase in firm A's prices.

Figure 2: Market configurations after firm A's price deviation



In Figure 2a), firm A decreases its prices low enough so that regions AB and BA disappear. Firm A's profit is now $(p_{1,A} + p_{2,A})AA$, where area AA is the shape $ACGHL$. I will argue that the straightforward application of the formulas used in step 1 to calculate the demand for firm A's products overstates the real demand for the current situation. First, the value of x, computed according to the formula in step 1, is the length CF; the value of y is AD. The formula for AB, which is equal to $x(1-y)$, is, then, the area of rectangle $CDEF$ taken with a negative sign (since y is greater than 1). The area of AA computed using the formula from step 1, is the area $ADEJK$. Therefore, the demand for firm A's product in the first category computed using the formulas from step 1, is the sum of areas AA and AB, which is equal to the area $ACFEJK$. It is larger than the real demand for the first product—area $ACGHL$. Similarly, the demand for the second product computed using the formulas in step 1, which is equal to the area $ADEJIL$, is larger than the real demand for the second product—area $ACGHL$. Thus, the real demand for firm A's products is smaller than the demand computed using the formulas in step 1. Consequently, the real profit of firm A when it decreases its price is smaller than the profit computed using the formula in step 4. Since it was unprofitable for firm A to decrease its prices for the profit function computed using the formula in step 4, it is also unprofitable to decrease its prices for the real profit function.

In Figure 2b), firm A increases its prices high enough so that regions AB and BA disappear. This means that $x = \frac{p_B - p_A + t - d_B}{2t}$ from Figure 1 becomes less than or equal to zero. This happens when p_A is greater than or equal to $p_B + t - d_B$. Going back to Figure 2b), take the consumer located at the point $(x; 0)$ who is indifferent between bundles AA and BB. Using the utility functions from (1), we get $B - t(x+0) - 2p_A + d_A = B - t(1 - x + 1) - 2p_B + d_B$. From here, $x = \frac{2p_B - 2p_A + 2t + d_A - d_B}{2t}$. Firm A's profit is $\pi_A = 2p_A \frac{1}{2} x^2$. The derivative of this profit function with respect to p_A is $\frac{\partial \pi_A}{\partial p_A} = x^2 + 2p_A x \frac{\partial x}{\partial p_A} = x^2 - \frac{2p_A x}{t} = x(x - \frac{2p_A}{t}) = x\left(\frac{2p_B - 6p_A + 2t + d_A - d_B}{2t}\right)$.

This derivative is equal to zero when either x or $\frac{2p_B-6p_A+2t+d_A-d_B}{2t}$ is equal to zero. Solving $x=0$ and $\frac{2p_B-6p_A+2t+d_A-d_B}{2t}=0$ for p_A, we get $p_A=p_B+t+\frac{d_A-d_B}{3}$ and $p_A=\frac{p_B+t}{3}+\frac{d_A-d_B}{6}$, respectively. These are the two roots of the equation $\frac{\partial \pi_A}{\partial p_A}=0$. Since the profit function is a cubic polynomial with a positive leading coefficient, the smaller of the two roots, $p_A=\frac{p_B+t}{3}+\frac{d_A-d_B}{6}$, is a local maximizer of a profit function.

To summarize, in order to switch to the market structure depicted in Figure 2b), p_A has to be at least p_B+t-d_B, and the profit function corresponding to this situation is maximized at $\frac{p_B+t}{3}+\frac{d_A-d_B}{6}$. Then, if we can show that $p_B+t-d_B \geq \frac{p_B+t}{3}+\frac{d_A-d_B}{6}$, it would mean that for the prices that are high enough to result in a new market structure depicted in Figure 2b), the profit function is decreasing, so firm A can not increase its profit. The inequality $p_B+t-d_B \geq \frac{p_B+t}{3}+\frac{d_A-d_B}{6}$ is equivalent to $4p_B \geq 5d_B+d_A-4t$. Since the price of firm B is fixed, we can substitute $p_B=\frac{d_B-d_A}{6}+\frac{2t^2}{d_A+d_B+2t}=\frac{d_B^2-d_A^2+2td_B-2td_A+12t^2}{6(d_A+d_B+2t)}$ into the inequality to obtain $\frac{2(d_B^2-d_A^2+2td_B-2td_A+12t^2)}{3(d_A+d_B+2t)} \geq 5d_B+d_A-4t$. This inequality simplifies to $2d_B^2-2d_A^2+4td_B-4td_A+24t^2 \geq 15d_B^2+3d_A^2+18d_Ad_B+18td_B-6td_A-24t^2$, which further simplifies to $48t^2-13d_B^2-5d_A^2-14td_B+2td_A-18d_Ad_B \geq 0$. Since both d_A and d_B are smaller than or equal to t, this inequality holds true ($48t^2 \geq 13d_B^2+5d_A^2+14td_B+16d_Ad_B$ and $2td_A \geq 2d_Ad_B$). ■

Proof of Proposition 2. In order to find Nash equilibria, we first compute the best reply of firm A. Since the game is symmetric, the best reply of firm B is the same. When firm B chooses $d_B=0$, if firm A chooses $d_A=0$, it gets a profit of t. If firm A chooses $d_A=d$, it gets a profit of $\frac{(d^2+2td+12t^2)^2}{72t^2(2t+d)}$. I will find the conditions on d for which $t \geq \frac{(d^2+2td+12t^2)^2}{72t^2(2t+d)}$. This inequality is equivalent to $144t^4+72dt^3 \geq d^4+4d^2t^2+144t^4+4d^3t+24d^2t^2+48dt^3$, which after simplifying and dividing by d becomes

$$d^3 + 4d^2t + 28dt^2 - 24t^3 \leq 0. \tag{7}$$

The cubic equation $d^3 + 4d^2t + 28dt^2 - 24t^3 = 0$ has 2 complex roots and one real, $\overline{d} \approx 0.76t$. Since the left-hand side of inequality (7) increases in d, (7) holds true when $d \leq \overline{d}$. This means that firm A's best reply to firm B's strategy $d_B = 0$ is to use $d_A = 0$ when $d \leq \overline{d}$ and to use $d_A = d$ when $d \geq \overline{d}$.

When firm B chooses $d_B = d$, if firm A chooses $d_A = 0$, it gets a profit of $\frac{(-d^2-2td+12t^2)^2}{72t^2(2t+d)}$. If firm A chooses $d_A = d$, it gets a profit of $\frac{t^2}{t+d}$. We will find the conditions on d for which $\frac{t^2}{t+d} \geq \frac{(-d^2-2td+12t^2)^2}{72t^2(2t+d)}$. This inequality is equivalent to

$144t^5 + 72dt^4 \geq (t+d)(d^4 + 4d^2t^2 + 144t^4 + 4d^3t - 24d^2t^2 - 48dt^3)$, which simplifies to

$144t^5 + 72dt^4 \geq d^5 + 5d^4t - 16d^3t^2 - 68d^2t^3 + 96dt^4 + 144t^5$. After further simplification and division by d, this inequality becomes

$$d^4 + 5d^3t - 16d^2t^2 - 68dt^3 + 24t^4 \leq 0. \tag{8}$$

The quartic equation $d^4 + 5d^3t - 16d^2t^2 - 68dt^3 + 24t^4 = 0$ has four real roots: $-5.53t$, $-3.53t$, $0.33t$, and $3.73t$. Since $d \leq t$ and d is nonnegative, the only relevant root is $\underline{d} \approx 0.33t$. Since the left-hand side of inequality (8) is negative at $d = t$, inequality (8) hold true when $d \geq \underline{d}$. In summary, firm A's best reply to firm B's using $d_B = d$ is to use $d_A = 0$ when $d \leq \underline{d}$ and to use $d_A = d$ when $d \geq \underline{d}$.

Taken together, when $d \leq \underline{d}$, $d_i = 0$ is a dominant strategy for both firms, and the Nash equilibrium involves both firms choosing $d_i = 0$. When $d \in [\underline{d}, \overline{d}]$, the best reply of both firms is to use $d_i = 0$ against the rival choosing 0 and to use $d_i = d$ against the rival choosing d. Therefore, there are two Nash equilibria: $(0; 0)$ and $(d; d)$. Finally, when $d \geq \overline{d}$, $d_i = d$ is a dominant strategy for both firms, and the Nash equilibrium involves both firms choosing $d_i = d$. ■

Proof of Proposition 3. The profit of firm A is equal to $\frac{\left(d_A^2 - d_B^2 + 2td_A - 2td_B + 12t^2\right)^2}{72t^2(2t + d_A + d_B)}$. Labeling $d_A^2 - d_B^2 + 2td_A - 2td_B + 12t^2$ as Q, the derivative of this profit function with respect to d_A is

$$\frac{2Q \cdot (2d_A + 2t) \cdot 72t^2(2t + d_A + d_B) - Q^2 \cdot 72t^2}{\left(72t^2(2t + d_A + d_B)\right)^2} = \frac{72t^2 Q \left(4(d_A + t)(2t + d_A + d_B) - Q\right)}{\left(72t^2(2t + d_A + d_B)\right)^2}. \tag{9}$$

The term in the parenthesis in the numerator of (9) simplifies to $4d_A^2 + 4d_A d_B + 12td_A + 4td_B + 8t^2 - d_A^2 + d_B^2 - 2td_A + 2td_B - 12t^2 = 3d_A^2 + 10td_A + 4d_A d_B + d_B^2 + 6td_B - 4t^2$. We label this quadratic polynomial as R. Then, the sign of the derivative of the profit function of firm A with respect to d_A is equal to $sign(Q) \cdot sign(R)$. Since d_A and d_B are both less than or equal to t, $Q = d_A^2 - d_B^2 + 2td_A - 2td_B + 12t^2$ is always positive. Therefore, the sign of (9) is the same as the sign of R. The solutions to the quadratic equation $R = 0$ are

$$\begin{aligned} d_A &= \frac{-10t - 4d_B \pm \sqrt{100t^2 + 16d_B^2 + 80td_B - 12d_B^2 - 72td_B + 48t^2}}{6} = \\ &= \frac{-5t - 2d_B \pm \sqrt{d_B^2 + 2td_B + 37t^2}}{3}. \end{aligned}$$

Since d_A has to be positive, we only consider the larger root $d_A^* = \frac{-5t - 2d_B + \sqrt{d_B^2 + 2td_B + 37t^2}}{3}$. Then, R is positive for $d_A > d_A^*$. The numerator of d_A^* is nonpositive when $\sqrt{d_B^2 + 2td_B + 37t^2} \leq 5t + 2d_B$. Squaring both sides and simplifying, we get $d_B^2 + 6td_B - 4t^2 \geq 0$. This inequality holds when $d_B \geq \frac{-6t + \sqrt{52t^2}}{2} = (\sqrt{13} - 3)t$. Hence, if $d_B \geq t(\sqrt{13} - 3)$, then R is always positive for positive d_A, and if $d_B < t(\sqrt{13} - 3)$, then R is negative for d_A smaller than $\frac{-5t - 2d_B + \sqrt{d_B^2 + 2td_B + 37t^2}}{3}$

and is positive for d_A greater than $\frac{-5t-2d_B+\sqrt{d_B^2+2td_B+37t^2}}{3}$. ■

Proof of Proposition 4. We assume that when $d_B \geq d_A$, firm A gets a smaller region of demand. After solving for the optimal prices, we will check that this is indeed the case. This situation is illustrated in Figure 2b). Since the firms sell only matching bundles, just the total price of the bundle matters. Therefore, we can assume that $p_{1,A} = p_{2,A} = p_A$ and $p_{1,B} = p_{2,B} = p_B$. In Step 5 of Proposition 1, we considered a consumer whose located at $(x;0)$ and is indifferent between bundles AA and BB. We found that $x = \frac{2p_B-2p_A+2t+d_A-d_B}{2t}$.

Now, we will compute the profit of firm B. It is equal to $\pi_B = (1-\frac{x^2}{2}) \cdot 2p_B = (2-x^2)p_B$. Taking the derivative of this profit function with respect to p_B and setting it equal to zero, we get

$$\frac{\partial \pi_B}{\partial p_B} = (2-x^2) - p_B \cdot 2x\frac{\partial x}{\partial p_B} = 2 - x^2 - \frac{2xp_B}{t} = 0. \tag{10}$$

In step 5 of Proposition 1, we also found that the best reply function of firm A is $p_A = \frac{p_B+t}{3} + \frac{d_A-d_B}{6}$. We can substitute p_A into x to get $x = \frac{2p_B-\left(\frac{2p_B+2t}{3}+\frac{d_A-d_B}{3}\right)+2t+d_A-d_B}{2t} = \frac{6p_B-2p_B-2t-d_A+d_B+6t+3d_A-3d_B}{6t}$, which simplifies to

$$x = \frac{2p_B + 2t + d_A - d_B}{3t}. \tag{11}$$

Now, we can substitute (11) into (10) to obtain

$$\frac{18t^2 - (2p_B + 2t + d_A - d_B)^2 - 6p_B(2p_B + 2t + d_A - d_B)}{9t^2} = 0.$$

After taking p_B out of the brackets, the equation becomes

$$18t^2 - 4p_B^2 - 4p_B(2t + d_A - d_B) - (2t + d_A - d_B)^2 - 12p_B^2 - 6p_B(2t + d_A - d_B) = 0,$$

which simplifies to

$$8p_B^2 + 5p_B(2t + d_A - d_B) + \frac{1}{2}(2t + d_A - d_B)^2 - 9t^2 = 0.$$

The roots to this quadratic equation are $p_B = \frac{-5(2t+d_A-d_B)\pm\sqrt{25(2t+d_A-d_B)^2+288t^2-16(2t+d_A-d_B)^2}}{16} = \frac{-5(2t+d_A-d_B)\pm 3\sqrt{32t^2+(2t+d_A-d_B)^2}}{16}$. The corresponding values of p_A are $\left(\frac{-5(2t+d_A-d_B)\pm 3\sqrt{32t^2+(2t+d_A-d_B)^2}}{16} + t\right)/3 + (d_A - d_B)/6 = \frac{6t-5d_A+5d_B\pm 3\sqrt{32t^2+(2t+d_A-d_B)^2}}{48} + (d_A - d_B)/6 = \frac{6t+3d_A-3d_B\pm 3\sqrt{32t^2+(2t+d_A-d_B)^2}}{48} = \frac{2t+d_A-d_B\pm 3\sqrt{32t^2+(2t+d_A-d_B)^2}}{16}$. Since $d_A \leq d_B$ and $2t < 3\sqrt{32t^2 + (2t + d_A - d_B)^2}$, the smaller root is negative, thus the only relevant solution is $p_A = \frac{2t+d_A-d_B+3\sqrt{32t^2+(2t+d_A-d_B)^2}}{16}$ and $p_B = \frac{-5(2t+d_A-d_B)+3\sqrt{32t^2+(2t+d_A-d_B)^2}}{16}$.

Substituting p_B into (11), we get $x = \frac{\frac{-5(2t+d_A-d_B)+3\sqrt{32t^2+(2t+d_A-d_B)^2}}{8}+2t+d_A-d_B}{3t} = \frac{6t+3d_A-3d_B+3\sqrt{32t^2+(2t+d_A-d_B)^2}}{24t}$, which simplifies to

$$x = \frac{2t + d_A - d_B + \sqrt{32t^2 + (2t + d_A - d_B)^2}}{8t}. \tag{12}$$

Since $\sqrt{32t^2 + (2t + d_A - d_B)^2} > -(2t + d_A - d_B)$, we know that x is greater than zero. To confirm that x is less than or equal to 1, we need to check that $\sqrt{32t^2 + (2t + d_A - d_B)^2} \leq 6t - d_A + d_B$. Squaring both parts, we obtain $36t^2 + d_A^2 + d_B^2 + 4td_A - 4td_B - 2d_Ad_B \leq 36t^2 +$

$d_A^2 + d_B^2 - 12td_A + 12td_B - 2d_Ad_B$, which is equivalent to $d_A \leq d_B$.

The area of AA is equal to $\frac{x^2}{2}$ and x increases with d_A, therefore $\frac{\partial AA}{\partial d_A} > 0$. The area of BB is $1 - \frac{x^2}{2}$, thus, in order to show that $\frac{\partial BB}{\partial d_B} > 0$, it is enough to show that $\frac{\partial x}{\partial d_B} < 0$. The derivative of (12) with respect to d_B is equal to $\frac{1}{8t}\left(-1 - \frac{2(2t+d_A-d_B)}{2\sqrt{32t^2+(2t+d_A-d_B)^2}}\right)$. It is smaller than zero when $2\sqrt{32t^2 + (2t + d_A - d_B)^2} > -2(2t + d_A - d_B)$, which holds true.

Finally, both prices increase with their respective degrees of complementarity. For firm A, d_A enters p_A only positively. For firm B, $\frac{\partial p_B}{\partial d_B} = \frac{1}{16}\left(5 - \frac{3\cdot 2(2t+d_A-d_B)}{2\sqrt{32t^2+(2t+d_A-d_B)^2}}\right)$. It is greater than zero when $5\sqrt{32t^2 + (2t + d_A - d_B)^2} > 3(2t+d_A-d_B)$ or $25(32t^2+(2t+d_A-d_B)^2) > 9(2t+d_A-d_B)^2$, which holds true. ■

Proof of Proposition 5. Similarly to the proof of Proposition 1, consider, first, the consumer located at the point $(x;1)$ who is indifferent between bundles AB and BB. Using the utility functions from (2), we obtain $S - t(x + 0) - p_{1,A} - p_{2,B} - c_A - c_B = S - t(1 - x + 0) - p_{1,B} - p_{2,B}$. From here, $x = \frac{p_{1,B}-p_{1,A}+t-c_A-c_B}{2t}$. Similarly, we find that $y = \frac{p_{2,B}-p_{2,A}+t+c_A+c_B}{2t}$, $u = \frac{p_{1,B}-p_{1,A}+t+c_A+c_B}{2t}$, and $v = \frac{p_{2,B}-p_{2,A}+t-c_A-c_B}{2t}$.

Now, we note that these values of x, y, u, and v are exactly the same as the values of x, y, u, and v from the proof of Proposition 1 if we set $d_A = d_B = c_A + c_B$. Therefore, we can substitute $c_A + c_B$ for d_A and d_B in the final formulas from Proposition 1 to obtain $p_A = p_{1,A} = p_{2,A} = \frac{d_A-d_B}{6} + \frac{2t^2}{2t+d_A+d_B} = \frac{2t^2}{2t+2(c_A+c_B)} = \frac{t^2}{t+c_A+c_B}$ and $\pi_A = \frac{\left(d_A^2-d_B^2+2td_A-2td_B+12t^2\right)^2}{72t^2(2t+d_A+d_B)} = \frac{144t^4}{72t^2\cdot 2(t+c_A+c_B)} = \frac{t^2}{t+c_A+c_B}$. The same formulas apply to the prices and the profit of firm B. ■