On the Outside-Option Principle with One-Sided Options*

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Abstract

This note examines a bargaining game in which a single player has an outside option that can be taken in any period of time. If the outside-option value is close to the efficient frontier, then there exist equilibria that contravene the "outside-option principle." In particular, the player with the outside option may receive significantly less than his/her equilibrium payoff in the game without it. An example of option-contract renegotiation is provided. JEL code: C7.

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1. Introduction

In many settings of bilateral negotiation, parties can unilaterally terminate bargaining by taking an outside option. In the *one-sided* case, an outside option is available to only one of the parties, although it may yield value to both. Suppose, for example, that players 1 and 2 are negotiating over how to split a monetary surplus $L > 0$, they get a flow payoff of zero while bargaining, and player 1 has an outside option that would yield payoff vector $w = (w_1, w_2)$ where $w_1 + w_2 \leq L$. Let z be the payoff vector that we would predict if the outside option were not available. Under the Nash bargaining solution, $z = (L/2, L/2)$. In the setting where player 1's outside option is available, the *outside-option principle* predicts that if $w_1 \leq z_1$ then the outcome of negotiation will be z, and otherwise the agreement will be $(w_1, L - w_1)$. Thus, the outside option value merely imposes an inequality constraint on the solution, having no effect if w_1 is low.

Osborne and Rubinstein (1990) evaluate the outside-option principle in the setting just described, by studying an alternating-offer bargaining game with $w_2 = 0$.[2] Here $z = (1, \delta)/(1 + \delta)$ where $\delta \in (0, 1)$ is the discount factor. Osborne and Rubinstein find multiple equilibria but confirm the basic theme of the outside-option principle: If $w_1 \leq \delta^2 z_1$ then the equilibrium payoff is z, as though the outside option did not exist. If $w_1 > \delta^2 z_1$ then there are generally multiple equilibria; each yields a payoff above w_1 for player 1 and, in

*This exercise is the starting point for a project that will subsume Watson and Wignall (2010).

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[2]Osborne and Rubinstein call player 2 the one with the outside option and assume, for simplicity, that the outside option is available only after player 1 rejects an offer, but this is inconsequential to their results.

particular, $(w_1/\delta, L - w_1/\delta)$ is an equilibrium value, which is close to $(w_1, L - w_1)$ for large δ.

One-sided outside options arise naturally, such as in contract renegotiation where one of the parties can take an irreversible trade action under the existing contract. For instance, player 1 is a manufacturer who chooses whether to launch a new product in any period, whereas player 2 designed the product but has no manufacturing capability. Launching the product amounts to exercising an outside option; it concludes the productive relationship, with payoff w determined by the parties' existing contract (which specifies a payment from manufacturer to designer conditional on product launch).[3] But in general $w_2 > 0$, and in fact $w_1 + w_2$ may be close to L.

This note examines noncooperative bargaining with a one-sided outside option that player 1 can take in any period, with w unrestricted. I show that the outside-option principle is generally *invalid* in settings where $w_1 + w_2$ is close to L. Notably, for $w_1 < \delta z_1$ there are multiple equilibrium values, including one close to w (even though the outside option is available only to player 1) and one close to z (even if z_2 is far less than w_2). The model and theorem characterizing equilibrium are provided in the next section. The practical significance of the result is demonstrated in Section 3 for a setting of contract renegotiation and hold-up, and Section 4 contains a proof of the theorem.

The results found here for one-sided outside options contrast with results for the two-sided case. In the model of Ponsati and Sakovics (1998), where an outside option with general value w is available to both players in every period, all equilibrium payoffs weakly exceed w. Binmore, Shaked, and Sutton (1989), which originated the outside-option principle, examine a two-sided setting where players can take their outside options only when rejecting a proposal, finding that the equilibrium outcome is close to the Nash bargaining solution with the constraints $m_1 \geq w_1$ and $m_2 \geq w_2$.

2. The Model and Result

Consider the *two-player random-proposer 1-option bargaining game* with three parameters: a surplus amount $L > 0$, bargaining weights $\pi = (\pi_1, \pi_2) \in \mathbb{R}^2_+$ satisfying $\pi_1 + \pi_2 = 1$, and an outside-option value $w \in \mathbb{R}^2$ satisfying $w_1 + w_2 \leq L$. Interaction occurs in discrete periods $1, 2, 3, \ldots$. In a given period t, Nature randomly selects the *proposer* for this period, which is player 1 with probability π_1 and player 2 with probability π_2. The proposer then chooses a proposal $x^t \in \{m \in \mathbb{R}^2_+ \mid m_1 + m_2 \leq L\}$. The other player observes x^t and decides whether to accept or reject it. If the proposal is accepted, then the game ends with payoff vector x^t received in period t. If the proposal is rejected, then player 1 has an opportunity to take the outside option, which ends the game with payoff vector w in period t. If the proposal is rejected and the outside option is not taken, then the

[3]Even if it is feasible to give the designer a contractual option allowing her to demand product launch (exercise an option by way of sending a message), so that both players have a trigger, it may not be optimal to provide such an option.

interaction continues in period $t+1$. Players share discount factor $\delta \in (0,1)$.

This bargaining model is a random-proposer variant of the game studied by Osborne and Rubinstein (1990, section 3.12.2), allowing for asymmetric bargaining weights and any outside-option vector.[4] Using standard methods (Shaked and Sutton 1984), one can easily show that, in the related model with no outside option, the unique subgame-perfect equilibrium payoff vector is πL, which is the generalized Nash bargaining solution with bargaining weights π.

To keep things simple and to show that the finding here does not require a complicated equilibrium construction, let us focus on *stationary* subgame-perfect equilibrium (SSPE). In such an equilibrium, the specification of behavior in any given period does not depend on the date or on the history of prior-period interaction, so there is a single equilibrium continuation value from the start of any period. Let us assume a strong form of stationarity whereby player 1's choice of whether to take the outside option at the end of a given period does not depend on the proposal that was rejected earlier in the period.

The following result identifies how such equilibrium values relate to the outside-option value w. The regions noted in the theorem refer to Figure 1 and the equilibrium values are shown in Figure 2. Define $\phi \equiv \delta\pi_1/(1-\delta\pi_2)$.

Theorem: Consider the random-proposer 1-option bargaining model described above.

[A] If $w_1 \leq \delta\pi_1 L$ and $w_1 + \phi w_2 < \phi L$, then the unique SSPE value is πL.

[B] If $w_1 > \delta\pi_1 L$ and $w_1 + \phi w_2 < \phi L$, the unique SSPE value is $(w_1/\delta, L - w_1/\delta)$.

[C] If $w_1 \leq \delta\pi_1 L$ and $w_1 + \phi w_2 \geq \phi L$, then there are three SSPE values:
$w + \pi(L - w_1 - w_2)$, $(w_1/\delta, L - w_1/\delta)$, and πL.

[D] If $w_1 > \delta\pi_1 L$ and $w_1 + \phi w_2 \geq \phi L$, the unique SSPE value is $w + \pi(L - w_1 - w_2)$.

In Figure 2, for Region C, the middle equilibrium value $(w_1/\delta, L - w_1/\delta)$ is not pictured so as not to clutter the diagram. Osborne and Rubinstein's (1990) analysis applies to Regions A and B at the horizontal axis. Take special note of Region C, where the equilibrium value $w + \pi(L - w_1 - w_2)$ gives player 1 less than $\delta\pi_1 L$, counter to the outside-option principle.

3. Example: Contract Renegotiation with Trade Options

Consider an example of contracting with unverifiable investment and verifiable trade actions, along the lines of Watson (2007) and Buzard and Watson (2012), but here with a durable trading opportunity. Player 2 is an inventor who makes an unverifiable investment

[4] The random-proposer protocol is used here for simplicity. The alternating-offer protocol produces similar results but requires additional notation to keep track of separate continuation values from odd- and even-numbered periods.

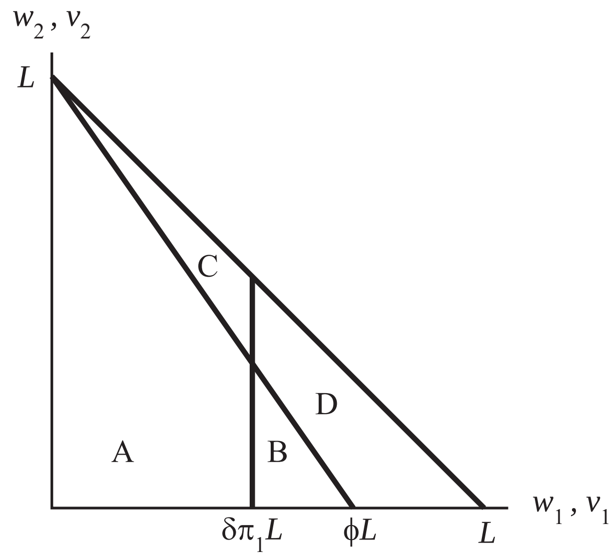

Figure 1: Regions of the space of outside-option values.

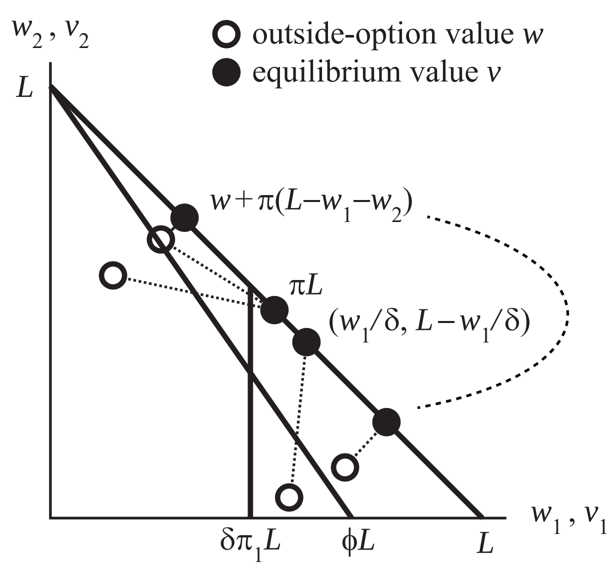

Figure 2: Agreement values related to outside-option values.

(low at cost 0, or high at cost c) to create a new product. Player 1 is a manufacturer who decides, verifiably, whether to bring the product to market (produce) or not. Producing in the case of high investment yields a monetary gain of 10 for player 1 and in the case of low investment yields a monetary gain of 2. Assume $c \in [0, 8]$ so high investment is efficient.

At time 0 the players form an externally enforced contract specifying a monetary transfer p from player 1 to player 2 to be paid if and when player 1 produces. Then player 2 chooses an investment level, which player 1 observes. Interaction in periods $1, 2, 3, \ldots$ occurs as in the bargaining model described in the previous section. An agreement constitutes renegotiating to a contract that forces player 1 to produce in this period and pay player 2 a specified amount, which implies an allocation x of the surplus $L = 2$ in the case of low investment and $L = 10$ if investment was high (the sunk investment cost not included). Player 1's outside option is to produce and pay the amount p specified originally. Thus, the contract formed in period 0 provides for a "trade-action based" option.

A key issue is whether player 2's investment incentive is limited due to the hold-up problem. With no initial contract and strong property rights for player 2's invention, player 2's investment return is whatever payment can be negotiated with player 1 in return for the right to produce. Assume $\pi_1 = \pi_2$. Player 2 obtains a payment of $2 \cdot \frac{1}{2} = 1$ in the case of low investment and $10 \cdot \frac{1}{2} = 5$ if investment was high. Therefore, player 2 would have the incentive to invest efficiently only if $c \leq 4$. Providing investment incentives for larger costs requires up-front contracting. However, one might expect that even then, player 1 could hold up production to get a large share of the production benefit, defeating the investment incentive. But thanks to the failure of the outside-option principle, an option contract specifying p below but close to 10 can induce the high investment for larger values of c. In the case of high investment, $w = (10 - p, p)$ in the renegotiation subgame starting in period 1 (Region C). There is an equilibrium of the subgame with value $(10 - p, p)$. In the case of low investment, $w = (2-p, p)$ in the renegotiation subgame and the equilibrium continuation value is $(1, 1)$. Setting $p = 9$ gives player 2 the full marginal benefit of the high investment, which yields the efficient outcome regardless of c.

4. Proof of the Theorem

For any SSPE, let v be the equilibrium continuation value from the start of each period. At the end of a period, after a rejected proposal, player 1 must not take the outside option if $w_1 < \delta v_1$, must take the outside option if $w_1 > \delta v_1$, and can randomize if $w_1 = \delta v_1$. Earlier in the period, the proposer must in equilibrium select an efficient proposal (achieving the joint value L) that makes the other player indifferent between accepting and rejecting, anticipating player 1's equilibrium decision regarding the outside option, and the other player accepts this proposal.

There are three cases regarding player 1's outside-option choice:

Case 1: $\boldsymbol{w_1 < \delta v_1}$. Player 1 always refuses the outside option, so the continuation value following rejection of any proposal is δv. The equilibrium proposals are $(L - \delta v_2, \delta v_2)$ from player 1 and $(\delta v_1, L - \delta v_1)$ from player 2. Because the former proposal will be made with probability π_1 and the latter with probability π_2, the equilibrium value satisfies

$$v = \pi_1(L - \delta v_2, \delta v_2) + \pi_2(\delta v_1, L - \delta v_1).$$

Solving this system of two equations (note that they imply $v_1 + v_2 = L$ and use this with the equation for v_1) yields $v = \pi L$. From this case's inequality, such an equilibrium exists if $w_1 < \delta \pi_1 L$.

Case 2: $w_1 > \delta v_1$. Player 1 always takes the outside option, so the continuation value following rejection of any proposal is w. The equilibrium proposals are $(L - w_2, w_2)$ from player 1 and $(w_1, L - w_1)$ from player 2. The equilibrium value is

$$v = \pi_1(L - w_2, w_2) + \pi_2(w_1, L - w_1) = w + \pi(L - w_1 - w_2).$$

Such an equilibrium exists if $w_1 > \delta\left(w_1 + \pi_1(L - w_1 - w_2)\right)$, which simplifies to $w_1 + \phi w_2 > \phi L$.

Case 3: $w_1 = \delta v_1$. Player 1 is indifferent between taking and refusing the outside option in any period. Suppose he takes the outside option with probability α. Then player 1's equilibrium proposal is

$$y^1 = (L - \alpha w_2 - (1 - \alpha)\delta v_2, \alpha w_2 + (1 - \alpha)\delta v_2),$$

player 2's equilibrium proposal is $y^2 = (w_1, L - w_1)$, and the equilibrium value is $v = \pi_1 y^1 + \pi_2 y^2$. Solve this system of two equations as in Case 1 and set δv_1 equal to w_1, which is the required equality for this case. Collecting terms yields:

$$(1 - \delta)(w_1 - \delta \pi_1 L) = \alpha \delta \pi_1(\delta L - w_1 - w_2).$$

This condition on α is valid if it implies $\alpha \in [0, 1]$. Consider two subcases:

- **(a)** $w_1 \geq \delta \pi_1 L$ and $w_1 + w_2 \leq \delta L$ (both sides positive). For $\alpha \leq 1$ we need $(1 - \delta)(w_1 - \delta \pi_1 L) \leq \delta \pi_1(\delta L - w_1 - w_2)$, which simplifies to $w_1 + \phi w_2 \leq \phi L$. This inequality along with $w_1 \geq \delta \pi_1 L$ implies $w_1 + w_2 \leq \delta L$, and so the equilibrium exists under the conditions $w_1 \geq \delta \pi_1 L$ and $w_1 + \phi w_2 \leq \phi L$.

- **(b)** $w_1 \leq \delta \pi_1 L$ and $w_1 + w_2 \geq \delta L$ (both sides negative). For $\alpha \leq 1$ in this subcase, we have the opposite condition than in the previous subcase: $w_1 + \phi w_2 \geq \phi L$. This inequality along with $w_1 \leq \delta \pi_1 L$ implies $w_1 + w_2 \geq \delta L$, and so the equilibrium exists under the conditions $w_1 \leq \delta \pi_1 L$ and $w_1 + \phi w_2 \geq \phi L$.

The equilibrium value in both subcases is $(w_1/\delta, L - w_1/\delta)$.

Consider the four regions described in the Theorem. Note that Region A satisfies the necessary conditions for only Case 1 and thus the unique equilibrium value here is πL. Region B is consistent with the necessary conditions of only Case 3a, implying a unique equilibrium value of $(w_1/\delta, L - w_1/\delta)$. Region D is consistent with the necessary conditions of only Case 2, and here the unique equilibrium value is $w + \pi(L - w_1 - w_2)$. Region C satisfies the necessary conditions of Cases 1, 2, and 3b, implying the three equilibrium values described in the Theorem.

References

Ken Binmore, Avner Shaked, and John Sutton. An outside option experiment. *The Quarterly Journal of Economics*, 104:753–770, 1989.

Kristy Buzard and Joel Watson. Contract, renegotiation, and hold up: Results on the technology of trade and investment. *Theoretical Economics*, 7:283–322, 2012.

Martin J. Osborne and Ariel Rubinstein. *Bargaining and Markets*. Academic Press, 1990.

Clara Ponsati and Jozsef Sakovics. Rubinstein bargaining with two-sided outside options. *Economic Theory*, 11(3):667–672, 1998.

Avner Shaked and John Sutton. Involuntary unemployment as a perfect equilibrium in a bargaining model. *Econometrica*, 52(6):1351–1364, 1984.

Joel Watson. Contract, mechanism design, and technological detail. *Econometrica*, 75(1): 55–81, 2007.

Joel Watson and Chris Wignall. Hold-up and durable trading opportunities. Unpublished manuscript, 2010.