Econ 202a Spring 2000
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Problem Set 2: Suggested Solutions

1 Question 1

In our economy all variables are given in logarithmic form. Aggregate demand is determined by

$$Y_t^d = M_t - P_t + V_t, \tag{1}$$

where the random variable V_t follows an AR(1) process with

$$V_t = \rho_1 V_{t-1} + \tilde{\epsilon}_t \quad \text{for } \tilde{\epsilon}_t \overset{iid}{\sim} N(0, \sigma_\epsilon^2) \text{ and } \rho_1 \in (0,1). \tag{2}$$

Aggregate supply results from a wage-setting process where wages are predetermined one period in advance for the following period. Let's denote wages for period t with ${}_{t-1}W_t$ for they are set at period $t-1$. So

$$W_t = {}_{t-1}W_t.$$

Rational individuals will anticipate the expected price level when they choose wages. Hence, let's assume that

$${}_{t-1}W_t = \mathbb{E}_{t-1}[P_t].$$

Then a natural aggregate supply function (similar to a Lucas supply function) becomes

$$\begin{aligned} Y_t^s &= \alpha + (P_t - W_t) + U_t \\ &= \alpha + (P_t - \mathbb{E}_{t-1}[P_t]) + U_t, \end{aligned} \tag{3}$$

where the random variable U_t also follows an AR(1) process with

$$U_t = \rho_2 U_{t-1} + \tilde{\eta}_t \quad \text{for } \tilde{\eta}_t \overset{iid}{\sim} N(0, \sigma_\eta^2) \text{ and } \rho_2 \in (0,1). \tag{4}$$

Finally, the central bank commits itself to the monetary rule

$$M_t = M_{t-1} + AU_{t-1} + BV_{t-1}. \tag{5}$$

1.1 [1a] Equilibrium price level

The prevailing price level in each period t will clear markets so that aggregate supply equals aggregate demand. Setting (1) equal to (3) and simplifying yields

$$P_t = \frac{1}{2}\left(M_t - \alpha + \mathbb{E}_{t-1}\left[P_t\right] - V_t - U_t\right). \tag{6}$$

If we want to express the price level at t in terms of past conditional expectations, we can use the facts that $\mathbb{E}_{t-1}[V_t] = \rho_1 V_{t-1}$ and $\mathbb{E}_{t-1}[U_t] = \rho_2 U_{t-1}$, along with (2) and (4), to rewrite (6) as

$$P_t = \frac{1}{2}\left(M_t - \alpha + \mathbb{E}_{t-1}\left[P_t\right] - \mathbb{E}_{t-1}\left[V_t\right] - \mathbb{E}_{t-1}\left[U_t\right] - (\tilde{\epsilon}_t + \tilde{\eta}_t)\right).$$

1.2 [1b] Expected equilibrium price level

When the rational individuals in this economy select the wage for next period, they form *conditional* expectations about the prevailing price level in the next period. Taking conditional expectations of both sides in (6), we find

$$\mathbb{E}_{t-1}\left[P_t\right] = \frac{1}{2}\left(\mathbb{E}_{t-1}\left[M_t\right] - \alpha + \mathbb{E}_{t-1}\left[P_t\right] - \mathbb{E}_{t-1}\left[V_t\right] - \mathbb{E}_{t-1}\left[U_t\right]\right)$$

or

$$\mathbb{E}_{t-1}\left[P_t\right] = M_t - \alpha - \mathbb{E}_{t-1}\left[V_t\right] - \mathbb{E}_{t-1}\left[U_t\right]. \tag{7}$$

In the derivation we have simplified $\mathbb{E}_{t-1}\left[\mathbb{E}_{t-1}[P_t]\right] = \mathbb{E}_{t-1}[P_t]$. Conditioning twice on the *same* information set is as if we only conditioned once. In addition, we made use of the fact that $\mathbb{E}_{t-1}\left[M_t\right] = M_{t-1} + AU_{t-1} + BV_{t-1} = M_t$ by the monetary rule (5). The fact that $\mathbb{E}_{t-1}\left[M_t\right] = M_t$ is the key property of the monetary rule. As a result, systematic monetary policy will play no role at all for the forecasting error in this model.

1.3 [1c] Equilibrium output

Aggregate output will depend on the forecasting error $P_t - \mathbb{E}_{t-1}\left[P_t\right]$ that the individuals make in an uncertain world. In particular, aggregate supply is given as

$$Y_t^s = \alpha + (P_t - \mathbb{E}_{t-1}[P_t]) + U_t$$

by (3). Subtracting the expected price (7) from the true equilibrium price (6), we find the forecasting error

$$P_t - \mathbb{E}_{t-1}\left[P_t\right] = -\frac{1}{2}\left(\tilde{\epsilon}_t + \tilde{\eta}_t\right). \tag{8}$$

So, aggregate output simply becomes

$$Y_t^* = \alpha - \frac{1}{2}\left(\tilde{\epsilon}_t + \tilde{\eta}_t\right) + U_t$$

in equilibrium. Using the fact that $\mathbb{E}_{t-1}[U_t] = \rho_2 U_{t-1}$ once more, aggregate output can also be expressed as

$$Y_t^* = \alpha + \mathbb{E}_{t-1}[U_t] + \frac{1}{2}\left(\tilde{\eta}_t - \tilde{\epsilon}_t\right). \tag{9}$$

1.4 [1d] Properties of the equilibrium

With this result for equilibrium output at hand, we want to know how the fluctuations of aggregate output (9) are affected by particular monetary rules. For this, we will be interested in the magnitude of output deviations from the trend (or expected output) and in the characteristics of the business cycle. In plain statistical terms, we want to know the unconditional variance and autocovariance of Y_t^*. Note that we take the perspective of an outside observer for this purpose. So, we do not care about conditional expectations, conditional variances or conditional autocovariances. We want to derive the purely unconditional relationships.

1.4.1 *Statistical properties of the underlying stochastic process U_t*

As the equation for aggregate output, (9), suggests, the statistical properties of the supply side disturbances will matter. So, let's first derive the unconditional expectation, variance and autocovariance for U_t. By the assumption that $\rho_2 \in (0,1)$, we know that U_t must be stationary. Therefore, $\mathbb{E}[U_t] = \mathbb{E}[U_{t-s}]$ for all s. Taking *unconditional* expectations of both sides of (4), we find $\mathbb{E}[U_t] = \rho_2 \mathbb{E}[U_{t-1}]$. Hence, U_t must have a mean of zero,

$$\mathbb{E}[U_t] = 0. \tag{10}$$

The variance of U_t can be derived along similar lines. Since U_t has zero mean, $\mathbb{V}\text{ar}(U_t) = \mathbb{E}[U_t^2] = \rho_2^2 \mathbb{V}\text{ar}(U_{t-1}) + \sigma_\eta^2$. And since $\mathbb{V}\text{ar}(U_t) = \mathbb{V}\text{ar}(U_{t-1})$, by stationarity of U_t,

$$\mathbb{V}\text{ar}(U_t) = \frac{\sigma_\eta^2}{1 - {\rho_2}^2}. \tag{11}$$

Finally, remember that the *unconditional* autocovariance function of an AR(p) process follows an AR(p) process as well [check for our case of an AR(1) or go back to problem set #1, question 1]. Hence,

$$\gamma_U(s) \equiv \mathbb{C}\text{ov}(U_t, U_{t-s}) = (\rho_2)^s \frac{\sigma_\eta^2}{1 - {\rho_2}^2}. \tag{12}$$

1.4.2 *Expected value of* Y_t^*

Taking *unconditional* expectations of both sides of the equilibrium output equation (9), we find

$$\begin{aligned} \mathbb{E}[Y_t^*] &= \alpha + \mathbb{E}\left[\mathbb{E}_{t-1}[U_t]\right] = \\ &= \alpha + \mathbb{E}\left[\rho_2 U_{t-1}\right] = \alpha. \end{aligned} \tag{13}$$

1.4.3 *Variance of* Y_t^*

With this at hand, the *unconditional* variance of Y_t^* becomes

$$\begin{aligned} \mathbb{V}\text{ar}(Y_t^*) &= \mathbb{E}\left[\left(\mathbb{E}_{t-1}[U_t] + \frac{1}{2}\left(\tilde{\eta}_t - \tilde{\epsilon}_t\right)\right)^2\right] = \\ &= \mathbb{E}\left[\left(\rho_2 U_{t-1}\right)^2 + \rho_2 U_{t-1}\left(\tilde{\eta}_t - \tilde{\epsilon}_t\right) + \frac{1}{4}\left(\tilde{\eta}_t - \tilde{\epsilon}_t\right)^2\right] = \\ &= {\rho_2}^2 \frac{\sigma_\eta^2}{1 - {\rho_2}^2} + \frac{1}{4}\left(\sigma_\eta^2 + \sigma_\epsilon^2\right) = \\ &= \frac{5{\rho_2}^2 - 1}{4(1 - {\rho_2}^2)}\sigma_\eta^2 + \frac{1}{4}\sigma_\epsilon^2. \end{aligned} \tag{14}$$

The standard deviation of Y_t^* from its mean α, that is the square root of (14), can be interpreted as the amplitude of the business cycle.

1.4.4 *Autocovariance of* Y_t^*

Finally, the unconditional autocovariance of Y_t^* is given by

$$\begin{aligned}
\mathbb{C}\text{ov}(Y_t^*, Y_{t-s}^*) &= \mathbb{E}\Bigg[\left(\mathbb{E}_{t-1}[U_t] + \frac{1}{2}(\tilde{\eta}_t - \tilde{\epsilon}_t)\right) \times \\
&\quad \left(\mathbb{E}_{t-s-1}[U_{t-s}] + \frac{1}{2}(\tilde{\eta}_{t-s} - \tilde{\epsilon}_{t-s})\right)\Bigg] \\
&= \mathbb{E}\left[\rho_2 U_{t-s-1} \cdot \rho_2 U_{t-s-2}\right] = {\rho_2}^2 \cdot \gamma_U(s) = \\
&= (\rho_2)^{s+2} \frac{\sigma_\eta^2}{1 - {\rho_2}^2} \qquad \text{for } s \geq 1. \qquad (15)
\end{aligned}$$

Hence, output is serially correlated. The wage-setting process, where wages must be predetermined one period in advance, causes a business cycle.

1.5 [1e] Impact of the monetary rule

The monetary rule (5) has no impact on output because the wage-setting process is not staggered, that is the wage choice is not dependent on any lagged expectations. As the derivations in section 1c) have shown, fluctuations of equilibrium output mainly depend on the forecasting error:

$$Y_t^s - \alpha = (P_t - \mathbb{E}_{t-1}[P_t]) + U_t.$$

A key property of the monetary rule is that $\mathbb{E}_{t-1}[M_t] = M_t$. Thus, the forecasting error

$$P_t - \mathbb{E}_{t-1}[P_t] = M_t - \mathbb{E}_{t-1}[M_t] - \frac{1}{2}(\tilde{\epsilon}_t + \tilde{\eta}_t) = -\frac{1}{2}(\tilde{\epsilon}_t + \tilde{\eta}_t)$$

is independent of the monetary rule, and hence aggregate output must be independent of any systematic monetary policy. Only a stochastic monetary 'rule' where $M_t - \mathbb{E}_{t-1}[M_t] = \tilde{\zeta}_t$ for some completely random $\tilde{\zeta}_t$ with mean zero could change this result. Then $\tilde{\zeta}_t$ would matter for aggregate output. But such a 'rule' is nonsensical since it only adds variance to output while we are trying to reduce variance.

2 Question 2

Again, let aggregate demand be determined by

$$Y_t^d = M_t - P_t + V_t,$$

as in (1), where the random variable V_t follows an AR(1) process with

$$V_t = \rho_1 V_{t-1} + \tilde{\epsilon}_t \quad \text{for } \tilde{\epsilon}_t \overset{iid}{\sim} N(0, \sigma_\epsilon^2) \text{ and } \rho_1 \in (0,1),$$

as in (2).

However, aggregate supply now results from a *staggered* wage setting process in which wages have to be predetermined more than one period in advance. Half of the wages in the economy are determined at even periods of time for the two periods in advance. The other half of the wages are determined at odd periods of time for the two periods in advance. As opposed to the Taylor model, however, individuals in the Fischer model can predetermine two different wages for the two following periods, one for each.

Let's use a similar notation as in question 1. Denote wages that are set two periods in advance with ${}_{t-2}W_t$ and wages that are set one period in advance with ${}_{t-1}W_t$. So, the prevailing economy-wide wage at period t is

$$W_t = \frac{1}{2}({}_{t-1}W_t +_{t-2} W_t).$$

Rational individuals will anticipate the expected price level when they choose wages. So, let's assume that

$${}_{t-i}W_t = \mathbb{E}_{t-i}[P_t].$$

Then our aggregate supply function becomes

$$\begin{aligned} Y_t^s &= (P_t - W_t) + U_t \\ &= (P_t - \frac{1}{2}(\mathbb{E}_{t-1}[P_t] + \mathbb{E}_{t-2}[P_t])) + U_t, \end{aligned} \tag{16}$$

where the random variable U_t follows the same AR(1) process

$$U_t = \rho_2 U_{t-1} + \tilde{\eta}_t \quad \text{for } \tilde{\eta}_t \overset{iid}{\sim} N(0, \sigma_\eta^2) \text{ and } \rho_2 \in (0,1)$$

as in (4) before. (For simplicity we choose $\alpha = 0$ from now on.)

Again, the central bank commits itself to the monetary rule

$$M_t = M_{t-1} + AU_{t-1} + BV_{t-1},$$

as in (5).

2.1 [2a] Equilibrium output

The prevailing price level at period t clears markets in the aggregate. Setting aggregate demand (1) equal to aggregate supply (3) and simplifying yields

$$P_t = \frac{1}{4}\left(\mathbb{E}_{t-1}[P_t] + \mathbb{E}_{t-2}[P_t]\right) + \frac{1}{2}\left(M_t - V_t - U_t\right). \tag{17}$$

In order to solve for P_t, we must first express $\mathbb{E}_{t-2}[P_t]$ and $\mathbb{E}_{t-1}[P_t]$ in terms of underlying variables. For this purpose, we can take expectations of (17) conditional on the information at $t-2$ and $t-1$, respectively.

2.1.1 *Expected price level at* $t-2$*:* $\mathbb{E}_{t-2}[P_t]$

For convenience, let's start with the expectations farthest in the past. Applying $\mathbb{E}_{t-2}[\cdot]$ to both sides of (17) yields

$$\begin{aligned}
\mathbb{E}_{t-2}\left[P_t\right] &= \frac{1}{4}\mathbb{E}_{t-2}\left[\mathbb{E}_{t-1}[P_t]\right] + \frac{1}{4}\mathbb{E}_{t-2}[\mathbb{E}_{t-2}[P_t]] \\
&\quad + \frac{1}{2}\mathbb{E}_{t-2}\left[M_t\right] - \frac{1}{2}\mathbb{E}_{t-2}\left[V_t\right] - \frac{1}{2}\mathbb{E}_{t-2}\left[U_t\right] \\
&= \frac{1}{4}\mathbb{E}_{t-2}\left[\mathbb{E}_{t-1}[P_t]\right] + \frac{1}{4}\mathbb{E}_{t-2}[\mathbb{E}_{t-2}[P_t]] \\
&\quad + \frac{1}{2}M_{t-2} + \frac{1}{2}\left[B\left(1+\rho_1\right) - \rho_1^2\right] V_{t-2} \\
&\quad + \frac{1}{2}\left[A\left(1+\rho_2\right) - \rho_2^2\right] U_{t-2}.
\end{aligned} \tag{18}$$

The second equality follows from the facts that

$$\mathbb{E}_{t-2}\left[V_t\right] = \rho_1^2 V_{t-2},$$

$$\mathbb{E}_{t-2}\left[U_t\right] = \rho_2^2 U_{t-2},$$

and

$$\mathbb{E}_{t-2}\left[M_t\right] = M_{t-2} + B\left(1+\rho_1\right)V_{t-2} + A\left(1+\rho_2\right)U_{t-2}.$$

Remember that, in general, all conditional expectations are functions of the variables on which we condition. So, to derive $\mathbb{E}_{t-2}\left[M_t\right]$, $\mathbb{E}_{t-2}\left[V_t\right]$, and

$\mathbb{E}_{t-2}[U_t]$, we use recursive relationships of the variables until we find a functional form that only involves lagged variables and no expectation operators.

Before we proceed, we need to further simplify (18) and express the conditional expectations $\mathbb{E}_{t-2}[\mathbb{E}_{t-2}[P_t]]$ and $\mathbb{E}_{t-2}[\mathbb{E}_{t-1}[P_t]]$ in simpler terms. Clearly, conditioning twice on the same information set is as if we only conditioned once. Hence, $\mathbb{E}_{t-2}[\mathbb{E}_{t-2}[P_t]] = \mathbb{E}_{t-2}[P_t]$. Things are more subtle for $\mathbb{E}_{t-2}[\mathbb{E}_{t-1}[P_t]]$. In order to simplify we need to argue that the forecasting error, which the individuals make at $t-1$, is *independent* of all previous observations. This is true in our model,[1] and we usually assume it. So $\mathbb{E}_{t-2}[\mathbb{E}_{t-1}[P_t]] = \mathbb{E}_{t-2}[P_t]$. (In general, you can take for granted that $\mathbb{E}_{s-i}[\mathbb{E}_s[P_t]] = \mathbb{E}_{s-i}[P_t]$ for $i \geq 0$ in all our models). Hence, we can further simplify (18) and obtain

$$\mathbb{E}_{t-2}[P_t] = M_{t-2} + [B + (B - \rho_1)\rho_1] V_{t-2} + [A + (A - \rho_2)\rho_2] U_{t-2}. \quad (19)$$

[1] In detail: First, note that we can generally write $P_t = \mathbb{E}_{t-1}[P_t] + \tilde{\xi}_t$, where $\tilde{\xi}_t$ is the forecasting error that we make when taking the conditional expectation at $t-1$. By the properties of $\mathbb{E}_{t-1}[\cdot]$, the forecasting error $\tilde{\xi}_t$ must have an unconditional mean of zero and must be *uncorrelated* with the previous realizations of all variables. This does not imply independence yet. But in our model both 'fundamental' error terms $\tilde{\epsilon}_t$ and $\tilde{\eta}_t$ are normally distributed. All endogenous variables (such as V_t, U_t, P_t, M_t, Y_t) are linear combinations of these 'fundamental' error terms. Since all linear functions of normally distributed random variables must be normally distributed random variables themselves, uncorrelatedness implies independence in our model. Hence we can write

$$\mathbb{E}_{t-2}[\mathbb{E}_{t-1}[P_t]] = \mathbb{E}_{t-2}\left[P_t - \tilde{\xi}_t\right] = \mathbb{E}_{t-2}[P_t].$$

The last step involved $\mathbb{E}_{t-2}\left[\tilde{\xi}_t\right] = \mathbb{E}\left[\tilde{\xi}_t\right] = 0$ and was permissible because stochastic independence implies mean independence (whereas uncorrelatedness would not be sufficient for mean independence). [You can check in the solution (22) later that $\mathbb{E}_{t-2}[\mathbb{E}_{t-1}[P_t]] = \mathbb{E}_{t-2}[P_t]$ is satisfied indeed.]

2.1.2 *Expected price level at* $t-1$*:* $\mathbb{E}_{t-1}[P_t]$

Similarly, applying $\mathbb{E}_{t-1}[\cdot]$ to both sides of (17) yields

$$\begin{aligned}\mathbb{E}_{t-1}\left[P_t\right] &= \frac{1}{4}\mathbb{E}_{t-1}\left[\mathbb{E}_{t-1}[P_t]\right]+\frac{1}{4}\mathbb{E}_{t-1}[\mathbb{E}_{t-2}[P_t]] \\ &\quad +\frac{1}{2}\mathbb{E}_{t-1}\left[M_t\right]-\frac{1}{2}\mathbb{E}_{t-1}\left[V_t\right]-\frac{1}{2}\mathbb{E}_{t-1}\left[U_t\right] \\ &= \frac{1}{4}\mathbb{E}_{t-1}\left[\mathbb{E}_{t-1}[P_t]\right]+\frac{1}{4}\mathbb{E}_{t-1}[\mathbb{E}_{t-2}[P_t]] \\ &\quad +\frac{1}{2}M_{t-1}+\frac{1}{2}BV_{t-1}+\frac{1}{2}AU_{t-1}, \end{aligned} \tag{20}$$

since $\mathbb{E}_{t-1}\left[M_t\right]=M_{t-1}+BV_{t-1}+AU_{t-1}$, $\mathbb{E}_{t-1}\left[V_t\right]=\rho_1 V_{t-1}$, and $\mathbb{E}_{t-1}\left[U_t\right]=\rho_2 U_{t-1}$. Again, conditioning twice on the same information set is as if we only conditioned once. Hence, $\mathbb{E}_{t-1}\left[\mathbb{E}_{t-1}[P_t]\right]=\mathbb{E}_{t-1}[P_t]$. Things are simpler for $\mathbb{E}_{t-1}[\mathbb{E}_{t-2}[P_t]]$ this time. Note that the conditional expectation inside, $\mathbb{E}_{t-2}[P_t]$, is some function of (potentially) all past realizations of our random variables and endogenous variables at $t-2$ or earlier. It cannot be a function of any later variable. Therefore, $\mathbb{E}_{t-1}[\mathbb{E}_{t-2}[P_t]]=\mathbb{E}_{t-2}[P_t]$. From above, (19), we know $\mathbb{E}_{t-2}[P_t]$ already. Using (19) in (20) and simplifying yields

$$\begin{aligned}\mathbb{E}_{t-1}\left[P_t\right] &= \frac{1}{3}\big(2M_{t-1}+M_{t-2}\big) \\ &\quad +\frac{1}{3}\big(2\left(B-\rho_1\right)V_{t-1}+\left[B+\left(B-\rho_1\right)\rho_1\right]V_{t-2}\big) \\ &\quad +\frac{1}{3}\big(2\left(A-\rho_2\right)U_{t-1}+\left[A+\left(A-\rho_2\right)\rho_2\right]U_{t-2}\big). \end{aligned} \tag{21}$$

2.1.3 *Equilibrium price level* P_t

Putting these results for $\mathbb{E}_{t-1}\left[P_t\right]$ and $\mathbb{E}_{t-2}\left[P_t\right]$ together, we can finally solve for the equilibrium price level. By (17), (19) and (21), we have

$$\begin{aligned} P_t &= \frac{1}{4}\big(\mathbb{E}_{t-1}[P_t]+\mathbb{E}_{t-2}[P_t]\big)+\frac{1}{2}\big(M_t-V_t-U_t\big) \\ &= \frac{1}{6}\big(3M_t+M_{t-1}+2M_{t-2}\big) \\ &\quad -\frac{1}{6}\big(3V_t-\left(B-\rho_1\right)V_{t-1}-2\left[B+\left(B-\rho_1\right)\rho_1\right]V_{t-2}\big) \\ &\quad -\frac{1}{6}\big(3U_t-\left(A-\rho_2\right)U_{t-1}-2\left[A+\left(A-\rho_2\right)\rho_2\right]U_{t-2}\big). \end{aligned} \tag{22}$$

2.1.4 *Equilibrium output* Y_t^*

Aggregate output will depend on the new kind of 'forecasting error' $P_t - \frac{1}{2}(\mathbb{E}_{t-1}[P_t] + \mathbb{E}_{t-2}[P_t])$ that the individuals make under staggered wage setting. In particular, aggregate supply is now given as

$$Y_t^s = (P_t - \frac{1}{2}(\mathbb{E}_{t-1}[P_t] + \mathbb{E}_{t-2}[P_t])) + U_t$$

by (16). Using (19), (21) and (22), we can find this new kind of 'forecasting error'

$$\begin{aligned} P_t - \frac{1}{2}(\mathbb{E}_{t-1}[P_t] + \mathbb{E}_{t-2}[P_t]) &= -\frac{1}{2}(\tilde{\epsilon}_t + \tilde{\eta}_t) + \frac{1}{3}(B - \rho_1)\tilde{\epsilon}_{t-1} \\ &\quad + \frac{1}{3}(A - \rho_2)\tilde{\eta}_{t-1}. \end{aligned} \tag{23}$$

So, the monetary rule clearly matters now. By choosing A and B properly, the central bank can reduce the kind of 'forecasting error' that the individuals are forced to make through the staggered wage-setting process. Since only half of the individuals can adjust wages in each period, they make larger mistakes.

After all, aggregate output becomes

$$\begin{aligned} Y_t^* &= (P_t - \frac{1}{2}(\mathbb{E}_{t-1}[P_t] + \mathbb{E}_{t-2}[P_t])) + U_t \\ &= \mathbb{E}_{t-2}[U_t] + \frac{1}{2}(\tilde{\eta}_t - \tilde{\epsilon}_t) + \frac{1}{3}[(B - \rho_1)\tilde{\epsilon}_{t-1} + (A + 2\rho_2)\tilde{\eta}_{t-1}] \end{aligned} \tag{24}$$

in equilibrium, using the fact that $\mathbb{E}_{t-2}[U_t] = \rho_2^2 U_{t-2}$.

2.2 [2b] Variance of equilibrium output

We want to explore the properties of the equilibrium from the perspective of an outside observer. As such, we do not want to condition on any period's information, and from now on all means, variances, and autocovariances are unconditional.

Before taking the variance of equilibrium output (24), note that the unconditional mean of output is zero, $\mathbb{E}[Y_t^*] = 0$ (since $\mathbb{E}[\mathbb{E}_{t-2}[U_t]] = \rho_2^2 \mathbb{E}[U_{t-2}] = 0$ by stationarity of U_t). Also note that the covariance between

U_t and any future disturbance is zero because U_t must be independent of future realizations of the errors. In particular, $\mathbb{E}[U_{t-2}\tilde{\eta}_{t-1}] = \mathbb{E}[U_{t-2}\tilde{\epsilon}_{t-1}] = 0$. Then, the variance of output becomes

$$\begin{aligned}
\mathbb{V}\text{ar}(Y_t^*) &= \rho_2^4 \mathbb{V}\text{ar}(U_{t-2}) + \frac{1}{4}[\mathbb{V}\text{ar}(\tilde{\epsilon}_t) + \mathbb{V}\text{ar}(\tilde{\eta}_t)] \\
&\quad + \frac{1}{9}\left[(B-\rho_1)^2 \mathbb{V}\text{ar}(\tilde{\epsilon}_{t-1}) + (A+2\rho_2)^2 \mathbb{V}\text{ar}(\tilde{\eta}_{t-1})\right] \\
&= \frac{1}{36}\left[9 + 4(B-\rho_1)^2\right]\sigma_\epsilon^2 \\
&\quad + \frac{1}{36}\left[9 + 4(A+2\rho_2)^2 + \frac{36\rho_2^4}{1-\rho_2^2}\right]\sigma_\eta^2, \qquad (25)
\end{aligned}$$

since $\mathbb{V}\text{ar}(U_{t-2}) = \mathbb{V}\text{ar}(U_t) = \sigma_\eta^2/(1-\rho_2^2)$ by (11).

2.3 [2c] Minimal variance of equilibrium output

The central bank wants to set a policy that minimizes the variance of output. In order to find the best rule, we can simply minimize the terms in front of σ_η^2 and σ_ϵ^2 in (25) with respect to A and B.[2] Note that both terms are quadratic and hence convex so that the problem is well behaved, and we need not worry about second-order conditions. The first-order condition for the optimal A, A^*, is $\frac{1}{36}8(A^*+2\rho_2) = 0$, so that

$$A^* = -2\rho_2. \qquad (26)$$

[2] Of course, the central bankers would like to set the terms in front of σ_η^2 and σ_ϵ^2 to zero. Trying that, however, the central bankers would find that there are complex roots for both A and B solving the respective quadratic equations $9 + 4(B-\rho_1)^2 = 0$ and $9 + 4(A+2\rho_2)^2 + 36\rho_2^4/(1-\rho_2^2) = 0$. The respective roots are

$$A_{1,2} = -2\rho_2 \pm \frac{3}{1-\rho_2^2}\sqrt{-1 + 2\rho_2^2 - 5\rho_2^4 + 4\rho_2^6}$$

and

$$B_{1,2} = \rho_1 \pm \frac{3}{2}\sqrt{-1}.$$

They are complex. The reason in the case of A is that $2\rho_2^2 - 5\rho_2^4 + 4\rho_2^6 < 1$ for $\rho_2 \in (0,1)$. Thus, the central bankers will have to pick the real parts of the roots in optimum: $A^* = -2\rho_2$ and $B^* = \rho_1$.

Similarly, the first-order condition for B is $\frac{1}{36}8\left(B^{*}-\rho_{1}\right)=0$, and

$$B^{*}=\rho_{1}. \tag{27}$$

By setting $B^{*}=\rho_{1}$, the central bank minimizes the kind of 'forecasting error' that the individuals make in their wage-setting process. The choice of $B^{*}=\rho_{1}$ removes the lagged aggregate demand disturbance, $\tilde{\epsilon}_{t-1}$, from (23) and hence from equilibrium output (24). For the lagged aggregate supply disturbance, $\tilde{\eta}_{t-1}$, the central bank takes an additional effect into account. The central bank knows that the 'forecasting error' $P_{t}-\frac{1}{2}\left(\mathbb{E}_{t-1}\left[P_{t}\right]+\mathbb{E}_{t-2}\left[P_{t}\right]\right)$ will depend on $\tilde{\eta}_{t-1}$ as stated in (23). But the central bank also knows that aggregate supply suffers a shock U_{t-1} over the course of two periods, and this shock stems from $\tilde{\eta}_{t-1}$, too, just as the 'forecasting error.' Hence, the 'forecasting error' of the individuals and the economy-wide aggregate supply shock are correlated. Instead of minimizing the 'forecasting error' *per se* (which would be minimal for $A^{*}=\rho_{2}$ as (23) shows), the variance of output can be further reduced by choosing $A^{*}=-2\rho_{2}$ and removing the correlation between the 'forecasting error' and output.

2.4 [2d] Effects of expected monetary policy

Expected monetary policy beyond the planning horizon of the individuals has no effect at all. For example, an announcement that the money supply will increase by five percent in two periods from now does not have any effect on output today. (This result is in contrast to Taylor's model, and in contrast to representative agent models with long planning horizons.)

Suppose that M_{t}, M_{t+1}, and M_{t+2} follow the monetary rule as given in (5): $M_{s}=M_{s-1}+AU_{s-1}+BV_{s-1}$. Suppose in addition that the monetary rule changes in period 3 for one period:

$$M_{t+3}=1.05M_{t+2}+AU_{t+2}+BV_{t+2},$$

but returns to the previous form (5) afterwards. When setting wages, the individuals in our model economy only form expectations over the next two periods. Hence, at time t, half of the individuals make their decision for the next two periods, and the other half has already chosen a period ago, at $t-1$. The group who is deciding at t forms expectations according to (19):

$$\mathbb{E}_{t}\left[P_{t+2}\right]=M_{t}+\left[B+\left(B-\rho_{1}\right)\rho_{1}\right]V_{t}+\left[A+\left(A-\rho_{2}\right)\rho_{2}\right]U_{t}.$$

Nothing has changed as compared to the situation where M_{t+3} followed the old rule.

At time $t+1$, however, there is, for the first time, a group of individuals who cares about M_{t+3}. When this group forms expectations about P_{t+3}, $\mathbb{E}_{t+1}[P_{t+2}]$, the new monetary rule matters. They will set

$$\begin{aligned}\mathbb{E}_{t+1}[M_{t+3}] &= \mathbb{E}_{t+1}[1.05M_{t+2} + AU_{t+2} + BV_{t+2}] \\ &= \mathbb{E}_{t+1}[1.05(M_{t+1} + AU_{t+1} + BV_{t+1}) + AU_{t+2} + BV_{t+2}] \\ &= 1.05M_{t+1} + B(1.05 + \rho_1)V_{t+1} + A(1.05 + \rho_2)U_{t+1}.\end{aligned}$$

Does the group who set wages at t care about the fact that the group at $t+1$ will take this decision? No, not in the Fischer model. Even though wages are *predetermined* two periods in advance by half of the individuals, they are not *fixed* across future periods. That is, the wage that the group at $t+1$ sets for period $t+2$ can be different from the wage that this group sets for $t+3$. Therefore, when the group at t makes its decision about $t+2$, it need not care about anything that is about to happen at $t+3$.

2.5 [2e] Effects of expected monetary policy in the Taylor model

This result would be different in the Taylor model. The reason is that in Taylor's model wages are not only *predetermined* two periods in advance, they are *fixed* in addition across both future periods. Consider the same expected change in the monetary supply at $t+3$ as in question 2d): $M_{t+3} = 1.05M_{t+2} + AU_{t+2} + BV_{t+2}$. The group at time $t+1$ will certainly care about M_{t+3} because monetary policy affects price levels and output. So, when the group at $t+1$ chooses its fixed wage for the next two periods, ${}_{t+1}\bar{W} = {}_{t+1}W_{t+2} = {}_{t+1}W_{t+3}$, the wage level at period $t+2$ is affected via M_{t+3}. The wage choice ${}_{t+1}W_{t+2}$ is going to be higher than if there were no change in M_{t+3}.

What about the group who sets wages at t? They do care about M_{t+3} now! The t-group rationally expects that the $t+1$-group will increase their wage choice ${}_{t+1}W_{t+2}$ due to the increase in M_{t+3}. But that means that the t-group better chooses its next two wages ${}_t\bar{W} = {}_tW_{t+1} = {}_tW_{t+2}$ taking into account that ${}_{t+1}W_{t+2}$ is higher. Thus, the t-group will, in turn, set somewhat higher wages already, and so forth. Thus, announced monetary policy changes far in the future have at least a little effect on the wage choice and hence the price level today when wages are not only *predetermined*, but *fixed* in addition.