

# Saving Your Home in Chapter 13 Bankruptcy

Michelle J. White and Ning Zhu

## Theoretical Appendix

This appendix gives derivations of the boundaries of the four default/bankruptcy regions shown in figure 1. Debtors' decisions to default on the mortgage and file for bankruptcy are considered separately for five ranges of housing values, starting from the lowest.

**Case (i):**  $V < M'_2 - R'$ . Here, the value of the debtors' homes in period 1 is so low that the cost of alternate rental housing in present value terms,  $R'$ , is less than the cost of owning  $M'_2 - V$ , even if debtors make no mortgage payments during the repayment period.

First consider debtors' bankruptcy decision when their incomes are below the state median income level. Because all of their income is exempt in bankruptcy, they receive no mortgage subsidy. They therefore prefer to default on the mortgage, regardless of whether they file for bankruptcy. Now consider their bankruptcy decisions. If they file under Chapter 13, the present discounted value of their net wealth is  $NY + V_a - A' - C_b - R'$ ; while if they do not file, it is  $NY + V_a - A' - P' - R'$ .  $V_a$  denotes the value of debtors' cars. Because  $P' > C_b$ , they prefer bankruptcy.

Now consider debtors' bankruptcy decision when their incomes are above the state median income level. If they file for bankruptcy, they receive the mortgage subsidy, which reduces the cost of owning from  $M' - V$  to as low as  $M'_2 - V$ . However even with this subsidy, they are better off defaulting on the mortgage because the cost of renting is lower than the cost of owning, i.e.,  $R' < M'_2 - V$ . They therefore choose between default/bankruptcy and default/no

bankruptcy. The present discounted value of their net wealth if they choose the former is  $(N - 5)Y + [5X_y + A'] + V_a - A' - C_b - R'$ ; while if they choose the latter it is

$NY + V_a - A' - P' - R'$ .  $Y_1^B$  denotes the income level where debtors in case (i) are indifferent between filing versus not filing for bankruptcy or,  $5Y_1^B - (5X_y + A') - C_b = P'$ . Here, debtors' gain from having their unsecured debt discharged in bankruptcy  $P'$  is just offset by the present

value of their disposable income plus bankruptcy costs,  $5Y_1^B - (5X_y + A') + C_b$ , or their costs of filing. Debtors file if their income is below  $Y_1^B$ , but not otherwise.<sup>1</sup>

Figure 1 shows debtors' period 1 income  $Y$  on the horizontal axis and their period 1 housing value  $V$  on the vertical axis. Case (i) is the lowest horizontal band. Debtors always default on their mortgages, but they file for bankruptcy if income is below  $Y_1^B$  and do not file otherwise.

**Case (ii):**  $M'_2 - R' < V < M' - R'$ . Here, housing value is higher, but the cost of rental housing  $R'$  is still less than the cost of owning,  $M' - V$ . In this range of values, the mortgage subsidy in Chapter 13 causes some debtors to change from defaulting on their mortgages to saving their homes.

Debtors whose incomes are below the income exemption face the same choice as those in case (i) and they still default and file for bankruptcy. However those whose incomes are above the exemption receive the mortgage subsidy if they keep their homes in bankruptcy and, in this case, their income during the repayment period becomes  $\min[5Y, 5X_y + A' + M'_1]$ . Consider debtors' choice whether to default. The discounted present value of their wealth if they choose default/bankruptcy is  $((N - 5)Y + \min[5Y, 5X_y + A'] + V_a - A' - R' - C_b)$ ; while if they choose no default/bankruptcy it is  $(N - 5)Y + \min[5Y, 5X_y + A' + M'_1] + V_a - A' + V - M' - C_b$ . When income during the repayment period is  $5X_y + A'$  or less, they prefer to default, and when it is  $5X_y + (M'_1 + A')$  or more, they prefer to keep their homes. The income level  $Y_2^B$  where case (2) debtors are indifferent between defaulting and not defaulting satisfies the condition

$5Y_{ii}^B - [5X_y + A'] = M' - V - R'$ . Here the extra cost to debtors of keeping their houses rather than defaulting (the right-hand side) just equals the amount of income they must use to repay unsecured debt in bankruptcy (the left-hand side). Debtors default if their income is less than  $Y_{ii}^B$  and they keep their homes otherwise.

Debtors may also be forced to default because they are liquidity-constrained, even when it is financially advantageous for them to keep their homes. The first year's cost of the repayment

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<sup>1</sup> This applied if  $5Y > 5X_y + A'$ . But the result remains the same if  $5X_y < 5Y < 5X_y + A'$ .

plan is  $C_b + (M'_1 + A')/5$  and we make the assumption that debtors are willing to use half of their income to repay the mortgage if it is financially worthwhile to do so. Debtors in case (ii) thus default if either they are liquidity-constrained or if their incomes are below the condition discussed above, so that  $5Y_{ii}^D = \max[5X_y + A' + M - V - R, 5C_b + M'_1 + A']$ .

Now consider debtors' bankruptcy decision when their incomes exceed the income exemption, or  $5Y > 5X_y + A' + M'_1$ . At a sufficiently high income level, debtors prefer not to file for bankruptcy. But if they do not file, they receive no mortgage subsidy and this means that they prefer to default rather than keep their homes. Therefore high-income debtors choose between the alternatives of default/no-bankruptcy and no-default/bankruptcy. The discounted present value of their wealth in the former situation is  $NY + V_a - A' - P' - R'$ ; while in the latter it is  $(N - 5)Y + [5X_y + M'_1 + A'] + V_a - A' + V - M' - C_b$ . Debtors are indifferent between these alternatives at the income level  $Y_2^B$  that satisfies the condition

$5Y_2^B - [5X_y + M'_1 + A'] + (M' - V) - R' + C_b = P'$ . Here, their net gain from debt discharge in bankruptcy (the right-hand side) is just offset by the cost of giving up their non-exempt income during the repayment period plus the extra cost of keeping their homes rather than renting (the left-hand side) plus the cost of filing. They choose no-default/bankruptcy if their incomes are below  $Y < Y_{ii}^B$  and they choose default/no-bankruptcy otherwise. Note that  $Y_{ii}^B$  increases as  $V$  rises, because debtors gain more from keeping their homes when home value is higher and this induces them to file for bankruptcy at higher income levels.

Assuming that  $Y_{ii}^D < Y_{ii}^B$ , debtors in case (ii) default and file for bankruptcy in the lowest income range, default but avoid bankruptcy in the highest income range, and save their homes in bankruptcy in the middle income range. However for some debtors, there is no middle income range where they save their homes (i.e.,  $Y_{ii}^B \geq Y_{ii}^D$ ). This is because they have little unsecured debt, so that the mortgage subsidy is too small to change their default decisions.

The second-lowest horizontal bar in Figure 1 shows the results in case (ii) when the middle sub-region exists.

**Case (iii).**  $M' - R' < V < M' + X_h + C_f$ . In case (iii), the cost of owned housing  $M' - V$  is less than the cost of rental housing  $R'$  and debtors therefore prefer to keep their homes. Their homes are not valuable enough to force them to repay any unsecured debt in bankruptcy.

Consider debtors whose incomes are below the state median level first. While they prefer to keep their homes, they may be liquidity-constrained. We assume that debtors are liquidity-constrained if  $5Y_{iii}^D < 5C_b + (M'_1 + A')$ . Debtors who default because they are liquidity-constrained also file for bankruptcy, since  $P' > C_b$ .<sup>2</sup>

Next consider debtors' decisions if they are not liquidity-constrained but have incomes below the income exemption. They still prefer to file for bankruptcy, so consider whether they default. The present value of their wealth if they choose default/bankruptcy is  $NY + V_a - A' - R' - C_b$ ; while the present value of their wealth if they choose no-default/bankruptcy is  $NY + V_a - A' + V - M' - C_b$ . They prefer not to default since  $V > M' - R'$ .

Finally consider debtors' bankruptcy decisions when their incomes exceed the income exemption including the mortgage subsidy. Since they do not default, the present value of their wealth if they choose no-default/bankruptcy is

$(N - 5)Y + [5X_y + M'_1 + A'] + V_a - A' + V - M' - C_b$ ; while the present value of their wealth if they choose no-default/no-bankruptcy is  $NY + V_a - A' + V - M' - P'$ . They are indifferent between the two choices at the income level  $5Y_{iii}^B = 5X_y + M'_1 + A' + P' - C_b$ .

Figure 1 shows the results for case (iii) in the middle horizontal band. Debtors default because they are liquidity-constrained and file for bankruptcy in the lowest income region and they do neither in the highest income region. In-between, they receive the mortgage subsidy and save their homes, but they would have kept their homes anyway. Compared to case (ii), fewer debtors default but more file for bankruptcy.

**Case (iv).**  $M' + X_h + C_f < V \leq M' + X_h + C_f + P' - C_b$ . Here home equity  $V - M'$  exceeds the cost of foreclosure plus the homestead exemption. Therefore in a foreclosure, the house

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<sup>2</sup> These debtors may file under Chapter 7 if they have already given up their homes. Or they might file under Chapter 13, but give up their homes when they realize that they cannot afford a repayment plan.

would sell for enough to pay  $V - M' - X_h - C_f > 0$  to unsecured creditors. Case (iv) is unlikely to occur in states with high homestead exemptions, but may occur in states with low homestead exemptions. We abbreviate  $V - M' - X_h - C_f$  as *NEHE*, for “non-exempt home equity.”

Consider debtors whose incomes are below the income exemption. Even though they have no disposable income, the “best interests of creditors” test requires that they repay unsecured creditors *NEHE*. And because they have no wealth other than their home equity, they must pay this amount from income even though their incomes would otherwise be exempt. Debtors are assumed to be liquidity-constrained if the first year’s cost of the repayment plan exceeds half of their incomes. The maximum income level at which debtors default because they are liquidity constrained is  $5Y_{iv}^D = 5C_b + (M'_1 + A' + NEHE)$ . Liquidity-constrained debtors also file for bankruptcy.

Now consider debtors who are not liquidity-constrained. They do not default, so consider their bankruptcy decisions. Their wealth if they do not file is  $NY + V_a - A' + V - M' - P'$ . Their wealth if they file is  $(N - 5)Y + \min[5Y, 5X_y + A' + M'_1] + V_a - A' + V - M' - NEHE - C_b$ , depending on whether their incomes are below or above the income exemption. In the former case, debtors prefer to file for bankruptcy as long as unsecured debt exceeds  $C_b + NEHE$ . In the latter case, debtors are indifferent between filing and not filing at an income level  $5Y_{iv}^B = 5Y_{iii}^B - NEHE$ . They file if their incomes are below  $Y_4^B$  and not otherwise.

Figure 1 shows debtors’ choices in case (iv) as the second-highest horizontal band. Debtors in case (iv) are more likely to be liquidity-constrained than those in case (iii), because case (iv) debtors must repay some of their unsecured debt in addition to repaying their mortgages. Debtors in case (iv) are less likely to file for bankruptcy as the value of their homes rises, because they must repay more to unsecured creditors in bankruptcy.

**Case (v).**  $V > M' + X_h + C_f + P' - C_b$ . In case (v), the value of the house is so high that selling it would generate enough to repay unsecured creditors in full. This means that debtors in case (v) never file for bankruptcy. They also never default on their mortgages. If they are liquidity-constrained, they prefer to sell their homes outside of bankruptcy and repay both the mortgage and their unsecured debt in full. The top horizontal bar in figure 1 shows the results in case (v).