Statistical Discrimination and Intergenerational Income Mobility

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Abstract

This paper develops a dynamic model of statistical discrimination that accounts for intergenerational income mobility. It is shown that when income is transmitted across generations through parental investments in the human capital of children, statistical discrimination can lead racial groups with low endowments of human capital to become trapped in inferior stationary equilibria. This result is surprising since it can occur when there is a unique equilibrium for each group in each generation and even though the distribution of human capital is allowed to evolve over time.

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1 Introduction

Racial wage inequality has been a persistent feature of the U.S. economy. The goal of this paper is to understand the mechanisms that have lead to ongoing disparities in the economic condition of different racial groups as well as the forces that perpetuate racial discrimination. As a framework in which to examine these issues, I develop a dynamic model of statistical discrimination that accounts for the transmission of earnings across generations.

This model builds upon the theoretical literature on racial wage inequality in several ways. First, existing models of persistent racial wage inequality consider discrimination primarily as a means to rationalize differences in either initial conditions or parameter values (e.g. Loury (1977) and Lundberg and Startz (1998)). As a result, these models provide little scope for analyzing how racial discrimination evolves over time. In contrast, in the model presented in this paper, racial discrimination arises endogenously. Thus, it is possible to examine the interaction between racial wage inequality and racial discrimination, and to ask whether we would expect both to persist across many generations.

In addition, substantial evidence suggests that, among children from similar family backgrounds, human capital accumulation is lower for blacks than for whites. For example, Neal and Johnson (1996) and Phillips et al. (1998) find that blacks' scores on standardized tests are significantly lower than those of whites, even after controlling for a broad range of family background characteristics. The model presented in this paper formalizes the idea that these patterns arise because the expectation of future discrimination lowers parents' incentives to invest in their children's human capital, even when they have the financial resources to do so. Further, it is shown that small racial differences in the return to investment are consistent with relatively large differences in observed investment patterns.

In the model, parents care about their own consumption and their children's future wage, which they can influence by investing in their children's human capital. A key feature of the model is that high-wage parents are more likely to invest than are low-wage parents. There are a variety of reasons we might expect parental investment to be increasing in parental income. Here, I focus on credit market constraints, but alternative explanations are discussed below.

Although these human capital investments are valuable the sense that they increase a child's future productivity, firms cannot perfectly observe parental investment decisions and may have different prior beliefs about the likelihood that parents from different racial groups invest. As a result, firms may use race as a proxy for unobserved productivity. In equilibrium, firms' prior beliefs are self-confirming. Thus, even if all workers are identical ex ante, in equilibrium, expected productivity and expected wages may vary by race. Further, since discrimination lowers a group's average wages, parents from disadvantaged groups will have fewer financial resources to invest in their children's human capital. As a result, firms will rationally discriminate against members of those groups in future generations.

In this setting, both racial discrimination and racial wage disparities can persist. Moreover, this can happen even when there is a unique equilibrium for each group in each generation. That is, in contrast to previous models of statistical discrimination, repeated coordination failures are not needed to generate persistent racial discrimination. In particular, it is shown that statistical discrimination changes the transmission of income across generation by leading parents' investment behavior to depend upon the distribution of income in the parents' racial group. As a result, racial discrimination and racial wage inequality can be self-reinforcing, and initial differences in human capital are sufficient to lead groups to converge to different steady state equilibria.

2 Related Research

The model in this paper builds upon the theory of statistical discrimination, pioneered by Arrow (1972, 1973), McCall (1972) and Phelps (1972). Models of statistical discrimination explain discrimination as a rational response to uncertainty in labor markets. In some models, uncertainty occurs at the search stage, and firms use their prior beliefs about worker-productivity to determine the intensity with which to search for workers from different racial groups (for example, Mailath, Samuelson and Shaked (2000))[1]. In other models, uncertainty occurs at the wage-setting stage, and firms use their prior beliefs to determine wages (for example, Aigner and Cain (1977) and Coate and Loury (1993)).

Although uncertainty is the root cause of discrimination in all models of statistical discrimination, uncertainty alone is not sufficient to generate discriminatory outcomes. Thus, these models depend upon other forces that, together with uncertainty, lead to discrimination. For example, recently, it has been shown that when workers decide whether or not to make productivity-enhancing investments in their human capital, discrimination can arise due to self-confirming

[1] This has sometimes been referred to as search discrimination.

negative stereotypes and the existence of multiple equilibria. That is, employers believe that workers from one racial group are less likely to be productive than are workers from some other group, and, in equilibrium, individuals invest in such a way that the firms' beliefs are confirmed, even though the groups are identical ex ante (Coate and Loury (1993), Fryer (2003), Mailath, Samuelson and Shaked (2000), Moro and Norman (2003, 2004), Norman (2003), Fang (2004), and Arcidiacono (2003)).

The model in this paper builds on Coate and Loury (1993), but wages are determined competitively, as in Moro and Norman (2003). Unlike previous research, however, the model presented in this paper considers the intergenerational dynamics of statistical discrimination; parents, rather than workers, make the human capital investments, and a parent's ability to invest depends on the wage that she earns. Thus, discrimination in one generation can affect human capital investment in following generations.

This research also builds upon a large body of theoretical literature on intergenerational income mobility. In many models of intergenerational income mobility, income is transmitted across generations through parental investments in the human capital of children (for example, Becker (1975), Blinder (1974), Becker and Tomes (1979), Loury (1981) and Mulligan (1997)). Since racial discrimination may alter the rate of return on human capital investment and the degree to which parents invest in their children, discrimination may also lead to racial wage differentials. However, standard models of intergenerational income mobility do not explain why racial discrimination exists in the first place. In this paper, racial differences in the returns to human capital investment arise endogenously.

Finally, this paper is related to a number of papers that examine the sources of persistent racial wage differentials. For example, Loury (1977) presents a model in which a group's ability to acquire human capital depends upon the resources available to members of that group. Thus, income differences may persist over long time horizons. Lundberg and Startz (1998) also develop a model of human capital accumulation in which the "social capital" of a community affects the human capital accumulation of individuals within that community. Within this context, they show that racial segregation can perpetuate the human capital and income gap created by racial discrimination. However, like the literature on intergenerational income mobility, these papers do not explicitly model the discriminatory process. As a result, it is not possible to consider whether and under what conditions discrimination will persist over time.

3 The Model

3.1 Individual Behavior

Consider a dynamic economy in which individuals are organized into families consisting of one parent and one child. Population growth is fixed at zero and the time horizon is infinite. In addition, individuals belong to one of two identifiable groups, blacks and whites, say, which will be denoted group b and group w respectively. Labelling the two groups as such is purely for convenience. However, it is worth noting that the model presented in this paper does not apply to all identifiable groups in the labor market. An essential feature of the model is that the labor market conditions facing a particular group in one generation influence the resources available to, and thus the labor-market conditions facing, that group in the next generation. Necessary for this type of intergenerational linkage is the transmission of group identity across generations. Thus, for example, this is not a model of discrimination based on gender. Although racial identity can become blurred across generations, to a large extent, racial distinctions persist. The model in this paper assumes that members of a particular group have children who are also members of that group.

Although the model spans an infinite number of generations, each individual lives for only three periods. In the first period, individuals are children and do not participate in the labor force. In the second period, they become parents and receive income by selling their leisure in the labor market, and, in the third period, individuals retire and observe the labor market outcomes of their children. Parents value their own consumption and their child's wage. A parent can influence her child's wage by providing a social and cultural environment that fosters the accumulation of skills. However, this human capital investment is assumed to be imperfectly observed by firms, and, thus, might correspond to purchases such as tutoring or a home in a good neighborhood. For simplicity, this investment is modelled as a binary choice. Let the utility function for a parent at time t be given by:

$$U_1(c_t) + \beta U_2(w_{t+1}) + \delta\nu$$

where c_t is the parent's consumption, w_{t+1} is the child's wage, δ is an indicator variable that equals one if the parent invests and zero otherwise, β is a discount factor and ν is a random variable that represents heterogeneity in parents' preferences for investment. Although ν varies across individuals, it is assumed that the distribution of ν is identical across racial groups. Let G

denote the c.d.f. of ν in the population. Finally, it is assumed that $dU_1/dc_t > 0$, $dU_2/dw_{t+1} > 0$, and $d^2U_1/dc_t^2 < 0$, so that the child's future wage, w_{t+1}, is a normal good.

A worker whose parent invests is "qualified"; otherwise, the worker is "unqualified". However, as mentioned above, parental investment is not perfectly observed by firms. Instead, when a worker enters the labor market, he or she receives a productivity signal, $\theta \in [0, 1]$. For example, θ might be a test score obtained at the time a worker is hired. For workers who are qualified, θ is drawn from the p.d.f. f_q (with the corresponding c.d.f. F_q), and for those who are unqualified, θ is drawn from the p.d.f. f_u (with the corresponding c.d.f. F_u). Without loss of generality, it is assumed that $\frac{f_u(\theta)}{f_q(\theta)}$ is strictly decreasing in θ. Since this (strict) Monotone Likelihood Ratio Property implies that higher values of the productivity signal are more likely if a worker is qualified, and since workers with higher productivity signals earn higher wages, parents have an incentive to invest in their children.

However, parents must balance this incentive with the cost of investment, $k > 0$. Parents from group j at time t solve the following problem:

$$\max_{\delta \in \{0,1\}} E_t^j \left[U_1(c_t) + \beta U_2(w_{t+1}) + \delta\nu | \delta \right]$$

$$\text{s.t.} \quad c_t + \delta k = w_t$$

where w_t is the parent's wage. Since the expected value of the child's future wage depends on whether the parent invests, then so does the parent's expected utility. In addition, the parent's expected utility is allowed to vary across groups since labor market discrimination may affect the child's expected future wage.

Implicit in the parent's maximization problem is the assumption that parents face a borrowing constraint and must finance investments in their children through reductions in their own consumption. Thus, under the assumption that the child's wage is a normal good, the likelihood that a parent invests increases with the parent's wage. In fact, it is well-known that if parents are borrowing constrained in this way, then parents' investments in their children will depend positively on parents' income (for example, Loury (1981), Becker and Tomes (1986) and Mulligan (1997)). These borrowing constraints are typically thought to be quite reasonable in an intergenerational context since societal factors make it difficult to write contracts that require children to repay debts incurred by their parents. However, borrowing constraints are not needed to generate the positive relationship between parental income and investments in children. For example, if parents are assumed to care about their children's human capital in addition to their children's future income, then higher levels of parental income will be associated with higher levels of investment, even in the absence of borrowing constraints. Indeed, there are many variants of the standard model of intergenerational income mobility that generate this positive relationship between parental income and investment in children[2]. For simplicity and in keeping with the standard literature on intergenerational income mobility, the model in this paper assumes that parents are borrowing constrained in the sense that they cannot incur debts that must be repaid by their children.

3.2 Firm Behavior

It is assumed that there are two identical, risk-neutral firms that compete for workers and that exist for only a single period. At each firm, qualified workers are more productive than unqualified workers. Qualified workers produce W_q, and unqualified workers produce W_u, where $W_u < W_q$.

If firms could perfectly observe worker productivity, they would be willing to pay qualified workers W_q and unqualified workers W_u. However, firms cannot observe whether a worker is qualified. In addition, output is assumed to be nonverifiable so that contracts cannot be written on output, and, as a result, qualifications cannot be indirectly inferred from output. Thus, a worker's wage is determined by the only two pieces of information available to firms: the worker's race and his or her productivity signal. Based on these two pieces of information and the firms' prior beliefs about the probability that workers from each group are qualified, firms form posterior beliefs about worker productivity. In particular, let π_t^j denote a firm's prior belief that a worker from group j at time t is qualified, and let $p(\theta, \pi_t^j)$ denote the firm's posterior belief that a worker with signal $\theta \in [0, 1]$ is qualified. Then

$$p(\theta, \pi_t^j) = \frac{\pi_t^j f_q(\theta)}{\pi_t^j f_q(\theta) + (1 - \pi_t^j) f_u(\theta)} \tag{1}$$

As expected, a firm's posterior belief that a worker is qualified is increasing in, π_t^j, its prior belief that the worker is qualified. In addition, since higher values of the productivity signal are more likely if a worker's parent invested, the firm's posterior belief is also increasing in θ.

[2] Another possibility is that parents value the income that children receive from labor market earnings differently than the income they receive from financial transfers.

3.3 Timing of the Game

As discussed above, the game in this model spans an infinite number of time periods. However, the model is greatly simplified by the assumption that parents care only about their children's wage (as opposed to, for example, their children's consumption) and by the assumption that firms live for only a single period (and, thus, have zero probability of being matched in the future with the children of current employees). As a result, there is an equilibrium within each generation in which optimization by both parents and firms depends upon the distribution of wages in the parents generation, but does not depend on the anticipation of what will happen in future generations. Taking the distribution of wages in the parents' generation as given, the timing of the game within a single generation of workers can be described as follows:

Stage 1. Parents have children, learn their investment cost and decide whether to invest in their children's human capital.

Stage 2. At the beginning of the next period, children become workers and enter the labor market. When they do so, they receive a signal $\theta \in [0, 1]$. If the worker's parent invested, then the signal θ is distributed according to the p.d.f. f_q. Otherwise, it is distributed according to the p.d.f. f_u.

Stage 3. Firms compete for workers by simultaneously announcing wage schedules. Wages are allowed to depend on the signal and group identity, so a pure action of firm i at time t is a pair of functions $w^j_{t,i} : [0, 1] \to \Re_+$, $j = b, w$.

Stage 4. Workers observe wage schedules and decide which firm to work for.

Stage 5. Payoffs are distributed.

This within-generation game is then repeated an infinite number of times.

4 The Equilibria

This section describes the equilibria of the model. As mentioned above, within each generation, the optimal actions of parents and firms depend only on the distribution of wages in the parents' generation. Thus, the wage distribution functions like a state variable, and of particular interest are the model's stationary equilibria in which the wage distribution is constant over time. If there exist multiple stationary equilibria, then even if the economy reaches a steady state, the equilibrium outcome may differ across racial groups. Thus, racial wage inequality and racial discrimination can persist.

In order to analyze the model's stationary equilibria, it is useful to first characterize the equilibria within each generation. Thus, Section 4.1 discusses the equilibria of the within-generation game, and then Section 4.2 discusses the model's stationary equilibria.

4.1 Within-Generation Equilibria

The equilibria of the within-generation game are found through backwards induction. In Stage 4, workers decide which firm to work for. A worker's best response is to work for the firm that offers him or her the highest wage.

In Stage 3, firms announce wage schedules. Suppose that, in equilibrium, workers are paid their expected marginal product. Thus, given the firm's belief that workers from each group invest, π_t^j, the wage for a worker from group j at time t with productivity signal θ would be given by:

$$w(\theta, \pi_t^j) = p(\theta, \pi_t^j)W_q + (1 - p(\theta, \pi_t^j))W_u. \tag{2}$$

Formally, a firm's action in Stage 3 (the wage offer schedule) is only a function of a worker's productivity signal. However, in equilibrium a firm's best response will depend on the firm's prior beliefs, and it is convenient to explicitly note this dependence by writing wages as a function of π_t^j. Following the Bertrand price competition logic, it can be shown that firms are playing best responses if and only if they choose wage schedules according to $w(\theta, \pi_t^j)$.[3]

In Stage 2, children receive productivity signals. Finally, in Stage 1, parents decide whether or not to invest. In this model, optimizing parents invest whenever expected utility of investing is greater than the expected utility of not investing. Let $B(w_t, \pi_t^j; \nu)$ denote the expected benefit of investing for parents with wage w_t and preference parameter ν when the firms have prior beliefs π_t^j, where

$$B(w_t, \pi_t^j; \nu) = U_1(w_t - k) - U_1(w_t) + \beta \int_{\Theta} U_2(w(\theta, \pi_t^j))[f_q(\theta) - f_u(\theta)]d\theta + \nu.$$

Thus, parents are playing best responses to the wage schedules set by firms if they invest whenever $B(w_t, \pi_t^j; \nu) > 0$. The benefit of investment is minimized when either $\pi_t^j = 0$ or $\pi_t^j = 1$,

[3]For a formal proof of this, see Moro and Norman (2003).

since if $\pi_t^j = 0$, then all workers are paid W_u, and if $\pi_t^j = 1$, then all workers are paid W_q, regardless of θ. Further, the monotone likelihood ratio assumption implies that the expected benefit of investing is a single-peaked function of firms' prior beliefs, π_t^j. In particular, when π_t^j is close to zero, $\frac{\partial B(w_t,\pi_t^j;\nu)}{\partial \pi_t^j} > 0$, and when π_t^j is close to one, $\frac{\partial B(w_t,\pi_t^j;\nu)}{\partial \pi_t^j} < 0$.

Since $\partial^2 U/\partial c_t^2 < 0$, the cost of investment in terms of the foregone consumption is decreasing in w_t so that $\frac{\partial B(w_t,\pi_t^j;\nu)}{\partial w_t} > 0$; that is, the benefit to investment is strictly increasing in parents' wages. Thus, for a given preference parameter, ν, there will be a cutoff wage level above which parents optimally invest in their children's human capital and below which they do not. Let $z(\pi_t^j;\nu)$ denote this cutoff wage level, where

$$z(\pi_t^j;\nu) = \{w_t | B(w_t, \pi_t^j;\nu) = 0\}.$$

Conveniently, since parents' only decision is whether or not to invest in their children's human capital, $z(\pi_t^j;\nu)$ summarizes all relevant information about parents' preferences.

Letting $H_{t-1}^j(w)$ denote the c.d.f. of wages in the parents' generation, the probability that optimizing parents invest in their children's human capital at time $t-1$ is given by:

$$I_{t-1}^j(\pi_t^j) = 1 - \int_{-\infty}^{\infty} H_{t-1}^j(z(\pi_t^j;\nu))dG(\nu). \tag{3}$$

An equilibrium of the within-generation game occurs when parents are playing a best response to the wage schedules set by firms and when firms are playing a best response to the distribution of parental strategies given the firms' beliefs. In other words, based on their prior beliefs about the probability that parents from a given group invest, the wage schedules set by firms must be such that parents from each group optimally invest at the rate postulated by firms. Such a self-confirming equilibrium is any π_t^j such that

$$\pi_t^j = I_{t-1}^j(\pi_t^j). \tag{4}$$

As long as U_1, U_2, f_q, f_u and H_{t-1}^j are continuous, then I_{t-1}^j is continuous on $[0,1]$. Thus, we know that there exists at least one solution to (4). Moreover, if the distribution of ν, $G(\cdot)$, is such that there is always some positive measure of parents who invest and some positive measure who do not (so that $0 < I_{t-1}^j(\pi) < 1$ for all π), then we know that the solution is non-trivial.

The fact that the benefit to investing is increasing over some ranges of π_t^j implies that there is a positive feedback between the proportion of parents who invest and the firms' prior beliefs. Thus, there may exist multiple within-generation equilibria, and even if two groups are identical ex ante, the equilibrium outcome for those groups may differ.

In the absence of an equilibrium selection rule, it is difficult to characterize the long-run dynamics of this model. Thus, the remainder of the paper focusses on situations in which the within-generation equilibrium is unique.

Note that there will exist a unique within-generation equilibrium for group j in time t, if there is no within-generation equilibrium such that $\frac{\partial I^j_{t-1}(\hat{\pi})}{\partial \pi} \geq 1$. Intuitively, this condition implies that as long as the positive feedback between the proportion of parents who invest and the firms' prior beliefs is not too strong, then there will exist at most one equilibrium within each generation. As show in the example in Section 5, it is not hard to find a set of parameter values that satisfy this condition.

4.2 Stationary Equilibria

In any equilibrium of the infinite-generation model, both individuals and firms act exactly as they do in the within-generation game. To see this, note that since individuals care only about the wage that their children earn, the decision about whether or not to invest is made without regard to the welfare of future generations. Similarly, firms are assumed to live for only one period and so do not consider the impact of their wage policies on the education levels of workers in future generations. Thus, in any equilibrium of the infinite-generation model, equation (4) must hold in each period.

Further, there may exist equilibria that are stationary in the sense that if one of these equilibria is ever realized, then it may be realized again in the following period. As mentioned earlier, of primary interest is whether there exist multiple stationary equilibria. If so, then even if the economy reaches a steady state, the equilibrium outcome may differ across groups. This section describes the intergenerational dynamics of the model and discusses the conditions under which multiple stationary equilibria arise.

In order to identify the stationary equilibria, it is useful to first specify how the wage distribution in one generation depends upon the within-generation equilibrium outcome in the previous generation. Suppose that the equilibrium outcome in time $t-1$ for group j is π^j_{t-1}, then the equilibrium wage distribution is given by:

$$H^j_{t-1}(w) = H(w|\pi^j_{t-1}) = \pi^j_{t-1}F_q(\theta^{-1}(w, \pi^j_{t-1})) + (1-\pi^j_{t-1})F_u(\theta^{-1}(w, \pi^j_{t-1})) \tag{5}$$

where

$$\theta^{-1}(w, \pi^j_{t-1}) = \{\theta | w = p(\theta, \pi^j_{t-1})W_q + (1 - p(\theta, \pi^j_{t-1}))W_u\}$$

Substituting (5) into (4), the probability that optimizing parents at time t invest in their children's human capital given the equilibrium outcome at time $t-1$ is given by:

$$I(\pi_t^j, \pi_{t-1}^j) = \int_\nu (1 - H(z(\pi_t^j; \nu) | \pi_{t-1}^j)) dG(\nu). \tag{6}$$

It is straightforward to verify that if $\pi' > \pi''$, then $H(\cdot|\pi')$ stochastically dominates $H(\cdot|\pi'')$. Thus, the higher is π_{t-1}^j, the more the parents will earn and the more likely it is that they will invest in their children's human capital.

A stationary equilibrium of this model occurs at any π^j such that

$$\pi^j = I(\pi^j, \pi^j).$$

We would like to know whether there are multiple solutions to the above equation. If there are, then even if the economy tends towards stationary equilibria, the equilibrium outcome may differ across racial groups. Proposition 1 establishes the conditions under which multiple, stable equilibria exist.

Proposition 1 *If U_1 and U_2 are continuous, F_q and F_u are continuously differentiable, and G is such that $0 < I(c,d) < 1$ for all $\{c,d\} \in [0,1] \times [0,1]$ and if there exists some stationary equilibrium $\hat{\pi}$ such that $\frac{\partial z(\hat{\pi},\nu)}{\partial \pi} < 0$, then there exists a function z such that there exist multiple, stationary, stable equilibria.*

The proof to this and all other propositions can be found in Appendix A.

Thus, even if the economy converges to a stationary point, the equilibrium outcome may differ across racial groups, and racial discrimination and racial wage inequality may persist. Thus, it is possible, for example, for both blacks and whites to be at a stationary equilibrium and for $\pi^b < \pi^w$, so that even among individuals with the same signal, θ, white workers will be paid higher wages than black workers.

As mentioned earlier, tracing out the dynamics of this model across generations is complicated by the fact that there may be multiple within-generation equilibria. However, Proposition 2 establishes that even if we restrict attention to situations in which there is a unique equilibrium within each generation, there still may exist multiple, stable stationary equilibria.

Proposition 2 *If the conditions of Proposition 1 hold, then there exists some z such that there* *are multiple, stable stationary equilibria even if there always exists a unique within-generation equilibrium.*

A direct implication of this proposition is that if two groups start out with different initial levels of human capital, then they may converge to different stationary equilibria, even if there is a unique equilibrium within each generation and even though human capital investments evolve over time.

Note that in standard models of intergenerational income mobility, a given parent's investment behavior has no impact on the investment behavior of others. As a result, under a fairly standard set of assumptions, these models have a unique, stationary distribution of income. In contrast, in this model, parents' optimal investment decisions depend upon the investment decisions of other parents. As a result, an individual's chances of economic success depend upon the distribution of income for the entire population in that individual's racial group. This positive externality can lead to multiple stationary equilibria. Thus, the model studied in this paper bears many similarities to the interactive Markov processes developed by Conlisk (1976).

5 Example

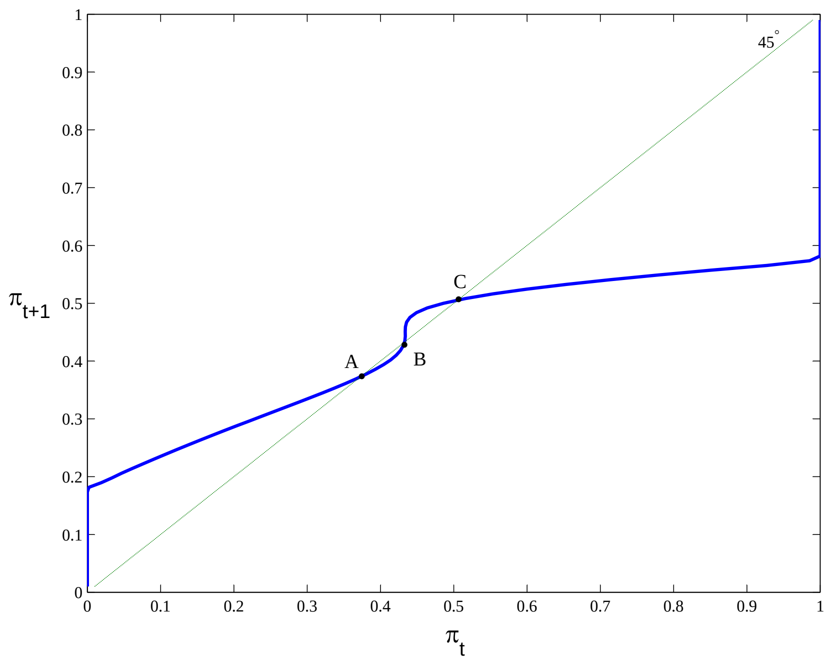
In order to demonstrate that it is possible to find a set of parameters that satisfy the conditions stated in Propositions 1 and 2, consider the following example in which the within-generation equilibrium outcome is always unique. Let the utility function of parents at time t be given by:

$$\ln(c_t) + \beta(w_{t+1} - \alpha)^{\frac{1}{3}} + \delta\nu,$$

where it is assumed that $\beta = .66$, $\alpha = 25$, and ν is distributed uniformly on $[0, .7]$. In addition, assume that $k = 8.82$, $W_u = 20$, $W_q = 30$, $F_u(\theta) = 1 - (1-\theta)^{1.12}$ and $F_q(\theta) = \theta^{1.12}$. In this example, α is intended to represent a "target wage" that parents have in mind for their children so that if $w_{t+1} < \alpha$, then the marginal utility of the child's wage is increasing, and if $w_{t+1} > \alpha$, then it is decreasing. This feature is needed to generate sufficient nonlinearities in parental investment patterns for there to exist multiple stationary equilibria.

In this example, the within-generation equilibrium outcome always unique. To see this, Figure 1 plots the within-generation equilibrium outcome in time $t+1$ as a function of the equilibrium outcome in time t. As the figure reveals, for each π_t, there is at most one equilibrium at time $t+1$.

Stationary equilibria occur at any π such that $\pi_t = \pi_{t+1}$. There are three such stationary equilibria in this example, given by points A, B and C, where π equals .3728, .4327 and .5053, respectively. Of these stationary equilibria, the equilibria at points A and C are also stable. Thus, if fewer than 43.27 percent of parents in time t are qualified, then future generations will converge to the stationary equilibrium at point A. On the other hand, if greater than 43.27 percent of parents are qualified, then future generations will converge to the stationary equilibrium at point C.



Stationary Equilibria

Thus, initially disadvantaged groups may become trapped even though there is always a unique within-generation equilibrium. That is, in contrast to standard models of statistical discrimination, repeated coordination failures are not needed to generate persistent discrimination. Rather, statistical discrimination changes the transmission of earnings across generations by leading parents' investment behavior to depend upon the distribution of income in the parents' racial group. Thus, statistical discrimination and racial inequality are self-reinforcing, and multiple equilibria can arise.

Interestingly, it is also possible to calculate the gap between the expected wages of qualified and unqualified workers. If $\pi^* = .3728$, this gap is \$0.1008, and if $\pi^* = .5053$, the gap is

\$0.1075. Thus, the return to investment (as measured by the gap in the expected wages) is almost identical in the two equilibria, highlighting the fact that small racial differences in the return to investment (\$0.1008 vs. \$0.1075) can generate relatively large differences in investment patterns ($\pi^* = .3728$ vs. $\pi^* = .5053$) and average wages (\$23.73 vs. \$25.05).

6 Conclusion

It is well-known that there exists persistent racial income inequality. Less is known about the sources of this persistence or the degree to which discrimination may explain ongoing racial inequality. As a step towards examining these issues, this paper presents a model of statistical discrimination that accounts for intergenerational income mobility. It is shown that statistical discrimination changes the transmission of income across generations by causing any given individual's chances of economic success to depend upon the economic success of other members of that individual's racial group. As a result, racial discrimination and racial wage inequality can be self-reinforcing, and groups that start out with low levels of human capital may become trapped. This result is surprising since it can occur even though the within-generation equilibrium is unique. Further, it is shown that small racial differences in the return to investment are consistent with relatively large racial differences in investment and average wages.

Appendix A: Proofs

Proof of Proposition 1:

Step 1: Given the assumptions, we know that $I(\cdot,\cdot)$ continuous on $[0,1]\times[0,1]$. Thus, there must exist some fixed point $\hat{\pi}\in(0,1)$ such that $\hat{\pi}=I(\hat{\pi},\hat{\pi})$.

Step 2: If there exists a stationary equilibrium $\hat{\pi}$ such that $\frac{\partial z(\hat{\pi},\nu)}{\partial \pi}<0$, then is possible to find a function z such that $\frac{dI(\hat{\pi},\hat{\pi})}{d\pi}>1$. To see this note that

$$\frac{dI(\pi,\pi)}{d\pi}=-\int_{\nu}\frac{\partial H(z(\pi,\nu)|\pi)}{\partial w}\frac{\partial z(\pi,\nu)}{\partial \pi}+\frac{\partial H(w|\pi)}{\partial \pi}\bigg|_{w=z(\pi,\nu)}dG(\nu),$$

where $\frac{\partial H(z(\pi,\nu)|\pi)}{\partial w}>0$ and $\frac{\partial H(w|\pi)}{\partial \pi}<0$. Thus, it is possible to pick some z such that $|\frac{\partial z(\hat{\pi},\nu)}{\partial \pi}|$ is sufficiently large to make $\frac{dI(\hat{\pi},\hat{\pi})}{d\pi}>1$.

Step 3: If there exists a stationary equilibrium $\widehat{\pi}$ such that $\frac{dI(\widehat{\pi},\widehat{\pi})}{d\pi}>1$, then it is possible to find a z in which there exist multiple, stable, stationary equilibria. To see this, note that since $0<I(c,d)<1$ for all $\{c,d\}\in[0,1]\times[0,1]$ and since $I(\cdot,\cdot)$ continuous on $[0,1]\times[0,1]$, then if there exists a stationary equilibrium $\widehat{\pi}$ such that $\frac{dI(\widehat{\pi},\widehat{\pi})}{d\pi}>1$, then there must exist at least two other non-trivial stationary equilibria, $\widehat{\pi'}$ and $\widehat{\pi''}$. In addition, it is possible to pick z such that $|\frac{dI(\widehat{\pi'},\widehat{\pi'})}{d\pi}|<1$ and $|\frac{dI(\widehat{\pi''},\widehat{\pi''})}{d\pi}|<1$.

Proof of Proposition 2: A sufficient condition for a unique within-generation equilibrium is that for all $\pi\in(0,1)$

$$\frac{dI(\hat{\pi},\overline{\pi})}{d\pi}=-\int_{\nu}\frac{\partial H(z(\hat{\pi},\nu)|\overline{\pi})}{\partial w}\frac{\partial z(\hat{\pi};\nu)}{\partial \pi}dG(\nu)<1$$

where $\overline{\pi}$ is the proportion of qualified workers in the parent's generation. In addition, we know from the proof of Proposition 1 that a sufficient condition for there to exist multiple stationary equilibria is that there must be some $\widehat{\pi}\in(0,1)$ such that

$$\frac{dI(\widehat{\pi},\widehat{\pi})}{d\pi}=-\int_{\nu}\frac{\partial H(z(\widehat{\pi},\nu)|\widehat{\pi})}{\partial w}\frac{\partial z(\widehat{\pi};\nu)}{\partial \pi}+\frac{\partial H(w|\widehat{\pi})}{\partial \pi}\bigg|_{w=z(\hat{\pi},\nu)}dG(\nu)>1.$$

Since $\frac{\partial H(w|\widehat{\pi})}{\partial \pi}<0$, it is possible for the above two equations to hold simultaneously.

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