Economics 246 — Fall 2005

International Macroeconomics

Problem Set 3

November 8, 2005

Due:	**Tue, November 29, 2005**
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1 Trending Fundamentals in a Target Zone Model (Froot and Obstfeld 1991)

Suppose the exchange rate follows

$$e_t = k_t + \frac{\eta}{h}\mathbb{E}_t\left[\mathrm{d}e_{t+h}\right],$$

where k_t is the fundamental value. As opposed to the model in class, suppose the change in the fundamental value follows the trending process

$$\mathrm{d}k_{t+h} = \mu h + h^{1/2}\nu\mathrm{d}z_{t+h},$$

where $\mathrm{d}z_{t+h}$ is drawn from a mean-zero i.i.d. normal distribution with unit variance. Under a free float $e_t = k_t$. Suppose the Home and Foreign central bank announce a target zone for the exchange rate: $e_t \in (\underline{e}, \overline{e})$.

This problem asks you to show step by step that the equilibrium exchange rate process must satisfy

$$e = G(k) = k + \eta\mu G'(k) + \frac{\eta\nu^2}{2}G''(k), \tag{1-1}$$

which has a solution of the form

$$G(k) = k + \eta\mu + b_1\exp\{\lambda_1 k\} + b_2\exp\{\lambda_2 k\} \tag{1-2}$$

for certain values of λ_1 and λ_2.

1. Verify that equation (1-2) is a solution to (1-1).
2. Suppose that $e = G(k)$ is a solution, where $G(\cdot)$ is twice continuously differentiable. Show that
$$G(k_t) = k_t + \frac{\eta}{h}\mathbb{E}_t\left[G(k_{t+h}) - G(k_t)\right]$$
3. Use a second-order Taylor approximation of $\mathbb{E}_t\left[G(k_{t+h}) - G(k_t)\right]$ around k_t to show that
$$\mathbb{E}_t\left[\mathrm{d}G(k_{t+h})\right] = \mu hG(k_t) + \frac{hv^2}{2}G''(k_t).$$
[*Hint*: Show that $\mathbb{E}_t\left[G'(k_t)\mathrm{d}k_{t+h}\right] = \mu hG'(k_t)$.]

4. Using these results, show that

$$G(k) = k + \eta\mu G'(k) + \frac{\eta\nu^2}{2} G''(k).$$

is the solution.

5. A general solution to a second-order differential equation of this form is $G(k) = k + \alpha + \alpha_1 \exp(\lambda_1 k) + \alpha_2 \exp(\lambda_2 k)$ for some constants α_1, α_2. Show that the previous results imply

$$G(k) = k + \eta\mu G'(k) + \frac{\eta\nu^2}{2} G''(k).$$

Verify that, for appropriately matched parameters, this is a solution. What quadratic equation determines the values λ_1, λ_2?

6. A particular solution to the general solution (1-2) can be found by imposing restrictions that e must satisfy at the limits of the band. Describe how the particular solution can be derived. [You do not need to derive it.]

2 Open-economy Bank Runs

The Diamond-Dybvig model of bank runs is extended to an open-economy setting, in which international borrowing and lending at a gross interest rate 1 is possible (net real interest rate 0). There are three periods and domestic agents live on a unit interval (with measure 1). Agents do not discount future consumption in their decision but face uncertainty as to what their preferences will be in the intermediate period 1. Impatient agents do not derive any utility from consumption after the intermediate period 1, whereas patient agents are indifferent between consumption in period 1 or 2.

1. Assume there are no banks, and no international borrowing or lending. In period 0, domestic agents invest their endowment z in the domestic technology, which yields a gross return r after one period and a gross return R after two periods. In period 1, each agent learns if he or she is "impatient", which occurs with probability π, or "patient," which happens with probability $1 - \pi$. Impatient agents liquidate their investments immediately for a gross return of $r < 1$ (a net return $r - 1 < 0$) and consume. Patient types let their investments mature until period 2 and consume zR units then, where $R > 1$. An agent derives utility $u(c)$ from consumption of c at any period and there is no discounting. What is the expected utility of a representative agent? What is expected output in periods 1 and 2?

2. Introduce a perfectly competitive banking sector, which earns zero profits in equilibrium. Banks have the ability to borrow and lend abroad at the net interest rate of 0 but their debt to foreign lenders can never exceed the limit $\overline{f}$. Assume that nonbank domestic residents have no direct access

to the international capital market, only through banks. In period 0, domestic agents deposit their endowments in banks. Banks can borrow and lend abroad, and invest in domestic technology. Suppose the banks simply lend πz abroad in period 0, drawing on these funds to pay off impatient depositors in period 1. Show that the expected utility of a representative agent is higher than in part 1. Why?

3. Banks can do better than the arrangement of part 2. Let x denote the (period 1) consumption of an impatient type and y the (period 2) consumption of a patient type. x^* and y^* denote first-best consumption levels. Suppose banks guarantee each impatient depositor x^* in period 1 and each patient depositor y^* in period 2. To do this, they borrow (or lend) the amount $\overline{f} - \pi x^*$ in period 0 and invest $k^* = z + \overline{f} - \pi x^*$ in the domestic technology. They draw on foreign balances to disburse πx^* in period 1, thereby paying off impatient depositors. In period 2 they repay any net borrowing from periods 0 and 1 and pay patient depositors a total of $(1-\pi)y^*$. Show that, under this banking arrangement, the economy faces the intertemporal budget constraint

$$\pi R x + (1-\pi) y = R w,$$

where $w \equiv z + \overline{f} - R/\overline{f}$.

4. Consider iso-elastic utility $u(c) = (c^{1-\rho} - 1)/(1-\rho)$. By maximizing expected utility in period 0, show that a representative domestic agent consumes the first-best levels x^* and y^*

$$x^* = \frac{\theta w}{\pi} \qquad \text{and} \qquad y^* = \frac{(1-\theta) R w}{1-\pi},$$

where

$$\theta \equiv \frac{\pi R^{\frac{\rho-1}{\rho}}}{\pi R^{\frac{\rho-1}{\rho}} + (1-\pi)}.$$

5. Verify that $y^* > x^*$ is satisfied so that patient depositors have no incentive to pretend they are impatient.

6. Show that, by the beginning of period 2, banks will always have borrowed up to their foreign credit limit $\overline{f}$.

7. Now consider bank runs in period 1. Let l be the amount of the investment k^* that banks must liquidate to pay off depositors who withdraw their funds in period 1. Suppose that banks can pre-commit to suspending domestic convertibility of their deposits once period 1 withdrawals force them to liquidate sufficient amounts of their investment that $R(k^* - l) = \overline{f}$. So, foreign creditors can be certain of being repaid in full, even if there is a run by domestic depositors and the banks close their doors to them.

Let $\hat{a}$ satisfy $R(k^* - \hat{a}) = \overline{f}$. Show that a run on the first-best banking arrangement is possible if

$$x^* > \pi x^* + r\hat{a}.$$

Here, x^* represents the bank's total potential short-term obligations, which it would be liable to satisfy if all depositors claimed to be impatient. Calculate x^* and k^* and show that a sufficient condition for a run to be possible is that $\rho \geq 1$.

3 Small-country Redux

The small-country case of a standard open-economy redux model can be derived without solving the intricacies of the full two-country model.

Assume that the small country consumes only a single imported good and a single exported good, over which it has some monopoly power. In particular, the demand curve that the small country faces is

$$y^d = p^{-\theta} C^W,$$

where p is the relative world price of the domestic good and C^W is the exogenous level of world demand. This price p has a negligible (no) effect on the foreign commodity price index P^*.

The small country faces an exogenous own-rate of interest r on the imported good. The small country has no effect on world prices or world aggregate variables.

Assume that absolute Purchasing Power Parity $P = EP^*$ and the Fisher Parity $1 + i_{t+1} = (1 + r)\frac{P_{t+1}}{P_t}$ hold.

1. Let p_y be the home-currency price of the single exported good and P the home-currency price of the imported good. Verify that world-demand can then be rewritten as
$$y^d = \left(\frac{p_y}{P}\right)^{-\theta} C^W.$$

2. Suppose the representative agent maximizes
$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[\ln C_s + \chi \ln \frac{M_s}{P_s} - \frac{\kappa}{2} y_s^2\right],$$
where C is consumption of the single imported good. The period budget constraint is
$$P_t B_{t+1} + M_t = (1+r) P_t B_t + M_{t-1} + p_{y,t} y_t - P_t C_t - P_t T_t.$$
Assume that $(1+r)\beta = 1$, and suppose the steady-state levels of the following variables satisfy $\overline{p}_y = \overline{EP}^*$, $\overline{C} = \overline{C}^W$ so that $\overline{p}_y = \overline{P}$. Standardize log

variables to be $c^w = p^* = 0$ so that $e_t = p_t$. Derive the agent's first-order conditions, log-linearize around the steady state, and show that

$$
\begin{aligned}
\hat{c}_{t+1} &= \hat{c}_t && \text{(3-1)}\\
\hat{m}_t - \hat{e}_t &= \hat{c}_t - \tfrac{1}{r}(\hat{e}_{t+1} - \hat{e}_t) && \text{(3-2)}\\
(\theta + 1)\hat{y}_t &= -\theta \hat{c}_t, && \text{(3-3)}
\end{aligned}
$$

where hats denote log deviations from initial steady state.

3. Show that $\hat{y}_t = \theta(\hat{e}_t - \hat{p}_{y,t})$ under the assumptions made so that

$$\hat{b}_{t+1} = (1{+}r)\hat{b}_t + (\theta - 1)(\hat{e}_t - \hat{p}_{y,t}) - \hat{c}_t,$$

where hats denote log deviations from initial steady state.

4. Assume that the small-country-currency price $p_{y,t}$ of the export good is set one period in advance, and reverts to its flexible-price level after a single period absent new shocks. Derive the responses of consumption, real money demand, and the terms of trade in the long term, that is between two steady states.

$$
\begin{aligned}
\hat{\overline{c}} &= \tfrac{\theta+1}{2\theta}\, r\hat{\overline{b}}\\
\hat{\overline{e}}_s - \hat{\overline{p}}_y &= -\tfrac{1}{2\theta}\, r\hat{\overline{b}}\\
\hat{\overline{m}} - \hat{\overline{p}} &= \hat{\overline{c}}
\end{aligned}
$$

5. Show that there cannot be overshooting following a Dornbusch-style unanticipated permanent monetary expansion $\hat{\overline{m}} = \hat{m}$.

6. Show that the equilibrium exchange rate must satisfy

$$e = \frac{r(1+\theta) + 2\theta}{\theta r(1+\theta) + 2\theta}\, m.$$

[*Hint*: Show that $e = \overline{e}$, $\overline{b} = (\theta - 1)e - c$ and $e = \frac{r(1+\theta)+2\theta}{r(\theta^2-1)} c$.]