Recessions and the Information Content of Unemployment

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February 21, 2012

Abstract

I present evidence that increases in the relative share of job losers (and decreases in that of job leavers) entering unemployment lower the quality of the unemployment pool. The share of job losers rises during recessions, and I analyze the causes and implications of this in an equilibrium labor market model. The theoretical analysis offers four main insights: (1) Employees remaining after a recession are disproportionately more productive than those who were terminated—this could play a role in the recently declining cyclicality of average labor productivity. (2) While workers fired during a recession are more productive (on average) than those fired at other times, the shift from job leavers to job losers during this time can still lower the “quality” of the unemployment pool during/after recessions. This decline in quality can slow hiring even after the economy has otherwise recovered to pre-recession levels, offering a potential explanation for “jobless recoveries.” (3) For hiring to return, the unemployment pool quality must rebound through inflows of workers who are not negatively selected (such as job leavers). Thus, if poor job-finding conditions motivate fewer workers to voluntarily enter the unemployment pool, this will further delay the recovery of employment. (4) While the new steady-state induced by a “permanent” recession would involve a higher quality unemployment pool than in the pre-recession economy, a transition between the two states yields a lower quality pool than in either state. Thus, conclusions about economic dynamics drawn from comparing the predictions of two static models can be misleading. The model presented addresses this—it rigorously incorporates employer learning and private information into an equilibrium framework for dynamic labor market analysis.

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I am deeply indebted to Simon Board, Maria Casanova, Maurizio Mazzocco, Moritz Meyer-Ter-Vehn, Marek Pycia, and Pierre-Olivier Weill for guidance in this project. I am also grateful to Guillermo Beylis, Mary Ann Bronson, Edgar Cortés, Hanno Lustig, Ichiro Obara, Joseph Ostroy, Mona Shangold, David Toniatti, Brian Waters, Kyle Woodward, and Yujing Xu for insightful comments and suggestions. As is generally the case, all mistakes are my own.
More than two years after the “official” end of this past recession, nearly one tenth of the U.S. labor force remains unemployed, and 45% of these unemployed have been jobless for over six months (BLS - CPS Labor Force Statistics).\footnote{The unemployment rate recently fell below 9% for the first time since the recession, but evidence suggests that has been caused not only by hiring but also by discouraged unemployed workers dropping out of the labor force.} Within the unemployment pool, the long-term unemployed workers (LTU) are least likely to find jobs, and policy-makers are concerned that the current mass of LTU could be trapped in this state. As such, LTU must be a primary target of any policy to reduce unemployment, but the effectiveness of such a policy depends on why these workers struggle to find jobs. In the existing literature, the lower reemployment probabilities of LTU are attributed to human capital depreciation during unemployment (Pissarides, 1992; Ljungqvist and Sargent, 2008; Möller, 1990), negative sorting induced by selective hiring from the unemployment pool (Lockwood, 1991), and even employer bias against LTU caused by the assumed presence of the previous two mechanisms (Jackman and Layard, 1991).\footnote{More generally, a substantial literature has aimed to separate the causes of these lower reemployment probabilities into two categories of mechanisms: (1) true duration dependence (caused by human capital depreciation, the stigma of long-term unemployment, etc.) and (2) unobserved heterogeneity among the LTU (which might be caused by negative sorting into long-term unemployment). See Heckman (1991) and Machin and Manning (1999) for surveys of such analyses.}

In this paper, I consider a new mechanism contributing to the current job-finding struggles of LTU: changes over time in the quality of workers entering the unemployment pool. Specifically, I argue that lower quality workers enter the unemployment pool during recessions, and that this results from more fired workers and fewer quitting workers entering unemployment at such times. Further, I present an equilibrium model of the labor market that explains this shift from quits toward fires, and I use this model to explore the short and long-term consequences of this shift on employment dynamics. I present this analysis in two parts:

Firstly, I consider a simple model in which firms hire selectively from the unemployment pool. I demonstrate that compositional changes in the quality of flows to unemployment should have different effects on the reemployment hazard rates of those who have been unemployed for different durations, and I use this to derive a test for the presence of these compositional changes and their impacts on hiring. I then confirm this presence using CPS monthly employment data and the Job Openings and Labor Turnover Survey (JOLTS); this empirical result is found in both data sources.

This test is particularly informative for two reasons: firstly, it relies directly on the link between hiring outcomes and the relative proportions of fired and quitting workers entering unemployment. Thus, even if the reader is skeptical of the precise mechanism at work, any alternative mechanism must generate the same empirical relationship, and such an alternative would likely have similar
implications for hiring dynamics as the mechanism discussed here. Secondly, the main explanations for long-term unemployment found in the literature (human capital depreciation, selection among those remaining unemployed, etc.) cannot—by themselves—generate the empirical patterns predicted by the model. While these alternative mechanisms are important for understanding unemployment, this analysis identifies a role for compositional change that is independent of these existing explanations.

Secondly, I develop an equilibrium labor market model to analyze the causes and consequences of the shift from quitting workers to fired workers in the flow to unemployment during recessions. In the model, firms learn privately during employment about the productivity of heterogeneous workers, and workers can reach the unemployment pool either voluntarily (by quitting) or via a targeted firing (due to low firm beliefs about productivity). The negative selection present among fired workers does not apply to those workers who quit their jobs. In equilibrium, the quality of the flow to unemployment results from a balance between the flow of disproportionately low quality fired workers and the flow of quits.

Using this framework, I study the labor market dynamics induced by a recession, in which workers become less productive relative to their costs of employment. This induces firms to raise standards for firing current workers and for employing new ones, so—consistent with the standard analysis in the literature (Nakamura, 2008; Kosovich, 2010; Lockwood, 1991)—workers fired during recessions are of higher average quality than those fired under other economic conditions. However, a recession also throws off the preexisting balance between fires and quits, and the dynamic labor market response to this provides four main insights:

(1) The employees who remain with a firm after a recession are disproportionately more productive than those employed beforehand. This mitigates the productivity decline that accompanies a recession, and may provide an explanation for the recently observed acyclicality/countercyclicality of average labor productivity (see Gali and van Rens, 2010).

(2) Because the flow of directed firings overwhelms that of quitting workers, a recession decreases the quality of the unemployment pool. This reverses the standard argument promoted by Nakamura (2008) and Kosovich (2010), among others. As a result, firms limit hiring, and they may continue to do so even after the economy has otherwise recovered to precession levels. (Thus, this mechanism may have a causal role in the "jobless recoveries" that have followed recent recessions).

(3) For hiring to resume, the unemployment pool quality must recover through inflows of workers who have not undergone the same negative selection, such as job leavers. Thus, if the poor job-finding conditions motivate fewer workers to voluntarily enter the unemployment pool, this will further delay the recovery of employment.

(4) More generally, the above results highlight the importance of explicitly modeling economic dynamics in this context, as conclusions drawn from comparing the predictions of two static models
may be misleading. For this purpose, the model developed in this paper offers a tractable equilibrium framework that can be used to study the evolution of employment and/or wages under changing economic conditions.

The paper proceeds as follows: Section 1 presents a simple model in which firms hire selectively (but imperfectly) from a pool of heterogeneous unemployed workers. A test is derived for the presence/influence of compositional changes in the flow to unemployment, and this is confirmed empirically. The next several sections develop the main theoretical model—Section 2 sets forth the structure of the economy, and Section 3 characterizes its steady-state equilibrium. Section 4 develops the dynamics of employment in response to a negative productivity shock (a "recession"), highlighting the causes and implications of the unemployment pool’s changing composition. Section 5 briefly considers several theoretical extensions and their implications, and Section 6 concludes.

1. Compositional Changes: Their Importance for Hiring Dynamics and a Test

1.1 Job Leavers, Job Losers, and the Impact of Compositional Change

If the quality of those who enter unemployment changes over the business cycle, the firm hiring response to these changes will affect the magnitude of fluctuations both in the labor market and in the economy as a whole. Countercyclical quality improvements would stabilize the economy, but procyclical improvements would magnify variations in employment, reducing (increasing) the value of a new hire when this value is already low (high). It is thus crucial to understand whether either of these conditions is true.

Previous studies have argued that unemployment pool quality improves during recessions (Kosovitch, 2010; Lockwood, 1991; Nakamura, 2008). These analyses begin with the assumption that all workers have less value to their firms during a recession (i.e. - they become less productive). As such, firms must raise standards for hiring new workers and for continuing to employ existing workers. Therefore workers fired during recessions are of higher "quality" on average than those fired under other economic conditions. Extending this logic to the unemployment pool yields the conclusion that the quality of the unemployed rises (and the corresponding stigma of unemployment falls) during recessions.

Of course, this reasoning implicitly assumes that employment can be terminated only by firms, and in turn, that the unemployment pool consists only of (disproportionately low quality) workers who have been fired. In reality, workers reach this pool by quitting their jobs as well (due to displeasure with the job, family location requirements, etc.), and the negative selection present among fired workers need

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3 As I will show, while the unemployment pool in the new steady-state equilibrium associated with a recession is of higher average quality than the pool in the pre-recession steady-state, the transition to the recession will generate a lower quality pool than that found in either steady-state.
not characterize those who voluntarily enter unemployment. Consistent with differences in quality between these groups, the distribution of unemployment duration for fires first-order stochastically dominates this distribution for quits; in other words, workers reaching the unemployment pool after being fired generally take significantly longer to find new employment than do unemployed workers who quit their previous jobs. This is demonstrated in Figure 1 below.

**Figure 1:** Unemployment Duration Distribution–Job Leavers vs. Job Losers (Source: CPS)

One might worry that the unemployment duration distribution for quits is skewed by those who quit with future employment already in place (and thus enter unemployment only for a brief time). Two factors address this concern. Firstly, the duration of unemployment for fires first-order stochastically dominates the duration for quits even when both distributions are truncated below at various durations from 1 to 25 weeks of unemployment, so unemployed workers with future jobs in place cannot explain these distributional differences alone. Secondly, the data generating these distributions include only workers who were unemployed during the monthly census sampling date, so most workers who quit with another job already in place would not have remained unemployed long enough to enter these data.\(^4\)

Given the observed differences between fires and quits, if the composition of the unemployment pool shifts from quits toward fires during recessions, this pool may actually decrease in quality at such

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\(^4\)Another concern about the implications of this pattern is the worry that, relative to fired workers, quitting workers disproportionately leave the unemployment pool by leaving the labor force, rather than by actually finding new jobs. Even if this is the case, though, unless there is positive selection among those who leave the labor force, this will not lower the perceived quality to firms of this pool of unemployed workers. In reality, there are many reasons to believe there is negative selection among those who leave the labor force (e.g. - if those who exit the labor force have received the most negative signals in the job market thus far, or if the perseverance required to continue job search also lends itself to performance on the job). If this selection is negative, then from an employer’s perspective, this selection is actually improving the pool of job applicants who quit previous jobs. As such, this would be consistent with firms preferring workers who quit previous jobs to those who were fired.
times. This pattern is empirically evident through the most recent recession.\textsuperscript{5}

![Figure 2: Evolution of firings vs. quits in the latest recession (Source: JOLTS)](image)

Given the suggestive evidence that (a) fired workers are, on average, less attractive to employers than those who quit and (b) flows to unemployment are increasingly composed of fired workers during recessions,\textsuperscript{6} it is crucial to understand the role of the changing composition of unemployment in employment dynamics over the business cycle. In particular, if firms are aware of these changing flows, they could use them to inform hiring decisions—a shift from quits toward fires could lead firms to limit hiring. Thus, these compositional changes could be contributing to the recent post-recession pattern of "jobless recoveries."

1.2 A Simple Hiring Model and a Test for Compositional Change

We now aim to evaluate the relevance of the aforementioned compositional changes for firm hiring decisions. Toward this end, we will consider a simple model of selective firm hiring from a pool of heterogeneous, unemployed workers.\textsuperscript{7}

**Worker Types and Unemployment**

There are two types of workers (type $\theta \in \{H, L\}$), and it is assumed that type $H$ workers are more productive than type $L$ workers. In total, there is a unit measure of these workers—at time $t$, measure $E_t$ of these workers are employed, while measure $1 - E_t$ are in the unemployment pool. This

\textsuperscript{5}If these intensities are computed instead using CPS employment data, the patterns are virtually identical to those displayed here in the JOLTS data (which are based on firm responses). Hence, these trends are robust to the exclusion of job-to-job transitions from the data.

\textsuperscript{6}Consistent with this, Baker (1992) finds evidence that changes in the numbers of job leavers and job losers entering unemployment play a role in the degree to which reemployment likelihood decreases with unemployment duration.

\textsuperscript{7}For this section, we consider a basic, two-type case of worker heterogeneity. This simple setup offers clear intuition for the model’s implications, and we will use the same two-type framework in the labor market equilibrium model developed later in the paper. Of course, worker heterogeneity is more complex in reality, but the basic predictions obtained in this section are robust to other specifications.
unemployment pool has proportion \( q_{U(t)} \) of type \( H \) workers at time \( t \). We will casually refer to this proportion as the "quality" of the unemployment pool.

For the purposes of our analysis, we will divide this pool into two groups according to durations of unemployment. Workers who have been unemployed for durations \( \tau \in [0, T) \) are defined as "short-term unemployed," while those unemployed for durations \( \tau \in [T, \infty) \) are "long-term unemployed" (where \( T \in \mathbb{R}_+ \)). Denote by \( q_{S(t)} \) and \( q_{L(t)} \) the type \( H \) proportions and by \( m_{S(t)} \) and \( m_{L(t)} \) the sizes of the short- and long-term groups, respectively.\(^8\)

### Hiring and Screening

Firms cannot observe specific worker types before hiring, but they can refine the pool of potential hires via a screening technology. For each firm, this screening technology instantly filters the unemployment pool into a hiring pool—type \( H \) workers pass through this filter with probability 1, while type \( L \) workers pass through with probability \( \alpha \in [0, 1] \). Thus, if the unemployment pool has proportion \( q_U \) of type \( H \) workers, the hiring pool after screening will consist of proportion \( q_H (q_U) = \frac{q_U}{q_U + (1-q_U)\alpha} \) of type \( H \) workers.\(^9\)

Intuitively, we can think of this screening technology as representing all the observable information that impacts firm hiring decisions (age, education level, employment/unemployment history, etc.). In turn, the value of \( \alpha \) reflects the amount of noise in this information (\( \alpha = 0 \): no noise, \( \alpha = 1 \): all noise).

### Unemployment Pool Dynamics and "Stocks" vs. "Flows" Models

Time is continuous, and the unemployment pool evolves in response to its inflows and outflows. Inflows are composed of fired workers, of whom constant proportion \( q_F \) are type \( H \), and workers who quit voluntarily, of whom proportion \( q_Q \) are type \( H \). At time \( t \), fired workers enter at intensity \( \eta_{F(t)} \), while quitting workers enter at intensity \( \eta_{Q(t)} \). Outflows from unemployment take the form of hiring—at time \( t \), these occur at intensity \( \eta_{H(t)} \) and contain proportion \( q_H (q_U(t)) \) of type \( H \) workers.

Given these terms, we can characterize the evolution of the unemployment pool quality with the following equation:

\[
q_{U(t+dt)} = \frac{[1 - E_t] q_{U(t)} - \eta_{H(t)} q_H (q_{U(t)}) + \eta_{F(t)} q_F + \eta_{Q(t)} q_Q}{1 - E_t - \eta_{H(t)} + \eta_{F(t)} + \eta_{Q(t)}}
\]  

(1)

It is easy to see here that hiring lowers the unemployment pool quality—because of screening, firms hire disproportionately many type \( H \) workers from the unemployed. Because better workers are continually removed from the pool, the proportion of type \( H \) workers decreases with the duration of unemployment. Further, screening implies that group-specific reemployment probabilities correlate with group quality, so these reemployment probabilities decline with unemployment duration. In this way, the standard mechanism for negative sorting into long-term unemployment is embedded in this

\(^8\)Note that we have the basic accounting identities \( 1 - E_t = m_{S(t)} + m_{L(t)} \) and \( (1 - E_t) q_{U(t)} = q_{S(t)} m_{S(t)} + q_{L(t)} m_{L(t)} \).

\(^9\)Note that \( q_H (q_U) = \frac{q_U}{q_U + (1-q_U)\alpha} > q_U \) for \( \alpha \in [0, 1] \).
model. While human capital depreciation does not appear explicitly in the model, adding it would merely steepen the rate at which quality declines with unemployment duration.\footnote{To incorporate human capital depreciation formally, we can simply assume that type $H$ workers become type $L$ workers at some Poisson intensity $\gamma$ during unemployment. (While this structure is merely a stylized representation of a richer process, it captures the main implications of human capital depreciation in a tractable way.) It is easy to see that this will simply compound the negative duration dependence of quality that is already present in this model.}

Both of these mechanisms use declines in quality during unemployment to generate a group of LTU with low hiring rates. In the context of this analysis, they have similar implications, and we can therefore group them together (along with other such mechanisms) for modeling purposes. We will refer to these as "stocks" models, since they take effect within the pool of unemployed workers. In contrast, the "flows" model will refer to the compositional change mechanism described earlier.

For the analysis to follow, we will formalize these groups into two representative models. The stocks model will involve no change in the quality of those entering unemployment, so we can think of firings and quits collectively as flows to unemployment. In this case, we can replace terms in equation (1) with $\eta_{EU(t)} \equiv \eta_{F(t)} + \eta_{Q(t)}$ and $\eta_{EU(t)Q_{EU}} \equiv \eta_{F(t)} q_F + \eta_{Q(t)} q_Q$. The relative flows of firings and quits have no explicit role in this model. In contrast, the flows model maintains equation (1) as it appears above and assumes $q_Q > q_F$.\footnote{Note that $\frac{\eta_{F(t)} q_F + \eta_{Q(t)} q_Q}{\eta_{F(t)} + \eta_{Q(t)}}$ must be greater than $\eta_{U(t)}$ for at least some $t$ if the unemployment pool quality is to remain above 0 (because otherwise there can be no positive force to balance out the effect of hiring). In the flows model, this implies that $q_Q$ must be greater than $q_U(t)$. For simplicity, we will assume that $q_Q$ in this model is always greater than $q_U(t)$, $q_S(t)$, and $q_L(t)$.

\footnotetext[12]{The result to follow does not require this; it will hold for all $t$ even if $q_F > q_S(t)$, as long as $q_F < q_S(t) + \frac{q_Q(t)}{q_F(t)} [q_Q - q_S(t)]$.}}

### Formal Testable Implication

Our goal in this section is to provide evidence for compositional change as in the flows model. Before developing our formal result, we make the following assumptions: $\alpha \in (0, 1)$, $m_{S(t)} \in (0, q_U(t))$; and $q_U(t)$, $q_S(t)$, $q_L(t) \in (0, 1)$. That is, screening is imperfect, and both unemployment duration groups have workers of each type. For simplicity, we also assume in the flows model that $q_F < q_S(t)$, meaning that the average quality of firings is lower than that of the short-term unemployed.

Finally, we assume that

$$\left\{ \partial \eta_{H(t)}, \partial \eta_{F(t)}, \partial \eta_{Q(t)} \right\} \perp \left\{ \partial \eta_{H(t-T)}, \partial \eta_{F(t-T)}, \partial \eta_{Q(t-T)} \right\}, \forall t$$

This implies that the effects of changes in flows to unemployment (which directly enter the pool of short-term unemployed workers) are not countered by changes in the composition of workers transitioning from short-term to long-term unemployment. Note that we allow correlation between $\left\{ \eta_{H(t)}, \eta_{F(t)}, \eta_{Q(t)} \right\}$ and $\left\{ \eta_{H(t-T)}, \eta_{F(t-T)}, \eta_{Q(t-T)} \right\}$, meaning that these intensities can correlate at two arbitrarily far apart points in time (with this distance reflecting the maximum duration in short-term unemployment); we merely restrict the correlation between changes in these intensities at two such points. There is no intuitive reason to believe such correlation exists in reality, but even if
there is correlation between these intensity changes, the results that follow will persist as long as this correlation is imperfect—we assume strict independence for simplicity and clarity, not out of necessity.

Given these assumptions, the objects of interest for our testable implication will be $\eta^S_{PH(t)}$ and $\eta^L_{PH(t)}$, the time $t$ reemployment hazard rates of short-term unemployed and long-term unemployed workers, respectively. If fired workers entering unemployment indeed have a lower type $H$ proportion than those who quit jobs, then increases in firings relative to quits should lower the quality of the unemployment pool. This decrease in quality should first affect the short-term unemployed; the long-term unemployed should see no change in quality until these workers have been unemployed for long enough to be classified in this group. Thus, relative to the overall quality of the unemployment pool, the quality of short-term unemployed workers should fall and that of LTU should rise.

Because of screening, the intensity of hiring from a given group should correlate with the group’s quality. Therefore, an increase in firings relative to quits should decrease $\eta^S_{PH(t)}$ relative to $\eta^L_{PH(t)}$.

This contrasts with the predictions of the stocks model, where changes in the relative flows of firings and quits should not affect $\eta^S_{PH(t)}$ and $\eta^L_{PH(t)}$ differently.

This is formalized below (in terms of elasticities):

**Proposition (i):** Define $\eta_{PH(t)} = \frac{\eta_{H(t)}}{1-E_t}$ to be the overall reemployment hazard rate of the unemployment pool at time $t$. Define $\eta_{PF(t)} = \frac{\eta_{F(t)}}{1-E_t}$ and $\eta_{PQ(t)} = \frac{\eta_{Q(t)}}{1-E_t}$ to be the intensities of firings and quits, respectively, that are scaled relative to the size of the unemployment pool (and denote the ratio of these by $\Omega_t = \frac{\eta_{PF(t)}}{\eta_{PQ(t)}}$). Then for a fixed scaled intensity of inflows to unemployment ($\eta_{PF(t)} + \eta_{PQ(t)}$), the mechanisms reflected in the stocks model imply:

$$\frac{\partial \ln \left[ \eta^S_{PH(t)} \right]}{\partial \ln [\Omega_t]} = \frac{\partial \ln \left[ \eta^S_{PH(t)} \right]}{\partial \ln [\Omega_t]} \quad \text{(STOCKS)}$$

In contrast, the flows model of compositional change implies:

$$\frac{\partial \ln \left[ \eta^L_{PH(t)} \right]}{\partial \ln [\Omega_t]} > \frac{\partial \ln \left[ \eta^L_{PH(t)} \right]}{\partial \ln [\Omega_t]} \quad \text{(FLOWS)}$$

(See Appendix A for proof)

This implies that, in the stocks model, an $x\%$ increase in the intensity of firing combined with an $x\%$ decrease in the intensity of quitting should not affect $\eta^S_{PH(t)}$ and $\eta^L_{PH(t)}$ differently. In the flows model, however, such a change should cause a decrease in $\eta^S_{PH(t)}$ relative to $\eta^L_{PH(t)}$.

Note that this result is informative specifically because it predicts different effects across durations of unemployment. For example, if we found that shifts from quits to firings lowered reemployment probabilities for the unemployment pool as a whole, this might simply reflect the fact that these shifts occur during periods when hiring decreases.\(^{13}\) This would not, however, explain why the job-finding probabilities of LTU improve relative to those of the short-term unemployed. Thus, the predictions will persist even if we allow correlation between the intensity of hiring ($\eta_{PH(t)}$) and the intensities of firing/quitting ($\eta_{PF(t)}$ and $\eta_{PQ(t)}$).

\(^{13}\) In the empirical analysis to follow, we control for changes in hiring intensity anyway.
1.3 Results

Empirical Framework

To assess this result empirically, we must quantify how the reemployment probabilities of the short- and long-term unemployed change in response to changes in the fires/ quits composition of flows to unemployment. To do this, it will be helpful for us to exploit individual-level variation that is available to us. Toward this end, we can extend the above theoretical result to incorporate individual-specific covariates (beyond the duration of unemployment). These covariates—which are observable to the econometrician—may correlate with type and, therefore, with hiring likelihood as well.

To formalize this somewhat, denote by \( x_i \) the \( k \)-dimensional vector of these covariates for individual \( i \). We can think of \( q_t(x_i) \) as the type \( H \) proportion among unemployed workers at time \( t \) with observables \( x_i \). In turn, \( \eta_{PH(t)}(x_i) \) is the time \( t \) probability of reemployment conditioning on \( x_i \). For the purpose of grouping individuals according to unemployment duration, define \( X_S \) to be the space of \( K \)-vectors such that \( x_i \in X_S \) for individuals who have been unemployed for durations \( \tau \in [0, T) \), and define \( X_L \) to be that for individuals with durations \( \tau \in [T, \infty) \). (For consistency in the following empirical analysis, I divide these duration groups between 12 and 13 weeks \( \approx 3 \) months; individuals with unemployment durations of 1-12 weeks are "short-term unemployed," while those with durations of at least 13 weeks are "long-term unemployed.")

If \( x_i \) consists only of whether or not an individual is among the short- or long-term unemployed, then \( q_t(x_i) \in \{q_S(t), q_L(t)\} \) and \( \eta_{PH(t)}(x_i) \in \{\eta^{S}_{PH(t)}, \eta^{L}_{PH(t)}\} \). More generally, we can write \( q_j(t) = \mathbb{E}[q_t(x_i)|x_i \in X_j] \) and \( \eta^{j}_{PH(t)} = \mathbb{E} \left[ \eta_{PH(t)}(x_i) \big| x_i \in X_j \right] \) for \( j \in \{S,L\} \). In this framework, our dependent variable is \( \partial \left( \ln \eta_{PH(t)}(x_i) \right) \), which we can calculate empirically through the following process:

First, we estimate equations of the form \( y_{it} = g_t(x_i) + \varepsilon_{it} \) for each \( t \), where \( y_{it} \) is a binary indicator for whether or not individual \( i \) (who was unemployed in period \( t \)) became employed in period \( t + 1 \).\(^{14}\)

From this, we obtain the optimal functions \( \hat{g}_t(\cdot) \) for each period \( t \), and \( \eta_{PH(t)}(x_i) \) is simply \( \hat{g}_t(x_i) \). We can then obtain our dependent variable through the approximation

\[
\partial \left( \ln \eta_{PH(t)}(x_i) \right) \approx \frac{\hat{g}_t(x_i) - \hat{g}_{t-1}(x_i)}{\hat{g}_{t-1}(x_i)}
\]

Our independent variables are obtained via similar approximations for \( \partial \ln \eta_{PH(t)} \), \( \partial \ln \eta_{PF(t)} \), and \( \partial \ln \eta_{PQ(t)} \).

Findings

I obtain the individual-specific \( \partial \left( \ln \eta_{PH(t)}(x_i) \right) \) using monthly CPS employment data from 2001 - 2011. For robustness of the results, proportional changes in intensities of hiring, firing, and quitting (corresponding to the values \( \partial \ln \eta_{PH(t)}, \partial \ln \eta_{PF(t)}, \) and \( \partial \ln \eta_{PQ(t)} \)) are obtained from

\(^{14}\)For transparency, the analysis here imposes linearity on \( g_t(\cdot) \), so we can write \( \hat{g}_t(x_i) \equiv \beta x_i \).
two independent sources: (1) I calculate these values directly using the monthly CPS data and (2) I use monthly JOLTS aggregate data to obtain a second set of these values.

Both sources are imperfect with regard to this specific analysis: JOLTS data reflect firm reports of labor turnover, and these figures may include job-to-job transitions, which do not involve the unemployment pool itself. If the intensity of these transitions fluctuates more or less than the inflows to and outflows from unemployment, then this may be a noisy (or even biased) representation of unemployment flows. In contrast, the CPS surveys individual workers, so it allows exclusion of those who never enter unemployment. Unfortunately, we can detect those in the unemployment pool only if they are in this pool at the time of the monthly survey, so the CPS may underestimate the relevant flows. To deal with these concerns, I obtain the results that follow using each of these data sources separately.

I estimate equations of the form

$$
\partial \left( \ln \left[ \eta_{PH(t)}(x_i) \right] \right) = \beta_0 + \beta_1 \partial \ln \left[ \eta_{PH(t)} \right] + \beta_2 \partial \ln \left[ \eta_{PF(t-1)} \right] + \beta_3 S I \left[ x_i \in X_S \right] \partial \ln \left[ \eta_{PF(t-1)} \right] + \beta_4 S I \left[ x_i \in X_S \right] \partial \ln \left[ \eta_{PF(t-1)} \right] + \varepsilon_{it}
$$

where firings and quits impact reemployment likelihood with a lag. This is because an individual’s first appearance in unemployment (in the data) might correspond—in reality—to his first or second week of joblessness. This offers little time for high quality unemployed individuals to distinguish themselves by reclaiming employment, so the effects of changes in the quality of flows to unemployment might be muted empirically. Because I consider individuals to be short-term unemployed for the first three months of joblessness, we can better detect changes in quality by introducing 1-2 months of lag to this estimation.

Our parameters of interest can then be expressed as

$$
\frac{\partial \ln \left[ \eta^S_{PH(t)} \right]}{\partial \ln [\Omega_t]} \approx \frac{\tilde{\beta}_2 + \tilde{\beta}_3 - \tilde{\beta}_4}{2}
$$

$$
\frac{\partial \ln \left[ \eta_{PH(t)} \right]}{\partial \ln [\Omega_t]} \approx \frac{\tilde{\beta}_2 - \tilde{\beta}_3}{2}
$$

Table 1 (below) provides estimates for these obtained under several regression specifications.\footnote{Under all specifications considered, our estimate \(\tilde{\beta}_0\) is extremely close to zero (in each of the models presented in the range of 1-4 weeks. I calculate the intensity of hiring as the fraction of unemployed workers who are successfully linked to the next month who gain employment in this period. (The match rate is quite high—see Rothstein (2011) for a more detailed discussion of matching consecutive months in the CPS).}

\footnote{To be as consistent as possible with the dependent variable \(\partial \left( \ln \left[ \eta_{PH(t)}(x_i) \right] \right) \) (from the CPS), I use values of \(\partial \ln \left[ \eta_{PH(t)} \right] \) from the CPS as well in the results that follow. Empirically, the impact of this choice on the results is trivial.}

\footnote{I measure flows of firings and quits by totalling unemployed job leavers and losers with unemployment durations in the range of 1-4 weeks. I calculate the intensity of hiring as the fraction of unemployed workers who are successfully linked to the next month who gain employment in this period. (The match rate is quite high—see Rothstein (2011) for a more detailed discussion of matching consecutive months in the CPS).}
Table 1: Responses of group reemployment probabilities to compositional changes (Sources: CPS, JOLTS)

Note that the sample above is restricted to men with no more than a high school education. Women are excluded to avoid the complications caused by weak labor market attachment (such as a greater willingness to respond to adverse shocks by substituting effort from the labor market toward family investment). In turn, those without higher education faced the steepest increases in unemployment incidence during the recession, so the causes of this group’s rising unemployment are crucial to understanding aggregate employment dynamics. Further—regarding the mechanism suggested in this paper—education is a tool for signaling competence to prospective employers, so those with lower educational attainment may be less able to distinguish themselves from the unemployment pool. Therefore, increases in targeted firings may impact this group’s reemployment probabilities more severely than others.18

Additionally, I test the hypothesis that

\[
\frac{\partial \ln [\eta^{S}_{PH(t)}]}{\partial \ln [\Omega_{t}]} < \frac{\partial \ln [\eta^{L}_{PH(t)}]}{\partial \ln [\Omega_{t}]}
\]

\[
\Rightarrow \hat{\beta}_{2S} < \hat{\beta}_{3S}
\]

(as predicted in the flows model) against the null (of the stocks model) that

\[
\frac{\partial \ln [\eta^{S}_{PH(t)}]}{\partial \ln [\Omega_{t}]} = \frac{\partial \ln [\eta^{L}_{PH(t)}]}{\partial \ln [\Omega_{t}]}
\]

Table 1, we find \( \hat{\beta}_{0} < 0.01 \). Further, the parameters of interest are almost completely unchanged if we formally restrict \( \hat{\beta}_{0} = 0 \).

18The results above persist without gender and education-based restrictions on the sample, but the estimates have slightly more noise.
In all specifications *and* in both measures of quits/firings (CPS and JOLTS), there is evidence for effects of compositional change—shifts from quits toward firings tend to foreshadow worsening reemployment probabilities for the short-term unemployed relative to the long-term unemployed. This is certainly consistent with lower quality flows to unemployment accompanying these shifts.

**Implications for Recessions**

The analysis thus far was designed to detect compositional changes that accompany normal-sized economic fluctuations. However, the theoretical analysis in the following sections develops consequences of these changes in response to a significant economic downturn, so it will be useful to connect this "equilibrium" empirical analysis to this sort of large economic shock (such as the latest recession) and the ensuing labor market response. Of course, this discussion will be inherently speculative, and what follows should be interpreted as suggestive evidence, not as a conclusive quantitative characterization.

Note that the data used in Table 1 are restricted to the period January 2001 - August 2008. The months September 2008 - August 2011 are omitted to prevent overestimation of the disparities between $\frac{\partial \ln [h^S_{PH(t)}]}{\partial \ln [t]}$ and $\frac{\partial \ln [h^L_{PH(t)}]}{\partial \ln [t]}$. If changes in the flows of firings and quits are serially correlated (as they are during severe economic fluctuations), and if these changes influence $\frac{\partial \ln [h^S_{PH(t)}]}{\partial \ln [t]}$ and $\frac{\partial \ln [h^L_{PH(t)}]}{\partial \ln [t]}$ differently, then the estimates will magnify these differences.\(^{19}\) This is because the change in reemployment likelihood for short/long-term unemployed will capture the compounded effects of multiple periods of similar shifts in composition.

The boundary was drawn between August and September of 2008 so the analysis would avoid the effects of the surge in firings toward the end of the recession (this surge can be seen in Figure 2). If this concern is legitimate, we should see an exaggerated gap between the estimates for $\frac{\partial \ln [h^S_{PH(t)}]}{\partial \ln [t]}$ and $\frac{\partial \ln [h^L_{PH(t)}]}{\partial \ln [t]}$ in the January 2001 - August 2011 sample, and this should grow further for the September 2008 - August 2011 period by itself. This is precisely what we observe (see Table 2 in Appendix C for details)—using the CPS flow measures, we find $\frac{\partial \ln [h^S_{PH(t)}]}{\partial \ln [t]} = -0.153$ and $\frac{\partial \ln [h^L_{PH(t)}]}{\partial \ln [t]} = 0.303$ for the full sample, and this disparity expands for September 2008 and after ($\frac{\partial \ln [h^S_{PH(t)}]}{\partial \ln [t]} = -0.412$ and $\frac{\partial \ln [h^L_{PH(t)}]}{\partial \ln [t]} = 0.445$).

Additionally, there was a sustained shift toward firings during late 2008 and early 2009, and the model developed in this section suggests that the reemployment probabilities of the short-term unemployed should have risen significantly in comparison to those of the long-term unemployed. It is important to note that this relative improvement would be brief—the lower quality workers entering

\(^{19}\)Because this serial correlation is not present in all months of the data, these measured effects will vary across months. As a result, estimates obtained using data for the entire time period will be noisier.
unemployment would initially lower the relative outcomes of the short-term unemployed, but they would reach long-term unemployment in 13 weeks and depress the outcomes of this group thereafter.

Figure 8 (in Appendix C) displays the probabilistic advantage of the short-term unemployed over six month intervals through the recession (the monthly flows of firings and quits appear as well). It is easy to see that this sustained shift toward firings was accompanied by a sharp, relative decline in reemployment likelihood for the short-term unemployed. Indeed, this decline had disappeared by the following six-month interval (and many of those who lost jobs during the surge of firings had joined the group of long-term unemployed by the following six-month period).

With this evidence established, we will proceed to a more detailed model of labor market equilibrium. Sections 2 - 4 will demonstrate how the mechanism behind the above empirical findings arises endogenously from optimal firm behavior, and these sections will explore the implications of this mechanism for the labor market response to a recession. In particular, a recession-induced decline in the quality of the unemployment pool may slow hiring from this pool even after economic conditions have otherwise recovered from this recession.

2. The Formal Model

2.1. Economic Environment

The economy consists of a unit measure of "firms" and a unit measure of "workers." Time is continuous and infinite, and firms discount the future at rate $r > 0$. All firms are identical, but workers are distinguished by a type $\theta \in \{H, L\}$. Of the unit measure of workers in the economy, the proportion $Q \in (0, 1)$ are type $H$.

**Employment, Payoffs, and Wages**

Firms make hiring and firing decisions—each can employ at most 1 worker at a time and must pay the instantaneous flow cost $w$ to do so. Firms receive a payoff $Y$ with Poisson intensity $\lambda$ from a type $H$ worker, and they receive no payoffs from type $L$ workers. In turn, workers face a binary choice between working (for wage rate $w$ and at effort cost $e_w$) and effortless unemployment. (The value $e_w$ is known to both workers and firms).

At Poisson intensity $\pi$, workers experience a shock that drives the flow cost of effort at the current firm from $\tilde{w}$ to $\tilde{w} + b$. Such a shock is meant to represent a living situation change (such as a need to move geographically for family reasons), a developing distaste for the tasks of the current job, or another personal reason for wanting to leave. For the purposes of this model, we will consider only the situation in which $b > \lambda Y - \tilde{w}$, so that no firm can retain a worker under these circumstances.

Wages are determined endogenously, but we will soon see that equilibrium forces drive wages down to the reservation wage $w = \tilde{w}$ (implied by the effort cost of labor).\(^{20}\)

\(^{20}\)We could instead interpret $\tilde{w}$ as a fixed flow cost of operation for the firm (rather than as a reservation wage for
hold for sufficiently high values of \( \tilde{w} \).\(^{21}\) Of course, in reality, labor markets can adjust through both employment and wages, but such wage rigidity is appealing in this case for two reasons: (1) There is an immense empirical literature on downward wage rigidity.\(^{22}\) Though debate continues regarding the mechanism for this rigidity, there is little doubt that it exists in the first place. Further, a wide range of evidence suggests that this rigidity is especially consequential during recessions. (2) This study aims to understand firm hiring and firing decisions, rather than bargaining, wage dynamics, and other labor market phenomena.\(^{23}\)

**Hiring and Contracts**

When out of the market, firms must pay cost \( c \) to hire a worker (who can come only from the unemployment pool). This cost \( c \) can be interpreted as including the search/interview/hiring costs associated with obtaining a new employee.

Firms cannot observe specific worker types before hiring. However, because all firms are ex ante identical (and thus have identical equilibrium strategies), firms can infer the fraction of the unemployed at time \( t \) with \( \theta = H \).

As in the model developed in Section 1, firms can use a *screening technology* to refine the pool of potential hires—the cost of screening is included in the hiring cost \( c \). Again, for an unemployment pool with type \( H \) proportion \( q_U \), the "effective" hiring pool after screening will have the type \( H \) proportion \( q_H (q_U) = \frac{q_U}{q_U + (1-q_U)\alpha} \). The intuition behind this structure is developed in Section 1.

Firms offer workers a fixed-wage contract, which will pay the flow value \( w \) at each instant while the worker is employed—firms reserve the right to terminate employment. As we will focus on employment (rather than on wages and contracting), we will assume that the realization of output \( Y \) is not observable to workers and is not contractible regardless. The wage level \( w \) is determined competitively; firms will attempt to outbid each other until it is no longer *ex ante* profitable to do so.

**Learning and Employment Termination**

Over time, firms learn about worker quality through payoff realizations. Suppose that, at time \( t \), a firm has belief \( p_t \) about the probability that its employee is type \( H \). Firms update according to Bayes’ workers. Alternatively, we could interpret \( \tilde{w} \) as a government-imposed minimum wage (if this minimum wage is greater than workers’ reservation wages). The only distinction between these cases is the source of the firm’s production cost—they are otherwise identical from the firm’s perspective. While these distinctions matter for welfare analysis, this is not the focus of the paper, so we will proceed with the "reservation wage" interpretation for now.

\(^{21}\)As we will see, if \( \tilde{w} \) is too low, employment will be full in equilibrium. In such cases, the hiring pool consists of those workers immediately exiting employment, and the quality of this flow will determine equilibrium wages. This "extreme" outcome is less desirable both for theoretical analysis and for applied insight, so we will ignore it in the analysis to follow.

\(^{22}\)This literature is far too diverse (and dense) for me to characterize adequately with a representative sample, but Bewley (1999) is among the most thorough and insightful analyses of this phenomenon.

\(^{23}\)Of course such phenomena exist, but the aim of this analysis is to understand the interplay between learning and hiring/firing behavior. As such, I omit these elements at present to preserve the model’s simplicity and clarity, but I briefly develop a more nuanced wage-setting process in Section 5.2.
rule; if the firm receives a payoff at time $t$, the firm changes its belief discretely by the amount

$$dp_t (1) = 1 - p_t$$

(so the firm updates to $p_{t+dt} = 1$ because a payoff could not have arrived with a type $L$ worker). If a firm receives no payoff at time $t$, the firm shifts its belief infinitessimally downward by

$$dp_t (0) = -\lambda p_t (1 - p_t) dt$$

Matches can end in two ways: (i) workers quit at Poisson intensity $\pi$ after receiving a shock to the effort cost of employment at the current firm and (ii) firms intentionally fire workers. Terminations of type (i) are meant to represent unforeseen, idiosyncratic worker shocks, such as a desire to geographically relocate one’s family or a newly-developed distaste for one’s current job. Obviously, type (ii) terminations correspond to targeted firings of low-productivity workers.

In making optimal decisions, firms face three options at each moment in time:

1. Retain its current employee, offering value $V(p_t)$
2. Fire its current employee and leave the market (indefinitely), offering value $0$
3. Fire its current employee and hire a new worker (at transition cost $c$) at random from the current stock of unemployed workers, offering value $V(q_{U(t)}) - c$ (where $q_{U(t)}$ is the proportion of type $H$ workers among the unemployed at time $t$). \(^{24}\)

A firm will prefer termination via option (2) or (3) when its belief about its worker’s type falls to some $p^*$ such that it expects greater value from either leaving the market or hiring a replacement—this will be developed in more detail in the next section.

### 2.2. The Firm’s Problem

Before analyzing this economy’s equilibrium (which we will do in the following section), we will consider the problem of a single firm, taking the proportion of high types among the unemployed as given. As described above, the firm constantly faces a choice between remaining with its current employee, leaving the market, and paying cost $c$ to hire a new worker (about whom it will have initial belief $q$ regarding the probability it is type $H$). Of course, the firm will choose the optimum of these, and it thus has the value function:

$$V(p_t) = \max \left\{ \lambda p_t Y - w \right\} dt + e^{-rdt} \mathbb{E} [V(p_t + dp_t)] , \ 0, \ V(q) - c \right\}$$

\(^{24}\)Note: after a match has been terminated, firms must pay $c$ to hire a new worker regardless of whether the termination was targeted or exogenous. In this sense, we exclude firing costs from those represented by $c$. Adding a parameter to capture these firing costs has no substantive impact on the results that follow.
To solve for the firm’s optimal strategy, we must first recognize that this strategy will involve a threshold rule for terminating the current match. In terms of the previously listed options, the firm will compare the value of (1) remaining in the match to the maximum value of (2) leaving the market (0) or (3) pursuing a new match \(V(q - c)\), and at some \(p^*\), the value of remaining in the current match will be equal to \(\max\{0, V(q - c)\}\). The firm will then choose the best alternative when its belief falls to this \(p^*\).

When the firm’s belief is above \(p^*\) (so that it chooses to remain with its current employee), its value function can be written as

\[
V(p_t) = [\lambda p_t Y - w] dt + \left(\frac{1}{1 + rd_t}\right) [\pi dt [0] + (1 - \pi dt) [\lambda p_t V(p_t + dp_t (1)) dt + (1 - \lambda p_t dt) V(p_t + dp_t (0))]]
\]

where

\[
V(p_t + dp_t (1)) = V(1) = \frac{Y\lambda - w}{r + \pi}
\]

and

\[
V(p_t + dp_t (0)) = V(p_t - \lambda p_t (1 - p_t) dt) \approx V(p_t) - \lambda p_t (1 - p_t) V'(p_t) dt
\]

From this, we can obtain the following linear, first-order ODE:

\[
[r + \pi + \lambda p_t] V(p_t) = Y\lambda p_t \left(\frac{r + \pi + \lambda}{r + \pi}\right) - w \left(\frac{r + \pi + \lambda p_t}{r + \pi}\right) - V'(p_t) p_t (1 - p_t) \lambda
\]

A particular solution to this is the expected value of committing forever to the current employee:

\[
\frac{\lambda p_t Y - w}{r + \pi}
\]

To capture the option value of being able to terminate the match, we must look to the solution of the homogeneous part of the ODE.\(^{25}\) Judiciously applying several techniques drawn from Bellman and Cooke (1963), Presman (1990), and Keller, Rady, and Cripps (2005), we obtain a solution of the form:

\[
V(p_t) = \frac{\lambda p_t Y - w}{r + \pi} + K (1 - p_t)^{\lambda + r + \pi \over \lambda} p_t r^{r - \pi \over \lambda}
\]

Here, \(K\) is a constant to be determined by a boundary condition:

\[
V(p^*) = \max\{0, V(q - c)\}
\]

Before proceeding, it is worth noting a property of this value function implied by the "learning" element of the problem. The value to the firm of employing a worker is greater than it would be if the firm did not consider the "option value" of employing the worker. This option value is generated

\(^{25}\)This homogenous solution will have the form \((1 - p_t)^{1+\mu} p_t^{-\mu}\) for some \(\mu\) to be determined.
by the opportunity to continue employing the worker, should he reveal himself to be the high type. If
the firm were to cut ties with this worker, this potential payoff would be lost.

To see this more clearly, consider the behavior of a "myopic" firm—one that considers the value of
an employee to be only the expected payoff predicted by its current belief about the employee's type.
For simplicity, let us normalize the firm's outside option to 0 (so assume that \( V(q) - c \leq 0 \)). Then
this firm will employ a worker only if \( \lambda p_t Y \geq w \), because otherwise the expected flow payoff associated
with the worker would be negative. Thus, such a firm would have a decision rule based on \( p^*_m = \frac{w}{\lambda Y} \),
which is higher than the \( p^* \) that an optimal firm would have (the corresponding optimal \( p^* \) will be
provided in the discussion of equilibrium in Section 3.2). This relationship can be seen in Figure 3
below, which compares the value functions and decision rules associated with the traditional optimal
firm and a "myopic" firm. Note that the x-axis covers \( p_t \) in the range between the firm's optimal \( p^* \)
and 1—both firms would pursue the outside option (with value 0) for \( p_t \) below this range.

![Figure 3: Firm value as a function of belief \( p_t \)](image)

We will not proceed toward an explicit solution (via the boundary conditions given above and
others) until our discussion of the steady-state equilibrium. Instead, we will digress briefly to consider
equilibrium in a full-information (no learning) environment.

3. Steady-State Equilibrium

3.1. Example: Equilibrium with Full-Information

For purposes of intuition, we will first consider the simple case in which there is no learning—employers
perfectly observe worker types even before hiring them.\(^{26}\) We will assume for convenience that \( V(0) < 0 \)
and \( \lambda Y - \bar{w} \geq 0 \), so that firms will either employ type \( H \) workers or leave the market. Type \( H \)
workers will never be fired intentionally, but they can still reach the unemployment pool by quitting.

\(^{26}\)Note that this full information equilibrium can be viewed as the extreme case in which the screening technology's
effectiveness parameter \( \alpha = 0 \).
Free entry will force wages for type $H$ workers up to $w = \lambda Y - (r + \pi) c$, so that the value of hiring a type $H$ worker is 0. As such, high type workers who reach the unemployment pool will be instantly hired at this wage.

In this full information equilibrium, employment (which we will denote $E$) will be $E = Q$. In order for hiring to occur in the equilibrium with unobserved types (before hiring), wages must be significantly lower to compensate firms for the risk of hiring a low type. As we will see in the following section, the steady-state employment level can be lower or higher than $Q$ (depending on the cost of effort $\tilde{w}$ and the corresponding reservation wage), but a significantly lower wage is needed to sustain hiring in this equilibrium.

### 3.2. Equilibrium with Learning

What happens when firms cannot perfectly observe worker types? To understand the impact of learning on employment dynamics in a competitive economy, we will start by considering equilibrium in a steady-state. We thus introduce the following concept:

**Definition 1:** A *steady-state employment equilibrium* $\{w^*, p_{ss}^*, q_{U(ss)}, q_{E(ss)}, E_{ss}\}$ consists of a market wage rate $w^*$, a threshold rule $p_{ss}^* \in [0,1]$, a proportion $q_{U(ss)} \in [0,1]$ of unemployed workers that are type $H$, a proportion $q_{E(ss)} \in [0,1]$ of employed workers that are type $H$, and a fixed proportion $E_{ss} \in [0,1]$ of workers that are employed such that:

1. **(i.a)** $V_{ss}(p_{ss}^*) = 0$ (firm optimality - value matching)
2. **(i.b)** $V_{ss}'(p_{ss}^*) = 0$ (firm optimality - smooth pasting)
3. **(ii)** $V_{ss}(q_{H}(q_{U(ss)})) \leq c$ (free entry)
4. **(iii)** $E_{ss}q_{E(ss)} + (1 - E_{ss})q_{U(ss)} = Q$ (market-clearing - employment/employee quality)
5. **(iv)** $q_{U(ss)} = \psi$ (constant type $H$ proportion among unemployed)
6. **(v)** $\eta_{ss} = \varphi$ (constant employment level)

where $\varphi$ is the measure of workers instantaneously entering unemployment, $\psi$ is the proportion of type $H$ workers among the unemployed at time $t + dt$, and $\eta_{ss}$ is the instantaneous intensity at which the unemployed are hired.

Further, the economic applications considered in this paper motivate us to focus on equilibria with neither full employment nor full unemployment. We will elaborate upon this later in the section.

**Definition 2:** Such an equilibrium is "nontrivial" if $E_{ss} \in (0,1)$.

Several aspects of these definition are worth noting:

- Firstly, no assumption is needed regarding whether firms remain in the market after firing workers (by hiring new workers) or whether these hirings are made instead by new firms entering the market. Even if firms out of the market attempt to hire workers, they can obtain only the value
\[ V_{ss} (q_{U(ss)}) - c = 0 \] in equilibrium, so conditions (i.a) and (ii) hold regardless of whether or not firms move between employing workers and leaving the market. As long as the proportions of firms in and out of the market are constant, these conditions will characterize equilibrium.

- Smooth pasting holds as a result of the standard arguments—only downward movement in the belief \( p_t \) will be differentiable, so this condition must hold when \( p_t \) reaches the threshold \( p_{ss}^* \).

- Firms apply the same screening technology to the same unemployment pool, so each firm begins with a prior belief of \( q_H (q_{U(ss)}) \) when hiring a worker. Similarly, firms share the same threshold level \( p_{ss}^* \). Thus, firms who fire workers after not receiving a payoff must all wait the same amount of time \( t_{ss}^* \) after hiring to terminate the match. We will characterize \( t_{ss}^* \) in terms of \( p_{ss}^* \) and \( q_{U(ss)} \) below (see Proposition 3.1).

- Condition (v) requires the measure of workers entering employment to exactly counter the measure of workers entering unemployment. The measure of workers instantaneously entering unemployment \( \varphi \) consists of those who quit jobs and those who are fired after duration \( t_{ss}^* \) of unproductive employment. Hence condition (v) implies

\[
\eta_{ss} = \left. \frac{E_{ss} \pi dt}{\text{hired}} \right|_{\text{quit}} + \eta_{ss} \left. \left[ (1 - q_H (q_{U(ss)})) e^{-\pi t_{ss}^*} + q_H (q_{U(ss)}) e^{-(\lambda + \pi) t_{ss}^*} \right] \right|_{\text{fired}}
\]

This condition will allow us to solve for the hiring intensity of the unemployed \( \eta_{ss} \) (see Proposition 3.2).

- Condition (iv) requires the type \( H \) proportion among the unemployed to be constant over time (note that we will casually refer to this proportion as the "quality" of the unemployment pool). Coupled with conditions (iii) and (v), this implies that the quality of the pool of employed workers should remain constant as well. The explicit implication of this condition is provided below:

\[
q_{U(ss)} = \psi = \frac{\left[ 1 - E_{ss} q_{U(ss)} \right] \eta_{ss} + \eta_{ss} q_H (q_{U(ss)}) - \eta_{ss} q_H (q_{U(ss)}) e^{-(\lambda + \pi) t_{ss}^*} + \left. E_{ss} q_{E(ss)} \pi dt \right|_{\text{quits}}} {1 - E_{ss} - \eta_{ss} + \eta_{ss} \left[ (1 - q_H (q_{U(ss)})) e^{-\pi t_{ss}^*} + q_H (q_{U(ss)}) e^{-(\lambda + \pi) t_{ss}^*} + \left. E_{ss} \pi dt \right|_{\text{quits}}} \right|_{\text{unemployed}} + \eta_{ss} \left[ (1 - q_H (q_{U(ss)})) e^{-\pi t_{ss}^*} + q_H (q_{U(ss)}) e^{-(\lambda + \pi) t_{ss}^*} + \left. E_{ss} \pi dt \right|_{\text{quits}}} \right|_{\text{hired}}}
\]

Intuitively, this means that the inflows to and outflows from unemployment must not impact the quality of this pool. Of course, combining this with conditions (iii) and (v) implies that both total inflows/outflows and those of only high types must be 0 in the steady-state employment equilibrium.
It is crucial to remember that, because hiring involves a selective screening process, hiring necessarily lowers the quality of the unemployment pool. For this reason, hiring plays a central role in preserving equilibrium. In particular, the intensity of hiring is the instrument by which the free entry condition maintains \( q_U(\text{ss}) \leq q_U^* (Y) \) (where \( V_{\text{ss}} (q_H (q_U^* (Y))) = c \)).

Intuitively, this market force reflects optimal firm equilibrium behavior—firms will continue to hire as long as it is profitable to do so. If too few firms were hiring, the unemployment pool quality would rise and more firms would find it profitable to hire. If too many firms were hiring, their screening process would deplete the quality of the unemployment pool, and it would no longer be profitable to hire. Thus, the economy can support only an intensity of hiring that will not drive \( q_U \) below \( q_U^* (Y) \).

For reasonable parameter combinations, there is a unique nontrivial steady-state employment equilibrium, and we can characterize these parameter combinations:

**Proposition 1:** For reservation values \( \bar{w} \in (w (Y, \lambda, \pi, r, Q, c, \alpha), \overline{w} (Y, \lambda, \pi, r, Q, c, \alpha)) \) (where \( w (\cdot) \) and \( \overline{w} (\cdot) \) are bounds determined by other parameters), there exists a unique nontrivial steady-state employment equilibrium satisfying the conditions in Definition 1.

(See Appendix A for proof)

Intuitively, for given parameters \( \{Y, \lambda, \pi, r, Q, c, \alpha\} \), \( w (Y, \lambda, \pi, r, Q, c, \alpha) \) is the highest reservation wage level at which \( E_{\text{ss}} = 1 \), and \( \overline{w} (Y, \lambda, \pi, r, Q, c, \alpha) \) is the lowest reservation wage level at which \( E_{\text{ss}} = 0 \). Wage levels strictly within this range will yield equilibria with positive masses in both employment and unemployment.

To better understand the boundary \( \overline{w} (Y, \lambda, \pi, r, Q, c, \alpha) \), note that, because there cannot be positive selection into unemployment, we must always have \( q_U(\text{ss}) \leq Q \). Thus, if \( \bar{w} \) is not low enough to allow firms to regain the hiring cost in expectation (meaning \( \bar{w} > \overline{w} (Y, \lambda, \pi, r, Q, c, \alpha) \)), we will have \( V (q_H (Q)) < c \), and there will be no employment in equilibrium.

In turn, regarding \( w (Y, \lambda, \pi, r, Q, c, \alpha) \), note that \( \bar{w} = w (Y, \lambda, \pi, r, Q, c, \alpha) \) corresponds to the reservation wage level at which there is a full employment equilibrium. The full employment steady-state equilibrium for reservation wages \( \bar{w} < w (Y, \lambda, \pi, r, Q, c, \alpha) \) will involve a lower termination belief level \( p_{\text{ss}}^* \). At such a termination threshold, the flow to unemployment will consist of disproportionately few fired workers (and disproportionately many quitting workers). Thus, this flow will have a higher proportion of type \( H \) workers \( q \). Workers entering unemployment are immediately hired again, and these hiring firms would obtain value \( V (q_H (q)) > c \) at \( w = \bar{w} \), so free entry would require wages

\footnote{Firms exert labor market externalities on other firms through both selective hiring and targeted firing. The implications of this will be developed later in the paper—in particular, in the discussion of cyclical equilibria in Section 5.1.}

\footnote{Appendix A also includes intuition regarding the impact of each of the parameters on the bounds \( w (Y, \lambda, \pi, r, Q, c, \alpha) \) and \( \overline{w} (Y, \lambda, \pi, r, Q, c, \alpha) \).}
$w > \bar{w}$ such that $V(q_H(q)) = c$. In other words, the hiring pool size would be of order $dt$, so these workers would be scarce, and firms would need to compete for their labor. With the pool quality at a level to support $w > \bar{w}$, firms would indeed bid up the wage until $V(q_H(q)) = c$. This is the only case with positive employment in which wages can take on a value other than $\bar{w}$, so we have the following result:

**Lemma 1:** In any nontrivial steady-state employment equilibrium, $w = \bar{w}$.

*(See Appendix A for proof)*

In what follows, we will consider only "nontrivial" steady-state employment equilibria, so we will focus on cases in which there is a positive mass of unemployed workers in equilibrium.

One more restriction will be imposed on the range of $\bar{w}$ we will consider in equilibrium—for some extreme parameter combinations, we will decrease the upper bound below $\bar{w}(Y, \lambda, \pi, r, Q, c, \alpha) - \delta$ to rule out cases in which $p_{ss} > q_{U(\bar{w})}$. Intuitively, we want to avoid equilibria in which the hiring cost is so low and the reservation wage level is so high that workers are being hired and fired in rapid succession.\(^{29}\) In such equilibria, the intensity of hiring is so great that all of the downward pressure on the unemployment pool quality comes from directed hiring—both random and directed firings exert positive pressure on the unemployment pool quality. This analysis considers a labor market steady-state in which job leavers and job losers are countervailing forces on the unemployment pool, so such extreme cases offer little, if any, applied insight.

The next step in our analysis is the use of the value-matching, smooth-pasting, and free entry conditions to solve explicitly for the firm’s value function.

**Proposition 2:** In a steady-state employment equilibrium, the firm’s value function can be written explicitly in terms of the threshold rule $p_{ss}^*$

$$V_{ss}(p_t) = \begin{cases} \left( \frac{1}{r+\lambda} \right) \left[ \lambda p_t Y - \bar{w} + (\bar{w} - \lambda p^t Y) \left( \frac{1-p_t}{1-p_{ss}^*} \right) \left( \frac{1-p_t}{1-p_{ss}^*} \right) \right] & \text{for } p_t \in [p_{ss}^*, 1] \\ 0 & \text{for } p_t \leq p_{ss}^* \end{cases}$$

Further, this threshold $p_{ss}^*$ is given by

$$p_{ss}^* = \frac{\bar{w}(r+\pi)}{\lambda [Y(r+\lambda+\pi) - \bar{w}]}$$

*(See Appendix A for proof)*

As can be seen in the expression for $p_{ss}^*$, firms will retain workers at lower beliefs with higher payoffs $Y$, faster learning (and more frequent payoffs) $\lambda$, less frequent worker quits $\pi$, less firm "patience" $r$ (or lower interest rates), and lower wages $\bar{w}$.

Further, we can see that the equilibrium distribution of firm beliefs will appear (approximately) as below:

---

\(^{29}\)One might want to consider this case in a model of the labor market for NFL coaches, but since this is not the intended topic of this analysis, we will ignore it here.
Because firms apply the screening technology to the unemployment pool, they can expect a new hire to be type $H$ with probability $p_0 = q_H \left(q_{U \left(s_s\right)}\right)$. While this individual is employed, the firm’s belief $p_t$ declines gradually unless the firm receives a payoff $Y$—if this occurs, the firm knows its worker must be of type $H$, so it updates to $p_{t+dt} = 1$ and employs the worker until random termination of the match (via $\pi$). The mass at $p_t = 1$ corresponds to such firms who know they have type $H$ employees. Without a payoff, the firm will employ the worker until its belief falls to $p^{*}_{ss}$ (where $V(p^{*}_{ss}) = 0$), at which point the firm will terminate the match intentionally and either hire a new worker or leave the market.

The following results characterize the aggregate implications of firm behavior in this equilibrium:

**Proposition 3:** In a steady-state employment equilibrium:

(i) Without receiving a payoff $Y$, a firm will wait for time $t^{*}_{ss}$ after hiring a worker before firing him, where

$$t^{*}_{ss} = \frac{1}{\lambda} \ln \left( \frac{1 - p^*_{ss}}{p^*_{ss}} \right) \left( \frac{q_{U \left(s_s\right)}}{\alpha \left(1 - q_{U \left(s_s\right)}\right)} \right)$$

(ii) Unemployed workers are hired at intensity:

$$\eta_{ss} = \pi E_{ss} dt \left[ \frac{q_{U \left(s_s\right)}}{q_{U \left(s_s\right)} \left[1 - e^{-\left(\lambda + \pi\right)t^*_{ss}}\right] + \alpha \left(1 - q_{U \left(s_s\right)}\right) \left[1 - e^{-\pi t^*_{ss}}\right]} \right]$$

(iii) The employment level can be written:

$$E_{ss} = \left[ \frac{Q - q_{U \left(s_s\right)}}{1 - q_{U \left(s_s\right)}} \right] + 1 \left( \frac{\alpha}{q_{U \left(s_s\right)}} \right) \left( \frac{1 - e^{-\pi t^*_{ss}}}{1 - e^{-\left(\pi + \lambda\right)t^*_{ss}} - \alpha \left[1 - e^{-\pi t^*_{ss}}\right]} \right)$$

(See Appendix A for proof)

We can use the above expressions as a framework for understanding the steady-state equilibrium. Consider how the equilibrium allocations shift as we vary the "effective" reservation wage $\bar{w}$. In line
with standard logic about the relationship between wages and employment, there exist high wage equilibria, in which employment and the intensity of hiring are low. Conversely, low wages will correspond to equilibria with high employment and hiring intensities.

In the following section, we will consider the response of this economy when unanticipated forces perturb the steady-state equilibrium.

4. Shocks and Employment Dynamics

4.1. Analysis and Results

The analysis thus far has considered a static economy, but this model has much to say about the dynamics of employment in response to economic shocks. In this section, we will explore these dynamics. We will first consider the impact of an unanticipated negative shock to the payoff $Y$—we can interpret this as a decrease in productivity, which much of the macro/search literature uses to represent a recession. (It can also be interpreted as a reduction in demand or as a combination of the two.)

Suppose that, at time $t = \hat{t}$, the payoff $Y$ falls unexpectedly from $Y_{ss}$ to $Y_{ss} - z$. Suppose also that, before this shock, the economy was set at its steady state employment equilibrium. The initial impact of this shock on firm behavior occurs through the threshold rule $p^*$, which rises from

$$p_{ss}^* = \frac{\bar{w} (r + \pi)}{\lambda [Y (r + \lambda + \pi) - \bar{w}]}$$

to

$$p_{\hat{t}}^* = \frac{\bar{w} (r + \pi)}{\lambda [(Y - z) (r + \lambda + \pi) - \bar{w}]} > p_{ss}^*$$

The decrease in $Y$ makes firms less patient in learning about worker types—the payoff associated with type $H$ workers has decreased (and also decreased relative to the payoff associated with type $L$ workers, which is 0), so the value of learning about worker type has also decreased. Hence, the threshold belief for terminating workers rises.

For notational purposes, let us define $t_z$ to be the new time after hiring associated with the economic shock. This $t_z$ applies to firms who hired workers before the shock occurred. Such firms will now be willing to wait $t_z$ after their initial hires without a payoff before cutting ties—if they have already waited for some time $t$ in the range $[t_z, t_{ss}^*]$, they will fire their workers immediately. Using the approach by which we obtained the result in Proposition 3.i, we can express this $t_z$ in terms of $p_{\hat{t}}^*$ and $q_{U(ss)}$:

$$t_z = \frac{1}{\lambda} \ln \left[ \left( \frac{1 - p_{\hat{t}}^*}{p_{\hat{t}}^*} \right) \left( \frac{q_{U(ss)}}{\alpha (1 - q_{U(ss)})} \right) \right]$$

With this notation established, we can begin to characterize the impact of this shock on the economy. The mass of firms who fire their workers immediately after the shock is visually depicted in Figure 5 (below).
These workers fired in response to the shock (in the belief region $[p^*_t, p^*_e]$) are of better average quality than those fired in the preexisting steady-state. Note that this is perfectly in line with the standard intuition promoted in Nakamura (2008), Kosovich (2010), and Lockwood (1991), among others. The standards for termination rise, so the quality of those terminated rises as well—this remains true in this model. The departure from this standard result is based not only on fired workers, but also on the mixture of fired workers and those who quit. This will be developed later in this section, when we characterize the evolving quality of the unemployment pool (see Proposition 5).

Next, consider the rise in the unemployed population—the unemployment pool will expand to include these newly fired workers:

**Proposition 4:** When the economy is settled at a steady-state employment equilibrium with a binding reservation value $\bar{w}$ in the range given in Proposition 1, and output falls unexpectedly from $Y$ to $Y - z$, the measure of unemployed workers rises from $1 - E_{ss}$ to

$$1 - E_{ss} + \frac{\eta_{ss}}{\pi} \int_{t}^{t_{ss}} \left[ qH \left( qU_{ss} \right) e^{-\left(\pi + \lambda\right)s} + \left(1 - qH \left( qU_{ss} \right) \right) e^{-\pi s} \right] ds$$

where $\eta_{ss}$ and $E_{ss}$ represent the quantities specified in Proposition 3 for the steady-state employment equilibrium.

**Proof:** The second component of this expression

$$\frac{\eta_{ss}}{\pi} \int_{t}^{t_{ss}} \left[ qH \left( qU_{ss} \right) e^{-\left(\pi + \lambda\right)s} + \left(1 - qH \left( qU_{ss} \right) \right) e^{-\pi s} \right] ds$$

describes the mass of workers fired in immediate response to the shock. To see the intuition for this expression, consider the path of beliefs followed by a firm hiring a worker at time $0$ in the steady state, where the firm neither realizes output nor has its worker quit before time $t^*_{ss}$. Of the workers hired in
the steady state, proportion $q_H \left( q_{U(ss)} \right)$ of these are type H. Because firms with type H workers can leave this belief path either through a worker quitting or through a realization of output, proportion $e^{-\left(\pi + \xi \right)t}$ of the firms hiring type H workers remain on this belief path after time $t$. In turn, firms with type L workers can leave this path only if this worker quits, so proportion $e^{-\pi t}$ of these firms remain on the path at time $t$. Thus, the above expression corresponds precisely to the mass of new fires depicted in Figure 5.

Of course, there is information in these firing decisions—the firms who fire workers in response to the shock have beliefs in the range $\left[ p^*_s, p^*_t \right]$. While these newly fired workers are better on average than those fired in the steady-state (at belief $p^*_s$), they still represent the lowest belief range of previously employed workers. As such, relative to the set of employed workers, this group has disproportionately few type H workers. Yet, this group’s quality alone is insufficient to lower the unemployment pool quality—the size of this group also plays a crucial role in the evolution of the unemployment pool.

In the steady-state, there was a delicate balance in the flow to unemployment between quitting workers (of which proportion $q_{E(ss)} > Q$ were type H) and fired workers (of which proportion $p^*_s$ were type H)—this balance maintained the quality of the unemployment pool. After the shock, the negative pressure on unemployment pool quality from the mass of directed firings overwhelms the positive pressure from the flow of quitting workers (which is always of order $dt$). Thus, even though the workers fired in response to this shock are better (on average) than those fired in the steady-state, the unemployment pool decreases in quality.

Proposition 5: $\exists \bar{z} > 0$ such that for $z \in (0, \bar{z})$, the proportion of type H workers in the unemployment pool immediately following the shock $(Y \rightarrow Y - z)$ falls to

\[
q_U(\bar{t}) = \frac{(1 - E_{ss}) q_{U(ss)} + \frac{\eta_{ss}}{\omega} \int_{t_z}^{t^*_s} q_H \left( q_{U(ss)} \right) e^{-\left(\pi + \lambda \right)s} ds}{1 - E_{ss} + \frac{\eta_{ss}}{\omega} \int_{t_z}^{t^*_s} \left[ q_H \left( q_{U(ss)} \right) e^{-\left(\pi + \lambda \right)s} + \left(1 - q_H \left( q_{U(ss)} \right) \right) e^{-\pi s} \right] ds} < q_H \left( q_{U(ss)} \right)
\]

(See Appendix A for proof)

To see why we must bound values of $z$ above for this to hold, consider the extreme case of $z > Y - \frac{w}{\lambda}$. Such a shock would be so large that even firms certain about having a type H worker would not continue to employ them, so we would have full unemployment, and $q_U(\bar{t})$ would rise to $Q > q_{U(ss)}$. Clearly then, for $z$ sufficiently close to this range, so many workers will enter unemployment that the unemployment pool quality will rise. We can sensibly ignore such cases as disconnected from reasonable applications of this analysis.

Of course, this contamination of the unemployment pool is not the only response to this shock—firm hiring standards rise along with firing standards:
Lemma 2: Define \( q_U(Y) \) to be the unemployment pool quality such that firms are indifferent between hiring and not \( V(q_H(q_U)) = c \) when the output level is \( Y \). Then \( q_U(Y) \) is strictly decreasing in \( Y \).

(See Appendix A for proof)

Since the economy was at its steady-state before the shock (and since there was free entry in this steady state), the value of hiring a worker from the pool of unemployed at \( Y_{ss} \) was 0 (meaning \( V(q_U(Y_{ss})) = c \)). Obviously, conditional on employing a worker with belief \( p_t > p^* \), the firm’s value is monotonically increasing in \( Y \). Thus, with \( Y \) now at \( Y - z \), \( V(q_H(q_U(Y_{ss}))) < c \) and firms will require a higher initial belief \( q_H(q_U(Y - z)) \) (and, in turn, a higher unemployment pool quality) to justify hiring a worker.

As we have seen, though, this increased standard is compounded by a drop in the unemployment pool quality—the rising standards for hiring and firing have forced a wedge between actual market conditions and those necessary for sustained hiring. As a result, hiring will cease completely. Hiring will resume only when the unemployment pool quality has recovered sufficiently to collapse this wedge—voluntary employee quits are the channel through which this will occur. Only after enough remaining employees have voluntarily entered the unemployment pool will hiring resume. Assuming that employee quits continue at a constant rate following the shock, we can analytically characterize the duration without hiring:

Proposition 6: After the output shock \( Y \rightarrow Y - z \), hiring will cease for the duration \( \hat{t}_H \) (an expression for \( \hat{t}_H \) appears in the appendix). If \( q_U(Y - z) < Q \), \( \hat{t}_H \) is finite and satisfies \( q_U(Y - z) = f_1(\hat{t}_H|t_{ss}, t_z) \) (an expression for \( f_1(\hat{t}_H|t_{ss}, t_z) \) appears in the formulation of this proposition given in Appendix A). If \( q_U(Y - z) \geq Q \), \( \hat{t}_H = \infty \).

(See Appendix A for proof)

First, note that, because there can be no positive selection into unemployment, \( q_U(t) < Q, \forall t \). Thus, if \( z \) is so large that \( q_U(Y - z) \geq Q \), then it is impossible for the unemployment pool quality to reach a level at which hiring can resume, and employment will converge to 0 over time.\(^{30}\)

Regarding the case in which \( q_U(Y - z) < Q \), the interested reader can verify that the expression provided in Appendix A for \( f_1(\hat{t}_H|t_{ss}, t_z) \) is visually horrendous. Despite this, the intuition behind the condition \( q_U(Y - z) = f_1(\hat{t}_H|t_{ss}, t_z) \) reduces to an accounting exercise. \( f_1(\hat{t}_H|t_{ss}, t_z) \) represents the proportion of type \( H \) workers in the unemployment pool when time \( \hat{t}_H \) has elapsed after the shock. To express this, we must account for workers who:

\( (i) \) were already unemployed at \( \hat{t} \)

\( (ii) \) were fired immediately after the shock

\( (iii) \) were fired upon beliefs reaching the new threshold \( p^*_t \) at some time \( t \in \left[ \hat{t}, \hat{t} + \hat{t}_H \right] \)

\(^{30}\)In the case where \( q_U(Y - z) = Q \), \( q_U(t) \) will converge to \( Q \) as \( t \rightarrow \infty \), but it will not reach \( Q \) in finite time.
(iv) quit their jobs at some time \( t \in [\hat{t}, \hat{t} + \hat{t}_H] \) while at firms with beliefs in the range \( p_t \in \left[qH \left(q_{U_{ss}}\right), p^*_t\right] \)

(v) revealed themselves to be type \( H \) workers before the shock and quit their jobs at some time \( t \in [\hat{t}, \hat{t} + \hat{t}_H] \)

(vi) revealed themselves to be type \( H \) workers after the shock and quit their jobs at some time \( t \in [\hat{t}, \hat{t} + \hat{t}_H] \)

These six groups are represented (in order) in both the numerator and denominator in the expression for \( f_1 \left( \hat{t}_H \big| t^*_s, t_z \right) \) given in Appendix A (in the numerator, only the type \( H \) workers are counted). To illustrate this further, the evolution of the employment level and the unemployment pool quality after the shock are depicted below.

![Figure 6: Evolution of employment level after shock](image)

![Figure 7: Evolution of unemployment pool quality after shock](image)

It is crucial to note that the duration obtained in Proposition 6 and depicted above in Figures 6 and 7 is computed assuming that workers continue to quit jobs voluntarily after the shock at the
same rate as they did in the previous steady-state. In reality, voluntary quits plummet during a recession (as shown in Figure 2). If this were to happen in the model, the duration without hiring would be magnified drastically. As voluntary quits are the main source of upward pressure on the unemployment pool quality, reducing the volume of these quits would slow the recovery in quality of the unemployment pool. As such, we can view this duration \( t_H \) as a lower bound on the duration for which hiring should cease.

Figures 6 and 7 make clear that the unemployment pool will grow after the shock—unmitigated by hiring—through the six channels listed above until the quality of this pool has risen to the new threshold required for hiring to resume. It is easy to see that the new, lower output equilibrium will have more unemployment and a higher unemployment pool quality. This is in line with the standard view about the composition of the unemployment pool in recession—more people are unemployed, and hiring/firing standards are higher, so the pool must be better. Of course, this model highlights the flaws with this logic: Firstly, the transition from the first equilibrium to the second involves a significant period with a lower quality unemployment pool. Secondly, the second equilibrium may not be a permanent state of the economy. A recession is commonly viewed as a temporary productivity shock—if productivity rebounds during the transitional period, this equilibrium with a better unemployment pool may never be reached in the first place.

To develop this concern, consider one further variation on this economic shock structure—suppose that, as before, there is an unanticipated output shock \( Y \rightarrow Y - z \) at time \( \hat{t} \), and that firms respond to this immediately. Rather than permanently remaining at this level, however, suppose that output will permanently return from \( Y - z \) to \( Y \) at some point in the future, and suppose that firms know this. In particular, suppose that the "recovery date" follows a Poisson distribution with parameter \( \gamma \), so that at each point in time during the "recession," the economy recovers with probability \( \gamma dt \). Then we can show that, for a range of recovery dates \( t \in \left[ \hat{t}, \hat{t} + t_{Y-z,Y} \right] \), there will be no hiring for some time even after the economy has recovered to the previous level.\(^{31}\)

**Proposition 7:** Consider an unanticipated transitory output shock \( Y \rightarrow Y - z \) to the steady-state at time \( \hat{t} \), immediately after which it is known that output will rebound \( Y - z \rightarrow Y \) at Poisson-distributed times with parameter \( \gamma \). Define \( p^* (Y - z, Y) \) to be the termination belief level after the shock but before the recovery, and define \( t^* (Y - z, Y) \) to be the associated time firms will wait without output before terminating a worker. Further, define \( q^*_U (Y - z, Y) \) to be the corresponding hiring threshold during this "recession," and again define \( q^*_U (Y) \) to be the hiring threshold after the recovery. After firms respond optimally to this shock at \( \hat{t} \), if the recovery occurs before time \( \hat{t} + t_{Y-z,Y} \) (where \( t_{Y-z,Y} \) satisfies \( q^*_U (Y) = f_2 \left( t_{Y-z,Y}, t_s, t_{Y-z} \right) \)) and an expression for \( f_2 \left( t_{Y-z,Y}, t_s, t_{Y-z} \right) \) is provided in the appendix), then the economy will remain without hiring for a positive duration of time even after the recovery. Further, \( \exists \Xi > 0 \) such that for \( z \in (0, \Xi) \), the range of recovery times \( t \in \left[ \hat{t}, \hat{t} + t_{Y-z,Y} \right] \) that will not immediately generate hiring is increasing in the magnitude of the

\(^{31}\)An expression for determining the duration \( t_{Y-z,Y} \) will be provided in the proof of Proposition 7 in Appendix A.
shock \( z \) (meaning \( t\frac{\partial z}{\partial Y} \) is increasing in \( z \) over this range).

(See Appendix A for proof)

Because the unemployment pool has been contaminated by the equilibrium response to the initial shock, the labor market may be unable to sustain hiring even after the recovery. In a sense, this result analyzes the impact of temporarily increased firing standards without the impact of increased hiring standards (as the hiring threshold returns to its steady-state level after the recovery).\(^{32}\) After the recovery, hiring will return sooner in this case than in the scenario considered in Proposition 6, but the key insight is the fact that stagnant hiring can persist even after other economic indicators have rebounded. As we will discuss in 4.2 below, this may be extremely relevant for understanding the "jobless recoveries" that have followed the past several recessions (especially the most recent one).

4.2. Implications of Theoretical Results

Certainly I do not intend the results of this section to translate precisely to reality—an actual recession takes the form of a gradual downturn, rather than a one-time productivity shock. Of course, it can be shown that even continuous, unanticipated declines in productivity (in which \( Y \) falls proportionally to \( dt \) at each increment \( dt \)) are sufficient to stop hiring in this model. I have represented a recession in this discrete way to preserve both intuitive and analytic simplicity, but the economy’s response to shocks is quite robust to the structure of these shocks.

Another obvious departure from reality is the starkness of the result in Proposition 6—hiring does not halt completely in recessions, it merely slows (although it can slow significantly). This is not meant to be interpreted as a quantitatively precise result,\(^{33}\) but the fact that such an extreme outcome can follow a small shock to \( Y \) is important. If this mechanism can stagnate the labor market so severely, then even a small-scale version of this occurring in reality may play a significant role in labor market dynamics during and after recessions.

In a similar vein, Proposition 7 is not meant to be taken as a precise representation of actual productivity shocks in the economy, but it is incredibly compelling that a contamination of the unemployment pool can stop hiring even after the economy has otherwise recovered to pre-recession levels. Because this result is so extreme, even a tiny analog of this mechanism in the actual labor market could have a significant impact on post-recession hiring dynamics. This may hold particular insight regarding the pattern of "jobless recoveries" that have followed recent recessions.

Further, these results are mitigated by the assumption that quits continue at the same rate during

\(^{32}\)In Appendix A, I also include an alternate formulation of this result (called Proposition 7.A) in which the initial unanticipated output shock \( Y \longrightarrow Y - z \) at time \( \hat{t} \) is followed almost immediately by another unanticipated output shock \( Y - z \longrightarrow Y \) at time \( \hat{t} + dt \), where firms have already responded to the first shock before the second occurs. Of course, a setting with two unanticipated shocks that are both expected to be permanent is farther from reality than the environment of Proposition 7, but it yields a similar result with simple, clear intuition.

\(^{33}\)At the cost of analytical tractability, we could generate a decrease (rather than a halt) in hiring by introducing firm heterogeneity. This complication adds little insight (if any), so it is omitted.
and after a recession. Changing this assumption to better reflect reality (e.g. - having the intensity of quits fall after the shock) would drastically magnify the results. Given that these stark results were obtained without using this assumption, we have all the more reason to take this mechanism seriously. As recent recessions have involved both significant reductions of employment and subsequent "jobless recoveries," more empirical research should focus on understanding the role this "lemons problem" plays in employer hiring decisions.

5. Extensions

5.1. Variable Employment Equilibria and a Cyclical Example

The results of Section 4 can be viewed as characterizing a dynamic balance between hiring and firing. Both selective hiring and targeted firing can be viewed as channels through which firms exert labor market externalities on each other. Specifically, in making both of these optimal employment decisions, firms lower the quality of the unemployment pool to be faced by other firms. If the combined downward pressure of these forces is too great, it will overwhelm the upward pressure from employee quits, and the steady-state equilibrium cannot be maintained. Thus, hiring and firing must be balanced to sustain this equilibrium.

In Section 4, the discrete output shock leads to a mass of targeted firings, and these firings exert such great downward pressure on the unemployment pool quality that hiring must stop altogether for some time to allow this quality to recover. This is a stark example of the volatile dynamics that can result when these forces fail to maintain balance. There is, however, a subtler way of capturing this intuition in the model.

Instead of the volatile environment above, let us consider a setting in which targeted firings and selective hirings repeatedly rise and fall in intensity to balance the economy. In the discussion to follow, I develop this setting to convey two related points about labor markets with private information and hiring/firing externalities:

1. Independently of the state of the economy, there must be an aggregate balance in firm restructuring decisions that impact the labor market. This market can support only so much combined hiring and firing at once. For example, a sector in which many firms are firing unproductive workers may experience a lull in hiring even if the marginal product of each quality of labor has not declined. This implies that fluctuations in employment can occur even without economic volatility.

2. Further, each fluctuation can help generate future fluctuations. Hence, the balance (or imbalance) between hiring and firing in the current labor market may continue to impact the labor market far beyond the immediate future.
Toward understanding these points, let us first formalize our environment in the following definition:

**Definition 3:** A variable employment equilibrium \( \{w^*, p^*_v, q_U(v), q_E(t), E_t\}_{t=0}^\infty \) consists of a market wage rate \( w^* \), a threshold rule \( p^*_v \in [0,1] \), a proportion \( q_U(v) \in [0,1] \) of unemployed workers that are type \( H \), an evolving proportion \( q_E(t) \in [0,1] \) of employed workers that are type \( H \) at each point in time, and an evolving proportion \( E_t \in [0,1] \) of workers that are employed such that:

(i.a) \( V(p^*_v) = 0 \) (firm optimality - value matching)

(i.b) \( V(p^*_v) = 0 \) (firm optimality - smooth pasting)

(ii) \( V(q_H(q_U(v))) \leq c \) (free entry)

(iii) \( E_t q_E(t) + (1 - E_t) q_U(v) = Q \) (market-clearing - employment/employee quality)

(iv) \( q_U(v) = \psi \) (constant type \( H \) proportion among unemployed)

where \( \varphi_t \) is the measure of workers instantaneously entering unemployment, \( \psi \) is the proportion of type \( H \) workers among the unemployed at time \( t + dt \), and \( \eta_t \) is the instantaneous intensity at which the unemployed are hired at time \( t \).

Note first that the constant employment level condition has been eliminated. In such equilibria, the quality of the unemployment pool will be constant, as it must always satisfy free entry (condition (ii)). In order to maintain this constant quality, the combined negative pressure on pool quality from selectivehirings and targeted firings must precisely counter the positive pressure on pool quality from quitting workers. In equilibrium, the economy preserves this balance by adjusting the intensity of hiring to account for the disparity between the positive pressure of quitting workers and the negative pressure of fired workers. For purposes of intuition, it is worth remembering that these conditions regarding the intensity of hiring reflect optimal firm behavior—firms will continue to hire as long as it is profitable to do so.

Of course, it is crucial to note that, in this model, the intensity of targeted firings at \( t \) is determined by the intensity of selective firings at \( t - t^*_v \). Hence, the intensity of hiring necessary to preserve equilibrium at time \( t \) is a function of the current employment level \( E_t \) and the hiring intensity \( t - t^*_v \) earlier (\( \eta_{t-t^*_v} \)).

Let us formalize this by defining \( \eta_v \left( \eta_{t-t^*_v}, E_t \right) : \mathbb{R}_+ \times [0,1] \rightarrow \mathbb{R} \). To preserve the type \( H \) proportion of the unemployment pool, the hiring intensity must be that at which the net flow into/out of unemployment will have proportion \( q_U(v) \) of type \( H \) workers—this is the same type \( H \) proportion

Note that the accounting condition (iii) requires \( E_t q_U(v) + (1 - E_t) q_E(t) = Q \), and that all equilibria we are considering involve the same unemployment pool quality level \( q_U(v) \). Hence, a given employment level \( E_t \) necessarily implies a unique type \( H \) proportion among the employed \( q_E(t) \). (Of course, if the level \( E_t \) implies a \( q_E(t) \notin [0,1] \), such an \( E_t \) level cannot be consistent with equilibrium. Further, an economic state with an implied \( q_E(t) \in [0,Q) \) could not be reached by any of the forces considered in this paper.)
as the unemployment pool itself. Thus, \( \eta_v \left( \eta_{t-t^*_v}, E_t \right) \) must satisfy:

\[
q_{U(v)} = \frac{\eta_{t-t^*_v} q_H \left( q_{U(v)} \right) e^{-(\lambda+\pi)t^*_v} + E_t q_E(0) \pi dt - \eta_v \left( \eta_{t-t^*_v}, E_t \right) q_H \left( q_{U(v)} \right)}{\eta_{t-t^*_v} \left[ 1 - q_H \left( q_{U(v)} \right) \right] e^{-\pi t^*_v} + q_H \left( q_{U(v)} \right) e^{-(\lambda+\pi)t^*_v} + E_t \pi dt - \eta_v \left( \eta_{t-t^*_v}, E_t \right) q_H \left( q_{U(v)} \right)}
\]

Obviously, there cannot be a negative intensity of hiring, so if \( \eta_v \left( \eta_{t-t^*_v}, E_t \right) < 0 \), the unemployment pool quality will fall below \( q_{U(v)} \) and hiring will stop for some amount of time until the pool quality again rises to \( q_{U(v)} \). In the extreme case we decided not to consider in Section (3), in which \( \alpha \) and \( c \) are sufficiently low (i.e. screening is effective enough and hiring costs are large enough) that \( p_{ss} > q_{U(ss)} \), then \( \eta_v \left( \eta_{t-t^*_v}, E_t \right) > 0 \) and free entry always holds with equality. However, in the main cases we considered in this paper, it is possible for the economy to have a sufficiently high hiring intensity at some point in time that conditions will require hiring to stop \( t = t^*_v \) later.

5.1.1. Example: Cyclical Equilibrium

In the general description above, we required no repeated cyclical employment pattern—employment evolved simply to preserve the free entry condition. To understand the implications of labor market externalities in variable employment equilibria, let us consider a simple cyclical example.

Imagine an equilibrium with 2 intensities of hiring, \( \eta_H \) and \( \eta_L \) (assume \( \eta_H > \eta_L \)). This equilibrium will consist of repeated cycles with an expansionary period of length \( t^*_v \) during which \( \eta_t = \eta_H \), followed by a contractionary period of length \( t^*_v \) with \( \eta_t = \eta_L \). Obviously, the employment level grows during the expansionary period and shrinks during the contractionary period.\(^{35}\) To preserve the cyclicity, the total growth while \( \eta_t = \eta_H \) must equal the total contraction while \( \eta_t = \eta_L \).

To understand why such an equilibrium can preserve the unemployment pool quality level, notice that whenever \( \eta_t = \eta_H \), it must also be true that \( \eta_{t-t^*_v} = \eta_L \). Similarly, these intensities are reversed during contractions. Hence, the hiring and firing intensities during an expansionary period must support a net flow into unemployment with type \( H \) proportion \( q_{U(v)} \), and these intensities during a contractionary period must support a net flow out of unemployment with type \( H \) proportion \( q_{U(v)} \). In the equation satisfied by \( \eta_v \left( \eta_{t-t^*_v}, E_t \right) \) given above, the numerator and denominator are both positive during expansionary periods and both negative during contractionary periods.

Because this equilibrium is not the central focus of the paper, I will stop the formal analysis here. For a more precise discussion of the aggregate conditions that this equilibrium must satisfy, see Appendix B. I will leave the development of further results regarding this equilibrium and the general class of variable employment equilibria to future work.

\(^{35}\)In turn, \( q_E(0) \) falls during expansions and rises during contractions. So, in line with standard intuition, the average quality of employed workers is greatest when employment is lowest—toward the end of a contractionary period.
5.1.2. Implications

Let us consider the insights of this cyclical equilibrium regarding the previously-mentioned main points of this extension. Regarding the first point, "exogenous shocks" obviously have no role in this equilibrium. Thus, in this setting, aggregate output can rise and fall without any changes in the structure of economy,\footnote{36In this equilibrium, only internal allocations of the economy vary over time—the aggregate composition does not.} and this output variation is driven by the evolving balance between the externalities imposed by selective hiring and firing.

Market forces work to limit the combined amount of hiring and firing, but the relative magnitudes of hiring and firing are constrained only in the long run (in this case, over a full cycle). In the short run, market forces allow fluctuation in the relative changes in hiring/firing intensities, and these relative changes can generate changes in aggregate output.

This equilibrium has particularly strong implications regarding the second point. Clearly, these cycles are self-sustaining. By starting the economy at a given employment level and distribution of firm beliefs about employed worker types, we can generate an infinite sequence of aggregate fluctuations.

I do not claim that this result constitutes an explanation for business cycles. Certainly, many other forces play a role in the evolution of the aggregate economy. Further, other learning structures (such as Brownian motion with known variance and unknown mean) might dampen these cycles over time, so there is reason to doubt that present conditions should impact labor markets infinitely far into the future. Notwithstanding, the proposed mechanism for present conditions to impact future labor markets in general seems relatively robust, so future research should aim to better understand this mechanism.

5.2. Spot Markets for "Poaching" and Tenure-Dependent Wages

As was explained earlier, the main model developed in Sections 2 – 4 avoids a detailed mechanism for division of the surplus. The market forces of free entry drive the ex ante firm surplus to 0, and workers are paid only their reservation values in equilibrium. So the only surplus obtained in equilibrium is claimed by the firms who realize ex post that they are employing type $H$ workers.

Though an explicit model of surplus division is tangential to the main arguments of this paper, it is worth mentioning that the equilibrium structure used in Sections 2 – 4 can be extended to analyze this aspect of labor markets and even to guide welfare analysis. To demonstrate this, I will extend the original model to incorporate the hiring of employed workers ("poaching"), and I will show that the model remains tractable for equilibrium analysis in this case.

5.2.1. A Simple Extension with Observable Tenure at the Current Firm

Assume that the model and equilibrium structure are precisely the same as described in Sections 2 and 3, except that unmatched firms now have the additional option of hiring a currently employed worker.
at hiring cost $c_E$. In attempting to hire these workers, firms cannot observe the worker’s output at her current job—this remains the private information of her current employer. The "poaching" firm can, however, view the worker’s duration of employment at her current firm, and all firms are identical, so equilibrium hiring/firing behavior is commonly known. It is easy to see that this employment duration can contain information about employee type.

Because firms fire workers after time $t^{*}_{ss}$, it is known that only type $H$ workers will be employed for any longer duration. Before time $t^{*}_{ss}$, however, workers only leave by quitting. Since these quits are uncorrelated with worker type, the type $H$ proportion among workers employed for a duration $t \in [0, t^{*}_{ss}]$ is the same as the type $H$ proportion among screened workers hired from the unemployment pool. Thus, there is no information contained in an employment duration of length $t \in [0, t^{*}_{ss}]$, and wages for employees with tenures in this range will remain at $\tilde{w}$—the wage level offered to newly hired workers in equilibrium. For workers with a tenure length $t > t^{*}_{ss}$ at the current firm, outside offers will bid up wages until the current employer retains only $c_E$ in total value. This is because, in addition to wages, other firms must pay this cost $c_E$ to poach the worker, but the current firm needs only to pay wages to retain the worker.

The current employer of a worker with tenure length $t > t^{*}_{ss}$ has an outside option of competing with other firms to hire a new worker either from another firm or from the unemployment pool, and both of these offer value 0 in expectation (due to free entry). Because the firm obtains the positive value $c_E$ from "defending" the current employee against competitors, this defense dominates the alternative. Thus, no poaching will occur in equilibrium.

Despite this, the threat of poaching has important effects on both wages and employment in equilibrium. Wages rise weakly with tenure—from the reservation value ($w = \tilde{w}$) for $t \in [0, t^{*}_{ss}]$ to expected output minus the firing cost ($w = \lambda Y - (r + \pi) c_E$) for $t > t^{*}_{ss}$. Further, because firms no longer obtain all of the surplus ex post, the option value of employing a type $H$ worker has decreased. As a result, firms will require a higher unemployment pool quality to motivate hiring, and this will require a lower employment level in equilibrium.

In the interest of brevity, I will confine further and more formal analysis to the Appendix and to future research.

### 5.2.2. Implications

Although the basic model developed in this paper is structured to focus on the dynamics of employment (rather than wages), this need not be the case. Via competition for productive employees, the extension developed here allows renegotiation of wages during employment. This framework might offer insight regarding wage dynamics and even welfare analysis of the labor market. Crucially, it remains tractable in this form, so it is possible that other nuances (such as saving and dynamic decision-making for workers in the model) can be incorporated to this framework in future theoretical analyses of the labor

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37In fact, it is possible to solve explicitly for the firm’s value function—a derivation is provided in Appendix B.
market. Hence, we should view this model as a baseline/starting point for this type of analysis, rather than as an upper bound for theoretical complexity.

6. Conclusions

In this study, I have considered a new mechanism, in which changes in the quality of those entering unemployment can generate both a long post-recession period with limited hiring and large numbers of individuals reaching long-term unemployment. If employers have private information about worker ability, then periods in which many firms make firing decisions will involve many low-ability workers entering the unemployment pool. Of course, previous research argued that these low-ability workers should still be better than those fired at other times (because firing standards are higher during recessions). What these studies have overlooked is the other impact of increasing firing standards—a significant increase in the number of workers fired. Workers fired before the recession were balanced in the unemployment pool by workers voluntarily quitting jobs, and these quitting workers need not have disproportionately low ability. During the recession, this balance was lost, and the unemployment pool may have worsened as a result. If true, this would at least partly explain the continued hesitancy of firms to hire, and this could grease the path to long-term unemployment for these disproportionately low-quality workers who would struggle to find work regardless.

To assess the impact of this compositional change on employment, I have provided an empirical strategy for detecting the role of changing flows to unemployment in firm hiring decisions. I have implemented this using CPS monthly employment data, and I have provided evidence linking these compositional changes to hiring decisions. The empirical patterns provided cannot be explained by human capital depreciation, negative selection of LTU, or other common explanations of persistent long-term unemployment, so this constitutes evidence that the compositional change mechanism impacts hiring independently of these other factors. As such, more investigation is warranted regarding the role of these changing flows on employment dynamics.

Building on this empirical motivation, I have formalized this mechanism in a dynamic framework that integrates employer learning and private information into a labor market equilibrium. The framework itself is a tool that merits further development and application. As I have demonstrated the danger of drawing conclusions about dynamic environments from comparisons of static models, the availability of a tractable dynamic framework for analysis of the labor market is an opportunity to investigate whether other analyses of this setting have been flawed.

In my present analysis, I show that an economic shock which raises firing standards can not only generate significant unemployment, but also discourage firms from hiring for a sustained period of time. The conditions amplify each other, and this will be worsened significantly if the stagnant labor market discourages workers who want to quit their jobs from doing so. (In generating my results, I have assumed that quits will continue regardless of labor market conditions, so I may be understating the potential impact of this mechanism).
In addition to this, I show that the stagnant labor market can persist even after the economy has otherwise recovered from the shock, so this may offer insight regarding the "jobless recoveries" that have followed the past several recessions. Further, I show that the model does not even require a shock to generate employment/output fluctuations. Firms necessarily impose externalities on the rest of the labor market via the acts of hiring and firing, but hiring and firing have different aggregate effects on labor markets and on the economy as a whole. As a result, aggregate fluctuations in hiring and firing (that are needed to maintain equilibrium balance in the labor market) can generate volatility throughout the economy.

These empirical and theoretical results offer strong motivation for future work investigating the role of this mechanism in recessions and in labor markets more generally.
References


APPENDIX A: Proofs of Theoretical Results

Proposition (i): Define \( \eta_{PH(t)} \equiv \eta_{PH(t)} \frac{1}{1-E_t} \) to be the overall reemployment hazard rate of the unemployment pool at time \( t \). Define \( \eta_{PF(t)} \equiv \eta_{PF(t)} \frac{1}{1-E_t} \) and \( \eta_{PQ(t)} \equiv \eta_{PQ(t)} \frac{1}{1-E_t} \) to be the intensities of firings and quits, respectively, that are scaled relative to the size of the unemployment pool (and denote the ratio of these by \( \Omega_t \equiv \frac{\eta_{PF(t)}}{\eta_{PQ(t)}} \)). Then for a fixed scaled intensity of inflows to unemployment (\( \eta_{PF(t)} + \eta_{PQ(t)} \)), the mechanisms reflected in the stocks model imply:

\[
\frac{\partial \ln [\eta_{PH(t)}]}{\partial \ln [\Omega_t]} = \frac{\partial \ln [\eta_{PH(t)}]}{\partial \ln [\Omega_t]} \quad \text{(STOCKS)}
\]

In contrast, the flows model of compositional change implies:

\[
\frac{\partial \ln [\eta_{PH(t)}]}{\partial \ln [\Omega_t]} > \frac{\partial \ln [\eta_{PH(t)}]}{\partial \ln [\Omega_t]} \quad \text{(FLOWS)}
\]

Proof: Due to the screening technology, the reemployment hazard rate of type \( L \) workers (\( \eta_{PH(t)} (L) \)) must be \( \alpha \) times this hazard rate for type \( H \) workers (\( \eta_{PH(t)} (H) \)). We can use this to solve for the type \( H \) hazard rate:

\[
\eta_{PH(t)} = \eta_{U(t)} \eta_{PH(t)} (H) + [1 - \eta_{U(t)}] \alpha \eta_{PH(t)} (H)
\]

\[
\implies \eta_{PH(t)} (H) = \frac{\eta_{PH(t)}}{\eta_{U(t)} [1 - \alpha] + \alpha}
\]

Similarly, the type \( L \) hazard rate is

\[
\eta_{PH(t)} (L) = \frac{\alpha \eta_{PH(t)}}{\eta_{U(t)} [1 - \alpha] + \alpha}
\]

Then the hazard rates of the groups of short- and long-term unemployed can be written:

\[
\eta_{PH(t)}^S = \eta_{PH(t)} \left( \frac{\eta_{S(t)} [1 - \alpha] + \alpha}{\eta_{U(t)} [1 - \alpha] + \alpha} \right) \quad \text{and} \quad \eta_{PH(t)}^L = \eta_{PH(t)} \left( \frac{\eta_{L(t)} [1 - \alpha] + \alpha}{\eta_{U(t)} [1 - \alpha] + \alpha} \right)
\]

and so:

\[
\ln \left( \eta_{PH(t)}^j \right) = \ln \left( \eta_{PH(t)} \right) + \ln \left( \eta_{j(t)} \right) \left[ 1 - \alpha \right] + \alpha - \ln \left( \eta_{U(t)} \right) \left[ 1 - \alpha \right] + \alpha \quad \text{for } j \in \{S, L\}
\]

It is easy to see here that—holding fixed the overall reemployment hazard rate—increases in the group-specific hazard rate reflect increases in the group-specific quality and/or decreases in the overall unemployment pool quality.

In the stocks model, relative changes in the flows of firings/quits do not impact these qualities, so \( \frac{\partial \ln [\eta_{PH(t)}^S]}{\partial \ln [\Omega_t]} = \frac{\partial \ln [\eta_{PH(t)}^L]}{\partial \ln [\Omega_t]} \). In the flows model, by differentiating \( \ln \left( \eta_{PH(t)}^S \right) \) and \( \ln \left( \eta_{PH(t)}^L \right) \) with respect to \( \ln [\Omega_t] \), we can show that:

\[
\frac{\partial \ln [\eta_{PH(t)}^S]}{\partial \ln [\Omega_t]} - \frac{\partial \ln [\eta_{PH(t)}^L]}{\partial \ln [\Omega_t]} = -\frac{1 - \alpha}{m_{S(t)}} \left( \eta_{F(t)} \left( \eta_{S(t)} - \eta_{F(t)} \right) + \eta_{Q(t)} \left( \eta_{Q(t)} - \eta_{S(t)} \right) \right)
\]

\[
< 0 \quad \text{(since } \eta_{S(t)} \equiv \eta_{F(t), \eta_{Q(t)}} \text{)}
\]
Proposition 1: For reservation values $\bar{w} \in (\underline{w}(Y, \lambda, \pi, r, Q, c, \alpha), \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha))$ (where $\underline{w}(\cdot)$ and $\overline{w}(\cdot)$ are bounds determined by other parameters), there exists a unique steady-state employment equilibrium satisfying the conditions in Definition 1.

Proof: To obtain the boundaries of this range $(\underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \text{ and } \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha))$, we must (1) use the value function and cutoff obtained in Proposition 2 to characterize the $p_{ss}^*$ and $q_{U(ss)}$ implied by parameters and (2) check that these $p_{ss}^*$ and $q_{U(ss)}$ imply values for $E_{ss}$ and $q_{E(ss)}$ that are in the range $(0, 1)$ via the overall and type $H$ unemployment inflow/outflow equations and the expressions provided in Proposition 3. The algebra involved in this is extreme and does not lead to closed-form expressions for $\underline{w}(Y, \lambda, \pi, r, Q, c, \alpha)$ and $\overline{w}(Y, \lambda, \pi, r, Q, c, \alpha)$ (they are analytically determined by equations, but cannot generally be made into closed-form expressions of only the parameters $\{Y, \lambda, \pi, r, Q, c, \alpha\}$). In addition, it is worth noting that (a) the process described above offers little insight about the conditions for existence of such equilibria and (b) the equations used in this process are all provided in Section 2. Given this, instead of repeating and combining these equations toward analytically characterizing these boundaries, I will supplement my above description of this process with intuition for the impact an increase in each parameter has on $\underline{w}(Y, \lambda, \pi, r, Q, c, \alpha)$ and $\overline{w}(Y, \lambda, \pi, r, Q, c, \alpha)$:

- If $Y$ rises, firm value functions rise, and firms have lower hiring and firing thresholds. Thus, wage levels for which there would have been zero employment can now sustain employment (so $Y \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \uparrow$), and wage levels for which there would have been an unemployment pool of positive mass will now generate full employment (so $Y \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \uparrow$).

- An increase in $\lambda$ also increases firm value functions, but the increased learning speed compounds this effect. For individual increases in $\lambda$ and $Y$ that lead to the same increase in the product $\lambda Y$ (which is the relevant measure of value for firms), the increase in $\lambda$ will increase value functions more, and in turn, it will increase hiring and firing thresholds more. It is easy to verify this analytically for the firing threshold $p_{ss}^*$—we can compare the derivatives

$$\frac{\partial p_{ss}^*}{\partial Y} \text{ vs. } \frac{\partial p_{ss}^*}{\partial \lambda}$$

(which are properly scaled to compare equivalent increases in the product $\lambda Y$), and we find that the firing threshold decreases more for increases in $\lambda$ whenever $\lambda Y > w$, and this must obviously be true in any equilibria with nonzero employment.

Thus, $\lambda$ has similar effects to $Y$ ($\lambda \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \uparrow$ and $\lambda \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \uparrow$), but these effects are proportionally stronger.

- An increase in $\pi$ lowers value functions by lowering the expected payoff than can be obtained from a type $H$ worker (because workers quit more frequently). In fact, it is easy to verify that $p_{ss}^*$ is increasing in $\pi$, and hiring standards are as well. However, a higher $\pi$ also implies more
upward pressure on the unemployment pool quality (via quits), and this works in opposition to the previous force.

The net effect will be to lower the maximal wage that can sustain positive employment \((\pi \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\) and lower the minimal wage that will sustain \(E_{sa} < 1\) \((\pi \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\), but it is worth noting that these forces will not be as strong as the effect on the value function alone would suggest.

- An increase in \(r\) (decreased patience) has almost identical effects on the value function as \(\pi\). However, the oppositional force improving the unemployment pool quality is not present here. As such, the effects of \(r\) will point in the same directions as the effects of \(\pi\) \((r \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow\) and \(r \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\), but without the mitigating equilibrium forces, these net effects will be stronger.

- If \(Q\) increases, the economy can accommodate positive employment at a higher range of equilibrium unemployment pool quality levels \(q_{U(\text{ss})}\). Thus \(Q \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \uparrow\). In turn, because the aggregate labor force quality is increasing, the range of wages supporting full employment will rise, so we must restrict the lower bound further: \(Q \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \uparrow\).

- If the hiring cost \(c\) rises, hiring standards must rise, so zero employment can occur at lower wage levels than before \((c \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\), and the range of wages that are sufficiently low supporting full employment shrinks \((c \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\).

- Increases in \(\alpha\) correspond to less effective screening technology. Hence, hiring from a given unemployment pool quality will yield firms a lower quality worker, so the lowest wage level at which there will be zero employment falls \((\alpha \uparrow \implies \overline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\), and the greatest wage level at which there will be full employment falls also \((\alpha \uparrow \implies \underline{w}(Y, \lambda, \pi, r, Q, c, \alpha) \downarrow)\).

Lemma 1: In any nontrivial steady-state employment equilibrium, \(w = \bar{w}\).

Proof: Obviously, we need only consider cases in which \(E_{ss} \in (0, 1)\). Clearly, no one would work at lower wages, so \(w \geq \bar{w}\). If firms could offer a higher wage in such cases, other firms would continue hiring until the quality of the unemployment pool falls to where firms cannot offer \(w > \bar{w}\). Thus, we have the result.\(^{38}\)
Proposition 2: In a steady-state employment equilibrium, the firm’s value function can be written explicitly in terms of the threshold rule $p_{ss}^*$

$$V_{ss}(p_t) = \begin{cases} \left( \frac{1}{r+\pi} \right) \left[ \lambda p_t Y - \bar{w} + (\bar{w} - \lambda p^* Y) \left( \frac{1-p_t}{p_{ss}} \right) \left( \frac{1-p_t}{1-p_{ss}} \right) \right] \ for \ p_t \in [p_{ss}^*, 1] \\ 0 \ for \ p_t \leq p_{ss}^* \end{cases}$$

Further, this threshold $p_{ss}^*$ is given by

$$p_{ss}^* = \frac{\bar{w} (r + \pi)}{\lambda [Y (r + \lambda + \pi) - \bar{w}]}$$

Proof: Value matching, smooth pasting, and free entry are used to solve for the constant $K$ (in equation (2)) in terms of $p_{ss}^*$. (Note that we could also solve for $K$ in terms of $q_{U(ss)}$ using these conditions). The above steady-state value function follows from a combination of these conditions.

To obtain $p_{ss}^*$, the three aforementioned conditions are used with equation (1) (the ODE).

Proposition 3: In a steady-state employment equilibrium:

(i) Without receiving a payoff $Y$, a firm will wait for time $t_{ss}^*$ after hiring a worker before firing him, where

$$t_{ss}^* = \frac{1}{\lambda} \ln \left( \frac{1-p_{ss}^*}{p_{ss}^*} \right) \left( \frac{q_{U(ss)}}{\alpha (1-q_{U(ss)})} \right)$$

(ii) Unemployed workers are hired at intensity:

$$\eta_{ss} = \pi E_{ss} dt \left[ \frac{q_{U(ss)} + \alpha (1-q_{U(ss)})}{q_{U(ss)} [1-e^{-\lambda \pi t_{ss}^*}] + \alpha (1-q_{U(ss)}) [1-e^{-\pi t_{ss}^*}]} \right]$$

(iii) The employment level can be written:

$$E_{ss} = \left[ \frac{Q - q_{U(ss)}}{1-q_{U(ss)}} \right] \left[ 1 + \left( \frac{\alpha}{q_{U(ss)}} \right) \left( \frac{1-e^{-\alpha \pi t_{ss}^*}}{1-e^{-\pi \pi t_{ss}^*} - \alpha [1-e^{-\pi t_{ss}^*}]} \right) \right]$$

Proof (i): If a firm receives no payoff at time $t$, the updating rules imply that $\frac{\partial (\ln (p_t))}{\partial t} = -\lambda (1-p_t)$ and $\frac{\partial (\ln (1-p_t))}{\partial t} = \lambda p_t$. Thus we can relate these two derivatives by

$$\frac{\partial (\ln (1-p_t))}{\partial t} = \frac{\partial (\ln (p_t))}{\partial t} + \lambda$$

Integrating both sides from 0 to $t_{ss}^*$ yields

$$\ln \left( \frac{p_{t_{ss}^*}}{p_0} \right) + \lambda t_{ss}^* = \ln \left( \frac{1-p_{t_{ss}^*}}{1-p_0} \right) - \ln (1-p_0)$$

where $p_0$ is the firm’s initial belief about its employee’s type. Note that (a) $p_{t_{ss}^*} = p_{ss}^*$, (b) $p_0$ must equal $q_H (q_{U(ss)})$ in equilibrium, and (c) $q_H (q_{U(ss)}) = \frac{q_{U(ss)}}{\alpha (1-q_{U(ss)})}$, and the result follows. ■
**Proof (ii):** In equilibrium, the instantaneous flow into employment among the unemployed \( (\eta_{ss}) \) must equal the instantaneous flow out of employment among the employed. This flow out of employment at time \( t \) consists of both the measure of workers who quit jobs \( (E_{ss} \pi dt) \) and the measure of workers hired at time \( t - t_{ss}^* \) whose employers received no payoff during that time:

\[
\eta_{ss} \left[ 1 - q_H \left( q_U(\eta_{ss}) \right) \right] e^{-\pi t_{ss}^*} + q_H \left( q_U(\eta_{ss}) \right) e^{-(\lambda+\pi)t_{ss}^*}
\]

Equating these inflows and outflows yields

\[
\eta_{ss} = \frac{\pi E_{ss} dt}{1 - e^{-\pi t_{ss}^*} \left( 1 - q_H \left( q_U(\eta_{ss}) \right) \right) - q_H \left( q_U(\eta_{ss}) \right) e^{-(\lambda+\pi)t_{ss}^*}}
\]

into which we can substitute \( q_H \left( q_U(\eta_{ss}) \right) \equiv \frac{q_U(\eta_{ss})}{\alpha(1-q_U(\eta_{ss}))} \) to obtain our desired result. ■

**Proof (iii):** In addition to the expression for \( \eta_{ss} \) obtained above (by equating inflows to and outflows from employment), we can also obtain \( \eta_{ss} \) by equating type \( H \) inflows to and type \( H \) outflows from employment (this must also hold in a steady-state employment equilibrium). From this, we obtain

\[
\eta_{ss} = \frac{E_{ss} q_E(\eta_{ss}) \pi dt}{q_H \left( q_U(\eta_{ss}) \right) \left[ 1 - e^{-(\lambda+\pi)t_{ss}^*} \right]} \text{ where } q_E(\eta_{ss}) = \frac{Q - (1 - E_{ss}) q_U(\eta_{ss})}{E_{ss}}
\]

Equating the two expressions for \( \eta_{ss} \) and solving for \( E_{ss} \) yields the result. ■

**Proposition 5:** \( \exists \tau > 0 \) such that for \( z \in (0, \tau) \), the proportion of type \( H \) workers in the unemployment pool immediately following the shock \( (Y \rightarrow Y - z) \) falls to

\[
q_U(t) = \frac{(1 - E_{ss}) q_U(\eta_{ss}) + \frac{\eta_{ss}}{dt} \int_{t_{ss}}^{t_{ss}^*} q_H \left( q_U(\eta_{ss}) \right) e^{-(\lambda+\pi)s} ds}{1 - E_{ss} + \frac{\eta_{ss}}{dt} \int_{t_{ss}}^{t_{ss}^*} [q_H \left( q_U(\eta_{ss}) \right) e^{-(\lambda+\pi)s} + \left( 1 - q_H \left( q_U(\eta_{ss}) \right) \right) e^{-\pi s}] ds} < q_U(\eta_{ss})
\]

**Proof:** This expression is the product of the same intuition given regarding the expression in Proposition 4 (the numerator consists of only the type \( H \) workers from this mass).

To show that the inequality holds for sufficiently small \( z \), we can substitute for \( E_{ss}, \eta_{ss}, t_{ss}^*, t_{ss}, p_{ss}^*, p_t^*, \) and \( q_H \left( q_U(\eta_{ss}) \right) \), and we can evaluate the expression. It can then be seen (just as it can from general intuition), that the inequality holds provided that \( p_{ss}^* < q_U(\eta_{ss}) \). We have, of course, restricted our attention only to equilibria satisfying this condition, so we are done. ■

**Lemma 2:** Define \( q_U^*(Y) \) to be the unemployment pool quality such that firms are indifferent between hiring and not \( V(q_H(q_U^*)) = c \) when the output level is \( Y \). Then \( q_U^*(Y) \) is strictly decreasing in \( Y \).

**Proof:** Clearly \( p_{ss}^* = \frac{\widehat{\omega}(\tau+\pi)}{\lambda Y(\tau+\lambda+\pi)-\omega} \) is strictly decreasing in \( Y \). Note that, for a given \( p_t \in [p_{ss}^*, 1] \), \( V(p_t) \) is strictly decreasing in the threshold \( p_{ss}^* \). In turn, the \( q_U^*(Y) \) satisfying \( V(q_H(q_U^*(Y))) = c \) is strictly increasing in \( p_{ss}^* \) (and \( q_H(q) \) is of course increasing in \( q \)). Thus, \( q_U^*(Y) \) must be strictly decreasing in \( Y \). ■
Proposition 6: After the output shock $Y \rightarrow Y - z$, hiring will cease for the duration $\hat{t}_H$. If $q_U^*(Y - z) < Q$, $\hat{t}_H$ is finite and satisfies

$$q_U^*(Y - z) = f_1 \left( \hat{t}_H \bigg| t_{ss}, t_z \right)$$

$$= (1 - E_{ss}) q_{U(ss)} + \eta_{ss} \int_{t_z}^{t_{ss}} e^{-(\pi + \lambda) s} q_H \left( q_{U(ss)} \right) ds + \eta_{ss} e^{-(\pi + \lambda) t_z} q_H \left( q_{U(ss)} \right) \hat{t}_H + \eta_{ss} \int_{0}^{\hat{t}_H} \pi e^{-\pi s} \int_{s}^{t_z} e^{-(\pi + \lambda) x} q_H \left( q_{U(ss)} \right) dx ds + \left[ 1 - e^{-\pi t_H} \right] m_{H(ss)}$$

$$+ \eta_{ss} q_H \left( q_{U(ss)} \right) \int_{0}^{\hat{t}_H} \left[ e^{-\pi s} - e^{-\pi t_H} \right] \lambda e^{-\lambda s} \int_{s}^{t_z} e^{-(\pi + \lambda) x} dx ds$$

where $m_{H(ss)} = \left( \frac{\eta_{ss}}{\pi} \right) q_H \left( q_{U(ss)} \right) \int_{0}^{t_{ss}} e^{-(\pi + \lambda) s} ds$ is the steady-state mass of employed workers who have already revealed themselves to be type $H$. If $q_U^*(Y - z) \geq Q$, $\hat{t}_H = \infty$.

Proof: As mentioned following Proposition 6 in the text, there can be no positive selection into unemployment, so $q_{U(i)} < Q, \forall t$. Clearly, then, the random inflows to unemployment from job quitters can—at most—bring the unemployment pool quality asymptotically toward $Q$. It can never reach any quality level $q_{U(i)} > Q$, and it can never reach quality level $q_{U(i)} = Q$ in finite time.

To see that this pool will reach any $q_{U(i)} < Q$ in finite time, first recall that, after the shock, targeted firings continue to occur at the new belief threshold $p^*_t$ for those workers who were hired in the previous steady state (before the shock). Suppose that we have reached time $\hat{t} + t_z$ (where $t_z$ again represents the time after hiring after the shock when workers who have not revealed themselves to be type $H$ will be fired) and that hiring has not yet begun. (Obviously, if hiring begins before this point, we have already reached $q_{U(i)} = q_U^*(Y - z)$, so we are done). Then the only remaining employed workers must have provided their employers a payoff, and these must be type $H$ workers. Then of course, the type $H$ proportion among the inflow to unemployment (which comes entirely through voluntary quits) must be 1. Over time, this flow to unemployment will raise the unemployment pool quality asymptotically toward $Q$, and by the structure of the Poisson distribution, it must surpass any $q < Q$ in finite time.

Regarding the expression pinning down $\hat{t}_H$—note that this is simply a mathematical translation of the aggregate type $H$ proportion among the six groups listed following Proposition 6 in the text. For clarity, the components are classified specifically according to these groups below:
Proposition 7: Consider an unanticipated transitory output shock $Y \rightarrow Y - z$ to the steady-state at time $t$, immediately after which it is known that output will rebound $Y - z \rightarrow Y$ at Poisson-distributed times with parameter $\gamma$. Define $p^*(Y - z, Y)$ to be the termination belief level after the shock but before the recovery, and define $t^*_{Y - z, Y}$ to be the associated time firms will wait without output before terminating a worker. Further, define $q_t^*(Y - z, Y)$ to be the corresponding hiring threshold during this "recession," and again define $q_t^*(Y)$ to be the hiring threshold after the recovery. After firms respond optimally to this shock at $t$, if the recovery occurs before time $\hat{t} + t_{Y - z, Y}$ (where $t_{Y - z, Y}$ satisfies $q_t^*(Y) = f_2(\hat{t} + t_{Y - z, Y})$, and an expression for $f_2(\hat{t} + t_{Y - z, Y})$ is provided in the appendix), then the economy will remain without hiring for a positive duration of time even after the recovery. Further, $\exists z > 0$ such that for $z \in (0, \overline{z})$, the range of recovery times $t \in [\hat{t}, \hat{t} + t_{Y - z, Y}]$ that will not immediately generate hiring is increasing in the magnitude of the shock $z$ (meaning $t_{Y - z, Y}$ is increasing in $z$ over this range).

Proof: $f_2(\hat{t} + t_{Y - z, Y}, t^*_{Y - z, Y})$ must express the proportion of type $H$ workers in the unemployment pool at time $\hat{t} + t_{Y - z, Y}$ (after the initial shock but before the recovery). The intuition for constructing
this expression will involve the same six groups used to construct \( f_1 \left( \hat{t}_{Y-z, Y} | t^*_s, t^*_Y \right) \) in Proposition 6. In fact, given the same arguments \( t_{Y-z, Y}, t^*_s, t^*_Y \), and \( t^*_Y \), the function \( f_2 \left( \hat{t}_{Y-z, Y} | t^*_s, t^*_Y \right) \) is identical to \( f_1 \left( \hat{t}_{Y-z, Y} | t^*_s, t^*_Y \right) \). To understand this, notice that a recovery occurring before \( t + t_{Y-z, Y} \) will decrease both the firing threshold \( p^* \) and the hiring threshold \( q^* \) (from \( q^*_H(Y - z) \) to \( q^*_H(Y) \)). The drop in \( p^* \) will result in delayed firings, but no immediate firm response. In turn, if hiring would not begin at the new, lower \( q^* = q^*_H(Y) \), this drop in \( q^* \) would cause no immediate firm response either. Thus, for a recovery at a time \( t \in \left[t, t + t_{Y-z, Y}\right) \), the unemployment pool and its quality will not change discretely in response to the recovery. Given this, we know that \( t_{Y-z, Y} \) must satisfy:

\[
q^*_H(Y) = f_2 \left( \hat{t}_{Y-z, Y} | t^*_s, t^*_Y \right)
\]

\[
(1 - E_{ss}) q_{U(ss)} + \eta_{ss} + \int_{t_{Y-z, Y}}^{t^*_s} e^{-(\pi + \lambda)s} q_H \left(q_{U(ss)}\right) ds + \eta_{ss} e^{-(\pi + \lambda)\hat{t}_{Y-z, Y}} q_H \left(q_{U(ss)}\right) \hat{t}_{Y-z, Y}
\]

\[
+ \eta_{ss} \int_0^{t_{Y-z, Y}} \pi e^{-\pi s} e^{-(\pi + \lambda)s} q_H \left(q_{U(ss)}\right) dx ds + \left[1 - e^{-\pi \hat{t}_{Y-z, Y}}\right] m_{H(ss)}
\]

\[
+ \eta_{ss} q_H \left(q_{U(ss)}\right) \int_0^{t_{Y-z, Y}} [e^{-\pi s} - e^{-\pi \hat{t}_{Y-z, Y}}] \lambda e^{-\lambda (t_{Y-z, Y} - s)} dx ds
\]

\[
\equiv \left(1 - E_{ss}\right) + \eta_{ss} \int_{t_{Y-z, Y}}^{t^*_s} e^{-(\pi + \lambda)s} q_H \left(q_{U(ss)}\right) ds + \eta_{ss} e^{-(\pi + \lambda)\hat{t}_{Y-z, Y}} q_H \left(q_{U(ss)}\right) \hat{t}_{Y-z, Y}
\]

\[
+ \eta_{ss} \int_0^{t_{Y-z, Y}} \pi e^{-\pi s} e^{-(\pi + \lambda)s} q_H \left(q_{U(ss)}\right) dx ds + \left[1 - e^{-\pi \hat{t}_{Y-z, Y}}\right] m_{H(ss)} + \eta_{ss} q_H \left(q_{U(ss)}\right) \int_0^{t_{Y-z, Y}} [e^{-\pi s} - e^{-\pi \hat{t}_{Y-z, Y}}] \lambda e^{-\lambda s} dx ds
\]

where \( m_{H(ss)} = \left(\frac{1}{\pi}\right) \left(\frac{\eta_{ss}}{\pi}\right) q_H \left(q_{U(ss)}\right) \int_0^{t_{Y-z, Y}} e^{-(\pi + \lambda)s} ds \) is the steady-state mass of employed workers who have already revealed themselves to be type H. If this \( f_2 \left( \hat{t}_{Y-z, Y} | t^*_s, t^*_Y \right) \) has not risen above \( q^*_H(Y) \) by the time the recovery occurs, this recovery will have no discrete effect on the unemployment pool quality. Thus, in this case, there will be a period without hiring even after the recovery.

To see that \( t_{Y-z, Y} \) is increasing in \( z \) for a range of \( z \), notice first that the post-shock unemployment pool quality \( q_{U(H)} \) is decreasing in \( z \) for \( z \) sufficiently small (since we consider only equilibria for which \( p^*_s < q_{U(ss)} \)). Increases in \( z \) on this range therefore lead to lower \( q_{U(H)} \), but we must consider also that the post-shock flow to unemployment will come from a combination of random job-quitters and workers fired at the higher threshold \( p^* (Y - z, Y) \). Because of this, we might worry that a faster rate of recovery might overcome the lower starting quality. Of course, this higher threshold will also yield a more intense flow of directed firings into the unemployment pool after the shock, and this will mitigate improvement in the rate of recovery caused by the higher threshold. In line with this reasoning, it turns out that the post-shock time required to reach the unemployment quality threshold \( q^*_H(Y) \) is increasing in \( z \) for small \( z \), which is our desired result.
This can be verified formally by explicitly evaluating the impact of $z$ on the expression for $f_2 \left( t_{Y-z,Y}^{*}, t_{ss}, t_{Y-z}^{*} \right)$, but demonstration of this involves pages of algebra with little (if any) intuitive insight. I have therefore given the previous intuition in place of this algebra, and I hope the insight provided was nontrivial. ■

In addition to the above result, I will provide an alternate formulation of this in which the initial unanticipated output shock $Y \rightarrow Y - z$ at time $\hat{t}$ is followed almost immediately by another unanticipated output shock $Y - z \rightarrow Y$ at time $\hat{t} + dt$. (Firms have already responded to the first shock before the second occurs.) Of course, a setting with two unanticipated shocks that are both expected to be permanent is farther from reality than the environment of Proposition 7, but the result is similar, and the simplicity of this setting allows us to connect the result to clear intuition.

**Proposition 7.A:** After an unanticipated output shock $Y \rightarrow Y - z$ at time $\hat{t}$, firm responses to this output shock, and an unanticipated perfect reversal of this output shock $Y - z \rightarrow Y$ at time $\hat{t} + dt$, hiring will cease for duration $\tH > 0$.39

**Proof:** First, we can bound $\tH$ above by $t_{ss}^{*} - t_{z}$. To see this, note that the mass of firings in response to the shock (in the belief range $[p_{ss}^{*}, p_{I}^{*}]$) will be of better quality than the directed firings that would have occurred at belief $p_{ss}^{*}$ without the shock. This is because some of those fired in response to the shock would have revealed themselves to be type $H$ workers during the subsequent period of length $t_{ss}^{*} - t_{z}$, but these type $H$ workers are instead included in the mass firings after the shock. Additionally, until hiring begins, this downward pressure on the unemployment pool quality (which would have been present without the shock) will be absent. These two facts imply that, after the elapsed time $t_{ss}^{*} - t_{z}$, without hiring beginning, the unemployment pool quality without hiring beginning must be strictly higher after the two-shock even than it would have been after no shock at all. The unemployment pool quality threshold for hiring ($q_U^{*}(Y)$) is identical in both cases (since $Y$ returns to the same level after the second shock), and the unemployment pool quality without any shock will remain at precisely $q_U^{*}(Y)$. Thus, after the elapsed time period $t_{ss}^{*} - t_{z}$ following the two-shock sequence, the unemployment pool quality would be greater than $q_U^{*}(Y)$ without hiring beginning, so hiring must begin again before this time $t_{ss}^{*} - t_{z}$ has elapsed.

Given this bound, we can provide an analogous expression to that from Proposition 6 for the time at which hiring will be renewed (note that group $\text{(iii)}$ is no longer included, since the threshold returns to $p_{ss}^{*}$, and since hiring must begin again before any workers who aren’t fired in the initial response to the shock reach this threshold):

$$q_U^{*}(Y) = f_2 \left( \widehat{tH} \mid t_{ss}^{*}, t_{z} \right)$$

Note that $\widehat{tH}$ satisfies the condition $q_U^{*}(Y) = f_2 \left( \widehat{tH} \mid t_{ss}^{*}, t_{z} \right)$, where $f_2 \left( \widehat{tH} \mid t_{ss}^{*}, t_{z} \right)$ is analogous to the expression $f_1 \left( \widehat{tH} \mid t_{ss}^{*}, t_{z} \right)$ in Proposition 6, but with terms adjusted to account for the firing and hiring thresholds immediately returning to their previous levels.
APPENDIX B: Extensions

Mechanics of Cyclical Employment Equilibrium Considered in 5.1.1

Here I detail more precisely the mechanics of the cyclical equilibrium example provided in Section 5.1. Toward this end, we will consider the aggregate conditions that must be satisfied in the economy. (In doing so, we will take optimal firm behavior and the corresponding thresholds \(p_v^*\) and \(q_{U(v)} = q_L^* (Y)\) as given). For ease of notation, define:

- the net change in the employment level from \(t\) to \(t + dt\):
  \[
  \Delta_E (t) = E_{t+dt} - E_t
  \]

- the mass of type \(H\) workers employed at time \(t\):
  \[
  E_H(t) = q_{E(t)} E_t
  \]

- the net change in the mass of type \(H\) workers employed from \(t\) to \(t + dt\):
  \[
  \Delta_{E(H)}(t) = q_{E(t+dt)} E_{t+dt} - q_{E(t)} E_t = E_{H(t+dt)} - E_{H(t)}
  \]

Further, index times in the cycle by \(t \in [0, 2t_v^*]\), where \(t \in [0, t_v^*]\) correspond to the expansionary part of the cycle, and \(t \in [t_v^*, 2t_v^*]\) correspond to the contractionary part. With this notation established, this cyclical employment equilibrium must satisfy the following:

The labor force must always be of unit mass, and it must have the proportion \(Q\) of type \(H\) workers, so the employment level and type \(H\) proportion of those employed must reflect this at all times:

\[
E_{H(t)} + (1 - E_t) q_{U(v)} = Q \quad \text{for}\ t \in [0, 2t_v^*]
\]

The employment level and the mass of type \(H\) workers employed must evolve according to the net flows into each. This net flow should consist of hirings minus directed firings and quits. Thus

\[
\Delta_E (t) = -E_t \pi dt + \eta_H - \eta_L \left[ (1 - q_H (q_{U(v)})) e^{-\pi t^*_v} + q_H (q_{U(v)}) e^{-(\lambda + \pi)t^*_v} \right] \quad \text{for} \ t \in [0, t_v^*]
\]

\[
\Delta_{E(H)}(t) = -\pi E_{H(t)} dt + q_H (q_{U(v)}) \left[ \eta_H - \eta_L e^{-(\lambda + \pi)t^*_v} \right] \quad \text{for} \ t \in [0, t_v^*]
\]

\[
\Delta_E (t) = -E_t \pi dt + \eta_L - \eta_H \left[ (1 - q_H (q_{U(v)})) e^{-\pi t^*_v} + q_H (q_{U(v)}) e^{-(\lambda + \pi)t^*_v} \right] \quad \text{for} \ t \in [t_v^*, 2t_v^*]
\]

\[
\Delta_{E(H)}(t) = -\pi E_{H(t)} dt + q_H (q_{U(v)}) \left[ \eta_L - \eta_H e^{-(\lambda + \pi)t^*_v} \right] \quad \text{for} \ t \in [t_v^*, 2t_v^*]
\]

Further, as explained above, the net flow into employment must always have proportion \(q_{U(v)}\) of type \(H\) workers (regardless of whether this net flow is positive or negative):

\[
q_{U(v)} = \frac{\Delta_E (t)}{\Delta_{E(H)}(t)} \quad \text{for} \ t \in [0, 2t_v^*]
\]
Finally, in order for the equilibrium to be truly cyclical, the net inflows to employment during the expansionary period must be exactly reversed by the net outflows from employment during the contractionary period:

\[ \int_0^{r^*_v} \Delta_E(s) \, ds + \int_{r^*_v}^{2r^*_v} \Delta_E(s) \, ds = 0 \]

Thus, for this "expansion/contraction" equilibrium to exist, the expansion/contraction periods must each last as long as any individual firm would wait without output before firing a worker, and the total growth of employment during this expansion must erode completely during the following contraction. Further, this must all occur with net flows to/from employment always having the same proportion of type \( H \) workers as the unemployment pool itself \( (q_{U(v)}) \).
Value Function from Extension with Observable Tenure and "Poaching" in 5.2.1

We will follow an approach similar to that used to solve for the value function in the main model from Sections 1 and 2. The tenure-dependence of wages will complicate things slightly, but we can deal with this by incorporating time into our value function. We will thus write the function for which we hope to solve as \( V(p_t, t) \).

While \( V(p_t, t) \) will depend on both beliefs \( p_t \) and tenure \( t \), these are not actually two separate state variables. In equilibrium, each belief \( p_t \) (aside from \( 1 \), which is a slightly different case) can be held at a unique employment tenure \( t \). We can thus write \( p_t \) as a function of \( t \), with \( p_0 = q_H (qV(\bar{s}_t)) \) and \( p_t \) evolving according to \( p_{t+dt} = p_t - \lambda p_t (1 - p_t) dt \).

Next, note that the firm can take advantage of the output produced by a type \( H \) worker only before tenure has reached \( t^*_{ss} \), so we must obtain a firm value \( V(1, t) \) associated with each \( t \in [0, t^*_{ss}] \). Intuitively, this will correspond to the output the firm expects to obtain minus the reservation wage for the next \( t^*_{ss} \) units of time, and total value \( c_E \) thereafter (when when \( t > t^*_{ss} \)). Thus, to calculate \( V(1, t) \), we must consider the expected present value of the surplus for the next \( t^*_{ss} \), and the discounted value of \( c_E \) forever beginning \( t^*_{ss} \) into the future. This yields:

\[
V(1, t) = \frac{1 - e^{-(r+\pi)(t^*_{ss}-t)}}{r + \pi} (\lambda Y - \bar{w}) + e^{-(r+\pi)(t^*_{ss}-t)} c_E
\]

Further, by applying an approach similar to that in Sections 1 and 2, we can obtain a differential equation in terms of \( V(1, t) \):

\[
(r + \pi + \lambda p_t) V(p_t, t) = \lambda p_t [Y + V(1, t)] - \bar{w} - \lambda p_t (1 - p_t) V_1(p_t, t) + V_2(p_t, t)
\]

We know that there will be a cutoff threshold \( p_{ss}^* \), so for any \( p_t < p_{ss}^* \), \( V(p_t, t) = 0 \) (although this can happen only for \( t > t^*_{ss} \)). Note that smooth pasting applies both to beliefs and to time. The argument for beliefs is standard, but to see why it applies to time, notice that movement in the firm’s value function before \( t^*_{ss} \) must converge to \( c_E \) as \( t \rightarrow t^*_{ss} \). This value function will decline as the window narrows for the firm to extract surplus from the worker’s output. The benefits of this window rely on Poisson realizations, but as \( t \rightarrow t^*_{ss} \), the length of this window shrinks until it is of order \( dt \). When this length is of order \( dt \), the realizations of output (which were of order \( dt \) at each instant) shrink to order \( dt^2 \), and so this time derivative must vanish—thus smooth-pasting applies to \( t \) as well. (Of course, because \( p_t \) and \( t \) are not truly separate states, this argument shouldn’t be that surprising, but it helps us understand the value function we hope to obtain).

Using these cutoff/smooth-pasting conditions, we find the threshold \( p_{ss}^* = \frac{\bar{w}}{\lambda [Y + V(1, t^*_{ss})]} \). Clearly \( V(1, t^*_{ss}) \) must equal \( c_E \), so this threshold becomes

\[
p_{ss}^* = \frac{\bar{w}}{\lambda [Y + c_E]}
\]
Returning to the differential equation above, we know that a particular solution is the value of committing to the current employee forever, which can be written
\[
\frac{pt \left(1 - e^{-(r+\pi)(t^*-t)}\right) \lambda Y - \left[1 - pt e^{-(r+\pi)(t^*-t)}\right] \bar{w}}{r + \pi} + pt e^{-(r+\pi)(t^*-t)} c_E
\]
And so we arrive at a solution of the form:
\[
V_{ss}(p_t, t) = \begin{cases} 
\frac{pt \left(1 - e^{-(r+\pi)(t^*-t)}\right) \lambda Y - \left[1 - pt e^{-(r+\pi)(t^*-t)}\right] \bar{w}}{r + \pi} + pt e^{-(r+\pi)(t^*-t)} c_E \quad & \text{for } p_t \in [p_{ss}^*, 1] \text{ and } t \in [0, t_{ss}^*] \\
c_E \quad & \text{for } p_t = 1 \text{ and } t \geq t_{ss}^* \\
0 \quad & \text{for } p_t \leq p_{ss}^* 
\end{cases}
\]
where \( K = \left( \frac{p_{ss}^*}{1 - p_{ss}^*} \right) \frac{r + \pi}{\bar{w}} \left[ \frac{\bar{w}}{r + \pi} - \left( \frac{p_{ss}^*}{1 - p_{ss}^*} \right) c_E \right] \)
and \( p_{ss}^* = \frac{\bar{w}}{\lambda [Y + c_E]} \)
### Table 2: Responses of group reemployment probabilities to compositional changes, including flows during recession (Source: CPS)

<table>
<thead>
<tr>
<th></th>
<th>Column (I)</th>
<th>Column (II)</th>
<th>Column (III)</th>
<th>Column (IV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial \ln \left[ \eta_{PH(t)}^g \right]}{\partial \ln [\Omega_e]} )</td>
<td>-0.153</td>
<td>-0.412</td>
<td>-0.153</td>
<td>-0.412</td>
</tr>
<tr>
<td></td>
<td>(0.148)</td>
<td>(0.405)</td>
<td>(0.080)</td>
<td>(0.205)</td>
</tr>
<tr>
<td>( \frac{\partial \ln \left[ \eta_{PR(t)}^L \right]}{\partial \ln [\Omega_e]} )</td>
<td>0.303</td>
<td>0.445</td>
<td>0.303</td>
<td>0.445</td>
</tr>
<tr>
<td></td>
<td>(0.281)</td>
<td>(0.523)</td>
<td>(0.152)</td>
<td>(0.258)</td>
</tr>
</tbody>
</table>

- **Standard errors clustered at year-month level?**
  - (I): N
  - (II): N
  - (III): Y
  - (IV): Y

- **Control for \( \partial \ln \left[ \eta_{PR(t)}^L \right] \)?
  - (I): Y
  - (II): Y
  - (III): Y
  - (IV): Y

- **Use sampling weights?**
  - (I): Y
  - (II): Y
  - (III): Y
  - (IV): Y

- **Condition on individual observables?**
  - (I): Y
  - (II): Y
  - (III): Y
  - (IV): Y

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>87,518</td>
<td>33,459</td>
<td>87,518</td>
<td>33,459</td>
</tr>
</tbody>
</table>

CPS data on unemployment flows are used to obtain estimates.
Sample used is restricted to unemployed men with no education beyond a HS diploma.

Table 2: Responses of group reemployment probabilities to compositional changes, including flows during recession (Source: CPS)
Figure 8: ST unemployed advantage in reemployment likelihood through the recession (Sources: CPS, JOLTS)