Economics 103 — Spring 2007
International Monetary Relations

COVERED INTEREST RATE PARITY

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Instructor: Marc-Andreas Muendler
E-mail: muendler@ucsd.edu

Covered Interest Rate Parity (CIP) relates the nominal interest rate in any economy, the United States say, to the nominal interest rate in any other economy, Europe say, and the forward premium on the nominal exchange rate between the two economies' currencies:

$$R_{\text{USD}} = R_{\text{EUR}} + f.$$

So, CIP states in a short equation that any nominal interest rate gain of USD cash deposits over EUR cash deposits, $R_{\text{USD}} - R_{\text{EUR}}$ will be wiped out completed by the depreciation of the USD against the EUR, as reflected in the forward premium f. Formally, the forward premium is defined as

$$f \equiv \frac{F - E}{E} = \frac{F}{E} - 1,$$

where F is the nominal forward exchange rate between the USD and the EUR and E is the nominal spot exchange rate between the USD and the EUR.

We say there *is* an arbitrage whenever

- there is no investment,
- there is no risk,
- but there is a profit.

In financial-market equilibrium, such a free lunch will not prevail for long. If there is an arbitrage, market participants will exploit the opportunity, and prices adjust until there is no more gain from an arbitrage.

So, financial-market equilibrium implies that there is *no arbitrage*. This means that whenever

- there is no investment,
- and there is no risk,
- then there must not be a profit.

The present short note on Covered Interest Rate Parity (CIP) shows that *no arbitrage* implies that CIP holds in any financial-market equilibrium. A typical argument in finance goes like this: "Dear reader, consider the following portfolio, an arbitrage portfolio. It involves no investment, and no risk. But we know, that there cannot be a profit to such a portfolio, therefore the following relationship must hold." In our case, the relationship that we want to hold is CIP. In a five step argument, we will look for an international arbitrage portfolio that is prohibited from yielding a profit and then presents us with CIP as a result.

One convenient portfolio to consider is the following one. It is not the only one; in fact, it is the counterpart to the arbitrage portfolio that you see in lecture. Say, our investor is an American citizen. She constructs her portfolio in five steps.

1. Borrow 1 EUR on the European capital market for 30 days. After 30 days, pay both principal and interest on this loan, $(1 + R_{\text{EUR}})$, to your European lender in EUR. (Note that there is a risk because the USD/EUR exchange rate in 30 days from now is uncertain. But our investor will conveniently get rid of the risk.)

2. Enter a forward contract. In this contract specify with your partner that you want him to pay $(1 + R_{\text{EUR}})$ EUR to you in exaclty 30 days from now. Also specify that you will pay him exactly $F \cdot (1 + R_{\text{EUR}})$ dollars, no more and no less, in 30 days from now. With the $(1 + R_{\text{EUR}})$ EUR that you will recveive from your forward contract partner, pay back the European lender in 30 days. (So, after entering this contract, our investor has gotten rid of all risk, the second dot in the no-arbitrage condition above is satisfied.)

3. Exchange the amount of EUR that you obtained (in step 1) for E USD on the spot market.

4. Give a loan of E USD to an American for 30 days, which yields $E \cdot (1 + R_{\text{USD}})$ in 30 days, including both principal and interest. (Since our investor has just borrowed the same amount from a European lender, she has not made any net investment. So, the first dot in the no-arbitrage above is now satisfied, too.)

5. Imagine the impossible for a moment. Imagine that the gain from this portfolio in 30 days, $E \cdot (1+R_{\text{USD}})$, exceeded our investor's payments to the European lender in 30 days, $F \cdot (1+R_{\text{EUR}})$. That would be an arbitrage for her. Unfortunately, such an arbitrage cannot exist. (Similarly, imagine a European investor had followed the reversed steps with an American resident, borrowing 1 USD, and so forth. Then he would get an arbitrage if $F \cdot (1 + R_{\text{EUR}})$ exceeded $E \cdot (1 + R_{\text{USD}})$.) We can conclude that $E \cdot (1 + R_{\text{USD}}) = F \cdot (1 + R_{\text{EUR}})$ must hold in equilibrium, where there is no arbitrage, and that is CIP after some reformulations.

The reformulations are:

$$\begin{aligned} E \cdot (1 + R_{\text{USD}}) &= F \cdot (1 + R_{\text{EUR}}) \\ (1 + R_{\text{USD}}) &= \frac{F}{E} \cdot (1 + R_{\text{EUR}}) \\ 1 + R_{\text{USD}} &= (1 + f) \cdot (1 + R_{\text{EUR}}) = 1 + R_{\text{EUR}} + f + f \cdot R_{\text{EUR}}. \end{aligned}$$

Note that $f \cdot R_{\text{EUR}} \approx 0$ because both f and R_{EUR} are small (percentage) numbers. So, multiplying them makes the term negligibly small, and we have CIP after all:

$$R_{\text{USD}} = R_{\text{EUR}} + f.$$