An Active-Contracting Perspective on Equilibrium Selection in Relational Contracts

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Abstract

Using a simple principal-agent model as an example, we highlight conceptual problems with the standard approach to equilibrium selection in relational contracting games. For instance, typical models give the principal the right to make a take-it-or-leave-it offer in each period, yet in the equilibria typically selected the principal does not have bargaining power to exert. More broadly, we show that this and other modeling features that seem to impose substantive structure actually do not affect the set of equilibrium payoffs in the model, absent an equilibrium selection by the analyst. We explain how contractual equilibrium, introduced by Miller and Watson (2013), solves these conceptual issues by axiomatizing the selection of equilibria in which the parties actively negotiate over how to play. We show how to compute a contractual equilibrium for the principal and agent. In contractual equilibrium, the agent’s equilibrium effort is increasing in her own bargaining power, and the way the parties play under disagreement depends on the history of the agent’s actions.

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1 Introduction

A relational contract is a plan for how the parties in a productive relationship should behave, such that all parties agree to the plan and none has an incentive to deviate from it, even though not every aspect of the plan can be enforced by an external legal authority. In the relational contracts literature, productive relationships are modeled as repeated games, where the requirement that no party has an incentive to deviate implies that the plan must form a subgame perfect equilibrium (or a refinement thereof). However, repeated games tend to have many equilibria, naturally raising the question of which equilibrium the parties should be expected to select. This question is what distinguishes the relational contracting literature from the larger repeated games literature: while the repeated games literature is largely concerned with characterizing the set of equilibria, the relational contracting literature focuses on equilibria that jointly benefit the contracting parties.

In this paper we address conceptual problems with the standard approach to modeling and equilibrium selection in the relational-contracting literature. Most importantly, the standard approach provides no way to represent how the parties meaningfully discuss what relational contract they should agree on, and there is no role for parties to exert bargaining power when negotiating over a contract. Moreover, common modeling elements, such as ultimatum offers, externally enforced wages, and automatic separation on disagreement, are effectively irrelevant, exerting influence over neither the set of equilibrium payoffs nor the selection from that set.

In 2013 we published “A Theory of Disagreement in Repeated Games with Bargaining” (Miller and Watson 2013), proposing the solution concept of contractual equilibrium for relational contracting. Contractual equilibrium is an axiomatic approach to equilibrium selection based on the idea that parties in a relationship can bargain over how to play, and in bargaining they can engage in meaningful discussion and exercise bargaining power. They can bargain not only at the start of the relationship, but also repeatedly as the relationship unfolds. In this article we explain how the contractual equilibrium approach can simplify and regularize the study of relational contracts, bringing new insights and stronger interpretation. We argue that contractual equilibrium can expand the scope of relational contracting analysis. Also, on the technical side, we describe how our theory extends to settings with outside options.

We illustrate our arguments starting with a widely known model of a principal and an agent, a perfect-monitoring special case of Levin (2003). This model features bargaining between parties in the form of a take-it-or-leave-it offer of a wage and bonus, which would seem to allocate all bargaining power to the party that makes the offer. However, reallocat-
ing the power to make the offer or even eliminating the bargaining phase does not change
the set of subgame perfect equilibrium payoffs. Levin’s model also features both externally-
enforced and voluntary transfers at the end of the period, but we show that the equilibrium
payoff set is unchanged by condensing all transfers into a voluntary phase immediately after
the bargaining phase. Finally, Levin’s model assumes that if the agent rejects the principal’s
offer then mechanically they must get their outside options for the period, but we show that
the equilibrium payoff set is unchanged if the outside options can be triggered voluntarily by
either party. These observations imply that giving one party (e.g., the principal) the power
to make offers, restricting the parties to take their outside options under disagreement, and
making the wage externally enforceable play no substantive roles in the analysis.

With regard to equilibrium selection, Levin (2003) and most of the subsequent literature
have largely adopted an intuitive approach, selecting equilibria on the Pareto frontier of
the equilibrium payoff set, because “it is natural to focus on contracts that maximize the
parties’ joint surplus.” In our view, this intuitive approach takes the selection decision out
of the hands of the contracting parties, leaving no role for meaningful discussion or exercise
of bargaining power. The bargaining phase in particular serves no real purpose; it merely
gives the players an opportunity to state their intention to play the equilibrium that the
analyst has already selected. Moreover, this opportunity does not imply that they must state
their intentions truthfully: because the proposer’s offer is cheap talk, the parties can simply
act according to the selected equilibrium regardless of what is said during bargaining.

To model how the parties jointly decide on their relational contract—that is, to model
active contracting—an axiomatic approach is most appropriate. In our view, an axiom
that restricts play to the Pareto frontier directly is tantamount to assuming the conclusion.
Instead, axioms should generate endogenously meaningful bargaining, connecting what the
parties choose to say in the bargaining phase to how they behave in the continuation game. If
the parties reach an agreement that is credible, they should follow through on it. Moreover,
in Miller and Watson (2013) and Watson (2013) we show that substantive refinement also
requires a “theory of disagreement” that constrains behavior after a responder rejects a
proposer’s offer. In Levin (2003), the theory of disagreement is that the parties receive
their outside options for the period. In contrast, contractual equilibrium employs a more
nuanced theory of disagreement, which allows the players to remain in the relationship. If
they disagree in the current period, contractual equilibrium specifies that they fall back on
behavior that they jointly arranged in their most recent agreement, and this behavior may

\footnote{Recall that the assumption that the wage is externally enforced at the end of the period is irrelevant because
it may as well be paid immediately.}
be responsive to their history.

To state the axioms simply, we first simplify the model as follows: in each period, first a random proposer is selected to make a take-it-or-leave-it offer; next the responder can agree or disagree; then voluntary transfers are exchanged; then the parties simultaneously decide whether to trigger the outside option for one period; finally the agent chooses effort to exert (if the outside option was not triggered). Rather than a wage and bonus, the proposer offers a suggested continuation payoff vector. This model has the same set of subgame perfect equilibrium payoffs as in Levin (2003).

Contractual equilibrium imposes three axioms on how offers and responses in the bargaining stage are interpreted. First, the Internal Agreement Consistency (IAC) axiom specifies that if the proposer offers a continuation payoff that is available at an alternative history in the same equilibrium, and the responder agrees, then this is considered a credible agreement and they should switch to playing as if at that alternative history. Second, No-Fault Disagreement (NFD) is our theory of disagreement: It specifies that if the responder rejects the proposer’s offer, then there should be no transfers that period, and their continuation play should depend neither on which player was selected to be the proposer nor on what proposal was offered. Together, IAC and NFD endow the bargaining phase with a well-defined disagreement point and bargaining set, enabling a standard bargaining analysis based on the Nash bargaining solution: the parties choose the maximal joint payoff from the bargaining set, through a transfer the proposer gets all the surplus relative to the disagreement point, and from the start of the period the expected outcome splits the surplus in proportion to the likelihood of being selected to propose.²

The third axiom is Pareto External Agreement Consistency (PEAC). It specifies that if the proposer offers a continuation payoff that is on the Pareto frontier among equilibria that satisfy IAC and NFD, and the responder agrees, then this is considered a credible agreement and they should switch to playing as if in that alternative equilibrium. PEAC allows the contracting parties to select among the best equilibria that satisfy IAC and NFD.

Unlike the intuitive approach of simply selecting the Pareto frontier, contractual equilibrium puts the decision in the hands of the contracting parties, by endowing their agreements and disagreements with meaning. Under the axioms, prepending a bargaining phase onto the stage game is no longer irrelevant—with meaningful discussion, it gives the parties the opportunity to plan out their relationship, thereby eliminating equilibria that are inconsistent with their joint incentives and relative bargaining power.

²Miller and Watson (2013) shows that the bargaining phase can be generalized to allow for arbitrarily many rounds of back-and-forth bargaining, with random proposer selection and random breakdown.
In addition to conceptual clarity, contractual equilibrium enables a straightforward analysis and generates new insights. In Miller and Watson (2013) we showed that contractual equilibrium identifies a unique joint payoff, which may be less than the highest feasible in the game, even if the parties are very patient. In general, the contractual-equilibrium value is less than the maximum joint payoff among all subgame-perfect equilibria (in the absence of our axioms). The theory thus provides a critical insight: We can predict that the parties will achieve a continuation value on a relevant Pareto frontier only if we model them as actively contracting, and this necessarily involves exercise of bargaining power with attendant effects on the attainable joint value.\footnote{Exercise of bargaining power can be interpreted as the party in position to make an offer being able to hold-up high-value cooperation to extract gains in the share of surplus.} What the parties can attain depends on their relative bargaining power and the properties of the stage game. We showed how to construct a contractual equilibrium in an arbitrary two-player repeated game, by solving three interrelated optimization problems.

With these results in hand, we can reinterpret contractual equilibria in the repeated game as reduced-form equilibria in a hybrid game, which in each period features first a cooperative bargaining phase with transfers, and then a non-cooperative stage game. The hybrid game is simpler to analyze, and generates an equivalent set of equilibria.

In this article we will first establish our negative results in the context of the Levin (2003) model. Then we will construct a contractual equilibrium and explain how it works. At the end we will discuss extensions to imperfect monitoring (studied in Miller and Watson 2013) and imperfect external enforcement (studied in Watson, Miller, and Olsen 2020), as well as open questions and opportunities for future work.

2 On the irrelevance of features in familiar relational contracting models

The basic relational-contracting framework has an infinite time horizon, discrete periods, and a fixed production technology that the players engage in repeatedly. The time period is denoted \( t = 1, 2, 3, \ldots \). Players discount future payoffs according to a common discount factor \( \delta \in (0, 1) \). That is, a player is indifferent between receiving a value \( g \) in the next period and receiving \( \delta g \) in the current period.

To make our points most forcefully, we focus on the Levin (2003) model of relational contracting, one of the standard models in the literature. There are two parties: the agent whom we call player 1, and the principal whom we call player 2. We generalize his model to
allow for random-proposer bargaining, but we specialize to the environment without private information and with perfect monitoring, and with a specific quadratic cost function. Our conclusions do not depend on these specializations; in Miller and Watson (2013) we show how our approach generalizes to arbitrary repeated games with imperfect public monitoring, which also implicitly nests the kind of private information modeled by Levin.

**Model 1** (Levin). In each period, the agent and principal interact in the following three phases, in the order shown:

- **Negotiation phase:** Nature selects one of the players to be the proposer. The agent is selected with fixed probability \( \pi_1 \) and the principal is selected with probability \( \pi_2 \), where \( \pi_1, \pi_2 \geq 0 \) and \( \pi_1 + \pi_2 = 1 \). (Levin assumes \( \pi_2 = 1 \).) The proposer offers a spot contract to the responder, consisting of an externally enforced net monetary transfer \( m = (m_1, m_2) \) to take place in the period and a suggested schedule of voluntary bonuses that the players may exchange at the end of the period. The transfer satisfies \( m_1 + m_2 \leq 0 \) and can be interpreted as a wage payment, along with “money burning” or transfers to a third party if \( m_1 + m_2 < 0 \). The responder either accepts or rejects the offer.

- **Production phase (stage game):** If the responder accepted in the negotiation phase, the players are engaged for the period, the agent selects an effort level \( e \geq 0 \), and the players receive stage-game payoff vector \( u = (-e^2, e + e^2) \). If the responder rejected in the negotiation phase, the players are disengaged for the period, they receive their outside-option payoffs given by vector \( u \).

- **Bonus phase:** If they are engaged, simultaneously the players make voluntary monetary transfers and can also burn money. Let \( b = (b_1, b_2) \) denote the vector of total monetary transfers received, where \( b_1 + b_2 \leq 0 \).

Total payoffs in the period are the sum of the stage game payoffs and monetary transfers, normalized by \( (1 - \delta) \), so that player \( i \) obtains \( (1 - \delta)(m_i + u_i + b_i) \) if engaged and \( (1 - \delta)u_i \) if disengaged. The normalization allows us to interpret the present value of a series of payoffs as the per-period average.

Although effort is unbounded, due to the quadratic cost and time discounting it turns out that the players cannot sustain arbitrarily high effort. We also assume \( u_1 + u_2 \geq 0 \), so the joint value of the receiving the outside option in a period exceeds the joint value of producing at minimal effort.

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4Operationally, each player chooses a non-negative amount to send to the other player, and also chooses a non-negative amount to burn. Since only the net transfers are important, for the rest of this article we ignore how simultaneous voluntary transfers are operationalized.
This modeling approach incorporates several special features that at first glance seem to impose structure with substantive implications, but turn out to be irrelevant when analyzing the model using subgame perfect equilibrium:

1. The inclusion of the bonus phase, which gives the parties a second opportunity to make monetary transfers in the period;

2. The assumption that the monetary transfers at the beginning of the period are externally enforced if the offer in the negotiation phase is accepted;

3. The assumption that the outside option is inextricably triggered if the offer in the negotiation phase is rejected;

4. The modeling of the negotiation phase using an ultimatum offer;

5. The inclusion of a recommended bonus schedule in the ultimatum offer.

The voluntary bonuses combined with the externally enforced wages suggest an environment of incomplete external enforcement, but in fact no external enforcement is needed to generate the same conclusions. The outside option directly constrains the set of equilibrium values, but in a subgame perfection analysis it does not matter whether it is triggered automatically upon rejection or voluntarily. We will later show that in a contractual equilibrium analysis, implications depend on whether the parties may elect not to trigger the outside option under disagreement. The ultimatum offer may seem to endow the proposer with bargaining power because it forces the responder to choose between the offered terms and the outside option, but in a subgame perfection analysis the privilege to make ultimatum offers is of no consequence. In fact, the set of payoffs attainable in subgame perfect equilibrium in Model 1 is identical to that of the Model 2, below, in which there is no negotiating and no external enforcement, and the outside option is literally optional:

**Model 2 (Basic).** In each period, the agent and principal interact in the following two phases, in the order shown:

- **Transfer phase:** The players simultaneously pay voluntary monetary transfers and can also burn money. Let \( m = (m_1, m_2) \) denote the vector of net monetary transfers received, where \( m_1 + m_2 \leq 0 \).

- **Production phase (stage game):** The players simultaneously decide whether to engage or disengage in the current period. If either player elects to disengage, then
there is no production in the period and the players receive their outside-option payoffs given by vector $u$. If they are engaged, the agent selects an effort level $e \geq 0$ and the players receive stage-game payoff vector $u = (-e^2, e + e^2)$.

To bound the joint value that can be generated in equilibrium, we must find the highest effort level that the agent could feasibly be given the incentive to choose. It is useful to define $g(\nu)$ as the highest value of $e$ that satisfies either $(1 - \delta)e^2 \leq \delta(e - \nu)$ or $e = 0$, which is an incentive condition explained as follows. Suppose $\nu$ is the sum of the players’ outside-option payoffs. If player 1 were supposed to choose effort $e > 0$ in the production phase of a given period, then the best deviation would be to choose zero effort, yielding a gain of $(1 - \delta)e^2$ in the current period. If $e$ were the highest joint value that the players could achieve in the continuation game from the next period, then the greatest punishment that could be imposed on player 1 is $\delta(e - \nu)$ in current-period terms. Some algebra reveals that

$$g(\nu) = \frac{\delta}{2(1 - \delta)} \left(1 + \sqrt{1 - 4 \left(\frac{1 - \delta}{\delta}\right) \nu}\right) \quad \forall \nu \leq \delta/4(1 - \delta);$$

while $g(\nu) = 0$ for all $\nu > \delta/4(1 - \delta)$. Note that $g$ is a decreasing function.

**Proposition 1.** In both Model 1 and Model 2, under subgame perfect equilibrium (SPE):

i. The highest effort level attainable is $e^* = g(u_1 + u_2)$;

ii. If $u_1 + u_2 \leq \delta/4(1 - \delta)$ then the set of equilibrium continuation values is

$$V^{SPE} = \{v \in \mathbb{R}^2 \mid v_1 \geq u_1, v_2 \geq u_2, v_1 + v_2 \leq e^*\},$$

whereas if $u_1 + u_2 > \delta/4(1 - \delta)$ then $V^{SPE} = \{u\}$.

The set $V^{SPE}$ is pictured in Figure 1 for the case of $u_1 = u_2 = 0$. We show this case for easy comparison with a set we construct in Section 4.

**Proof.** We start by supposing that $u_1 + u_2 \leq \delta/4(1 - \delta)$, and consider Model 2. The following specification of strategies is a SPE that achieves continuation value $u$. In every period, regardless of the history, the players are supposed to make no transfers. Regardless of the transfers actually made, the strategies prescribe that the players disengage in the current period. In the event that production occurs (due to both players deviating to engage rather than disengage), then the agent is supposed to select $e = 0$.

Continuation value $z^2 = (e^* - u_2, u_2)$ is achieved by a strategy profile that specifies the following actions on the equilibrium path, so long as both players behave as prescribed.
In the transfer phase, the principal is supposed to make a transfer of \( e^* + (e^*)^2 - u_2 \) to the agent, whereas the agent is supposed to transfer nothing. After this transfer, the players are supposed to remain engaged so that production occurs, and the agent is supposed to choose effort \( e^* \). If the principal deviates in the transfer phase, the players should disengage. Following this or any other deviation, from that point the players switch to the strategy profile that delivers \( z^1 = (u_1, e^* - u_1) \) if the agent deviated most recently, and \( z^2 \) if the principal deviated.\(^5\) It is easy to verify that this strategy profile is a SPE that attains continuation value \( z^2 \).

All other values in \( v \in V^{SPE} \) are achieved in equilibrium for appropriately prescribed first-period transfers and then conditioning the continuation as described. In particular, continuation value \( z^1 = (u_1, e^* - u_1) \) is obtained by having the principal make a transfer of \( e^* + (e^*)^2 - u_2 - \delta(e^* - u_1 - u_2) \) to the agent, after which they follow the strategy profile for \( z^2 \).

The strategy profiles for Model 1 are similar. To obtain \( u \) the proposer should offer no externally enforced or bonus transfers. The responder should reject this offer (triggering disengagement), but accept any deviant offer that would give the responder an externally enforced transfer greater than the responder’s outside option. If the parties are engaged, the agent should exert zero effort. The principal should never make any bonus payment, regardless of what was promised in the negotiation phase.

\(^5\)If both players deviated simultaneously, it doesn’t matter what they switch to, but for concreteness they can switch to the \( z^2 \) strategy.
To obtain $z^2$ or $z^1$, the proposer should offer as an externally enforced transfer equal to the corresponding voluntary transfer specified in the strategy profile for Model 2, and promise a zero bonus. The responder should accept this offer, and any deviant offer that gives the responder a higher externally enforced transfer. Other offers should be rejected, triggering disengagement. If there have been no deviations, then the agent is supposed to choose effort $e^\ast$. The principal should never make any bonus payments, regardless of what was promised in the negotiation phase. If either player ever deviates in the negotiation/engagement phase, then within that period the agent should exert zero effort if they are engaged. It is easy to verify that this strategy profile achieves the desired continuation value and is sequentially rational. All other values in $v \in V^{SPF}$ are achieved in equilibrium for appropriately prescribed first-period transfers from the principal and then conditioning the continuation as described for Model 2.

Finally, if $u_1 + u_2 > \delta/(4(1-\delta))$ then $g(u_1 + u_2) = 0$ and no effort can be sustained in any equilibrium of either model. In both models, the parties should always offer and pay zero transfers. In Model 2 they should always disengage. If in Model 1 the proposer deviates to offer the responder an externally enforced transfer greater than the responder’s outside option, then the responder should accept; otherwise the responder should reject. If the parties are engaged, the agent should exert zero effort. In each model this behavior constitutes a SPE within a single period, so repeating it unconditionally is a SPE in the repeated game.

Observe that our equilibrium construction for Model 1 does not make use of its special features. The players do not make any payments in the bonus phase, and the monetary transfers prior to the production phase are the same regardless of whether they are externally enforced. The outside option can be triggered voluntarily rather than automatically upon the rejection of an offer. The equilibrium proposal does not depend on the identity of the proposer, and any nonzero recommended bonus schedule in the negotiation phase is to be ignored or punished rather than carried out. Moreover, our constructions for $z^1$ and $z^2$ in both models satisfy the “strong optimality" refinement of Levin (2003) Corollary 1, which requires the continuation value to be on the equilibrium Pareto frontier at the start of the period after every history.

So we see that the familiar relational-contracting model provides no basis for the prediction that the players would coordinate on any particular equilibrium. (Imposing strong optimality assumes the conclusion: that the players will obtain a value on the Pareto frontier of the set $V^{SPF}$, if there exists an equilibrium that is on that frontier after every history.) One might invoke the idea of a social norm that coordinates the parties on a good equi-
librium, but there is no foundation for this either. Although Model 1 allows an ultimatum offer of an externally enforced transfer and a suggested bonus, the bargaining power we might expect an ultimatum proposer to exert is easily subverted by ordinary repeated-game rewards and punishments. So something important is missing.

The point of modeling “contracting” is that, through negotiation, the parties actively discuss and agree on a prescription for behavior in their relationship. Relational contracts rely on self-enforcement, so active contracting is about coordinating on a strategy profile in the continuation game. To represent discussion about the strategy profile, we need a language suitable for the players to convey proposals for equilibrium behavior, not merely the transfers that will be paid in the current period. The simplest approach is to allow them to propose a continuation payoff to receive, and let the continuation strategy profile that delivers it be implicitly understood. We have also seen that transfers and disengagement decisions do not need to be externally enforced, so we will model them as voluntary decisions. This leads us to our preferred noncooperative model, which augments Model 2 by adding a cheap-talk negotiation phase to the beginning of each period.

**Model 3** (Platform for Active Contracting). *In each period, the agent and principal interact in the following three phases, in the order shown:*

- **Negotiation phase:** Nature selects one of the players to be the proposer. The agent is selected with fixed probability $\pi_1$ and the principal is selected with probability $\pi_2$, where $\pi_1, \pi_2 \geq 0$ and $\pi_1 + \pi_2 = 1$. The proposer offers a continuation value $w \in \mathbb{R}^2$. The responder either accepts or rejects the offer. Both the proposer’s offer and the responder’s reply are cheap talk; i.e., they have no payoff consequences and do not constrain what the players may do in the continuation game.

- **Transfer phase:** The players simultaneously pay voluntary monetary transfers and can also burn money. Let $m = (m_1, m_2)$ denote the vector of net monetary transfers received, where $m_1 + m_2 \leq 0$.

- **Production phase (stage game):** The players simultaneously decide whether to disengage in the current period. If either player elects to disengage, then there is no production in the period and the players receive their outside-option payoffs given by vector $u$. If they are engaged, the agent selects an effort level $e \geq 0$ and the players receive stage-game payoff vector $u = (-e^2, e + e^2)$.

In subgame perfect equilibrium, the cheap-talk negotiation phase does not affect the set of SPE payoffs, because one can always construct an equilibrium in which cheap-talk messages are ignored.
Proposition 2. The SPE payoff predictions in Model 4 are the same as in Model 1 and Model 2.

In the next section we provide axiomatic foundations that enable the parties in Model 4 to actively negotiate over how to play the game, exerting bargaining power in the process. The resulting theory of contractual equilibrium yields specific, interpretable, and interesting behavior, where bargaining power matters, the Pareto frontier may be strictly worse than that of $V^{SPE}$, and after some histories the parties remain engaged even if they disagree.

3 Modeling active negotiation over self-enforced contracts

To model active contracting, we need to impart meaning to the parties’ negotiation process. The parties need to be able to discuss how to play, and coordinate on an understanding of how to play when they reach an agreement. Model 4 provides a language that the proposer can use to make an explicit suggestion for their continuation play, by suggesting a payoff vector for them to attain. Similarly, we interpret the responder’s acceptance as creating an “agreement”, and the responder’s rejection as generating a “disagreement”. But this is not enough, since the parties could simply ignore the content of their communication. So the next step is to impose axioms on endogenous meaning, to restrict attention to equilibria in which the parties treat credible agreements as meaningful. But even this is not enough, since the parties could still leverage disagreement play to punish deviant proposals. So the final step is to impose a “theory of disagreement” that continuation play under disagreement does not depend on how disagreement arose.

3.1 Implementing credible agreements is not enough

In active contracting, the parties mutually understand what is being proposed, and should follow through on their agreement if it is suitably self-enforceable. To make this idea precise, one can impose one or more agreement consistency axioms. Each such axiom deems a set of agreements as credible, and imposes the constraint that if a credible agreement is reached, then it is carried out. Or, more precisely, an agreement consistency axiom disqualifies subgame perfect equilibria in which credible agreements are not carried out. The agreement consistency axioms differ in which agreements they consider credible.

We start by considering the strongest possible agreement consistency axiom: the parties honor any agreement to obtain a continuation value that is attainable in subgame perfect equilibrium. This is the strongest notion of what makes an agreement credible, as it ignores the potential contradictions that could arise if the players had previously agreed to play
an equilibrium supported by Pareto-dominated punishments, and then had an incentive to
deviate by negotiating out of a punishment. Our next result shows that even with this strong
restriction, the same payoff set $V^{\text{SPE}}$ is still attainable in equilibrium.\(^6\)

**Axiom 1** (Universal Agreement Consistency, or UAC). *For every history at the start of a
period, if in the negotiation phase the parties reach an agreement $w \in V^{\text{SPE}}$, then their
continuation value from the start of the transfer phase is $w$.*

**Proposition 3.** The SPE payoff predictions of Model 4 under UAC are the same the SPE
payoff predictions of Model 1.

**Proof.** Given any $v \in V^{\text{SPE}}$, we construct a subgame perfect equilibrium that delivers
value $v$ while satisfying UAC. First, we describe the equilibrium path behavior. In period 1,
the proposer is supposed to offer $v$, and the responder is supposed to accept. In any period $t$,
suppose $w \in V^{\text{SPE}}$ is the standing agreement, i.e., the agreement reached in the negotiation
phase. The parties should exchange monetary transfers $m$ that satisfy

$$
\begin{align*}
    w = (1 - \delta) & \left( m - \left( (e^*)^2, e^* + (e^*)^2 \right) \right) + \delta z^2,
\end{align*}
$$

neither party should disengage, and the agent should exert effort $e^*$. In period $t + 1$ the
proposer should offer $z^2$. The responder should accept. Then they continue their play, with
$z^2$ as the standing agreement.

So far we have partially described a strategy profile that satisfies UAC on the equilib-
rium path, and delivers value $v$. Next we complete the description of the strategy profile and
explain why it is a subgame perfect equilibrium. Given the outcome of the random proposer
selection in a given period, label the proposer as player $i$ and the responder player $j$.

The key step addresses what happens if the proposer deviates to offer $w' \in V^{\text{SPE}}$ when
they were supposed to offer $w$ in the negotiation phase of any period $t$. Then the responder
should reject, and the parties should behave as if the standing agreement were instead $z^i$.
The reward of $z^i_j \geq w^i_j$ makes the responder willing to reject the valid offer, and the penalty
of $z^i_i \leq w_i$ discourages the proposer from offering it. Because the responder rejects the
offer, UAC does not constrain their continuation play. For other deviant offer $w' \notin V^{\text{SPE}}$,
the responder may simply accept, and the parties should behave as if the standing agreement
were instead $w$; this does not violate UAC, since it does not deem an agreement $w' \notin V^{\text{SPE}}$
credible.

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\(^6\)This result strengthens Theorem 1 in Miller and Watson (2013), which imposes a weaker set of agreement
consistency axioms.
If the responder deviates by rejecting an offer that was supposed to be accepted, then they can simply behave as if the offer was accepted, since UAC does not apply to behavior under disagreement. Thus the responder is willing to accept offers that are supposed to be made in equilibrium. (If the responder deviates by accepting an offer \( w \) that was supposed to be rejected, our construction implies that the offer satisfies \( w \in V^{SPE} \), so by UAC they must continue with \( w \) as the standing agreement.)

If either player deviates from the specified monetary transfer in period \( t \), then both parties should disengage, and in period \( t + 1 \) the proposer should offer \( u \) and the responder should accept. If either party deviates from the specified engagement decision in period \( t \), then in period \( t + 1 \) the proposer should offer \( u \) (and if they are engaged the agent should exert zero effort). Since the specified monetary transfer is always in service of obtaining some continuation value \( w \in V^{SPE} \), the threat of disengagement followed by \( u \) suffices to discourage deviant transfer and engagement decisions.

If the agent deviates in the production phase in period \( t \), then in period \( t + 1 \) the proposer should offer \( z^1 \), and the responder should accept. Then they continue their play, with \( z^1 \) as the standing agreement. The reward of getting \( z^2 \) rather than \( z^1 \) makes the agent willing to exert effort \( e^* \), as established in Proposition 1.

We see that even such a strong agreement consistency axiom as UAC does not eliminate any possible equilibrium payoffs. The problem is that even if the proposer makes a Pareto-improving offer, the responder can be rewarded for rejecting the offer, in a way that also punishes the proposer for making it. To obtain a refinement in which the parties can meaningfully discuss how to play, we need to rule out this kind of behavior. That is, we need a theory of disagreement.

### 3.2 Axioms of active contracting

To model active contracting, we introduce three axioms: two agreement consistency axioms that in combination are weaker than UAC, and a disagreement axiom. (We have slightly simplified the statement of each axiom compared to Miller and Watson (2013), but in a way that does not affect the characterization of contractual equilibrium.) The No-Fault Disagreement axiom embodies the idea that play under disagreement should not depend on how disagreement occurred. Disagreements can arise in a variety of ways: either player may be selected as proposer; then the proposer may make a deviant offer and the responder rejects it, or the proposer may make the equilibrium offer and the responder rejects it. The
axiom also embodies the idea that transfers should not be made if there is no agreement.\footnote{The intent here is to model a situation in which there is no external enforcement. Watson, Miller, and Olsen (2020) allows for long-term externally enforced contracts, in which case a legal contract signed in a prior period would be externally enforced even under disagreement in the current period.}

**Axiom 2 (No-Fault Disagreement, or NFD).** For every history at the start of a period \(t\), there exists a disagreement value \(w \in V^{\text{SPE}}\) such that if the responder rejects the proposer’s offer, then the continuation value from the start of the production phase is \(w\), regardless of the identity of the proposer, the content of the offer, and what monetary transfers were made in period \(t\).

The axiom still allows that the disagreement value \(w\) may differ depending on the history. For instance, the disagreement value may favor the agent if the agent exerted effort \(e^*\) in the prior period, but may favor the principal if the agent exerted deviant effort. The axiom also implies that under disagreement neither player will be willing to make a voluntary transfer, since the continuation value after the transfer phase does not depend on what transfers were made. By itself, No-Fault Disagreement does not materially affect the set of equilibrium payoffs, because it does not constrain play under agreement. This is shown in Theorem 2 of Miller and Watson (2013). We will shortly see that when NFD is combined with the agreement consistency axioms, the agreement the parties will reach in a given period is heavily influenced by what they would get if they disagreed. With the flexibility to condition their future disagreement values on the history of play, the parties will be able to condition their future agreements on the history as well, and thereby provide incentives for them to conform to their current agreement.

The agreement consistency axioms enable the parties to reach agreement on credible continuation play. Internal Agreement Consistency (IAC) deems an agreement \(w\) credible if it is “supported within the current equilibrium.” That is, for a given equilibrium there is a set \(W\) containing every value that can be obtained starting from the production phase, at some history. An offer \(w\) is supported within the current equilibrium if it can be obtained by first making an arbitrary transfer and then continuing from the production phase with a value in \(W\). We provide two examples for illustration.

**Example 1.** Consider an equilibrium in which the players are supposed to obtain a continuation value of \(u\) after every history. In this case \(W = \{u\}\). While the players could feasibly obtain other values by first exchanging a transfer and then continuing with this behavior, neither player would ever be willing to transfer a non-zero amount.

**Example 2.** In the proof of Proposition 1 we described a subgame perfect equilibrium in which at some histories to the production phase the agent exerts effort \(e^*\) and then obtains
In the following period, so \( W \) contains the value \((1 - \delta)(-(e^* )^2, 0) + \delta z^2\). At other histories (following a deviant transfer by the principal) the parties disengage and then continue with \( z^2 \), so \( W \) also contains the value \((1 - \delta)u + \delta z^2\). Other continuation values were obtained by first exchanging a transfer and then continuing with one of these values. Thus \( W = \{(1 - \delta)(-(e^* )^2, 0) + \delta z^2, (1 - \delta)u + \delta z^2\}\).

IAC allows the parties to discuss whether to switch to an alternative history in the same equilibrium. As the renegotiation-proofness literature has recognized, many commonly-studied repeated game equilibria employ Pareto-dominated punishments to deter deviations, but these punishments are vulnerable to renegotiation. Under IAC, if the parties find themselves negotiating at a history at which they are supposed to endure a Pareto-dominated punishment, the proposer has may instead propose switching to an alternative history at which they would receive a Pareto-superior value, and the responder may accept, upon which they should continue as agreed. IAC relates to internal or “weak” renegotiation-proofness notions studied by Bernheim and Ray (1989), Farrell and Maskin (1989), and Ray (1994). Unlike these renegotiation-proofness concepts, however, Model 4 explicitly models negotiation, allowing for the possibility of disagreement. So the IAC axiom enables players to renegotiate out of Pareto-dominated punishments, but does not require them to do so.

**Axiom 3** (Internal Agreement Consistency, or IAC). For a given SPE, let \( W \) be the set of equilibrium continuation values from the start of the production phase, across all histories. The equilibrium satisfies IAC if, for any history to a given period, if

1. the offer \( w \) satisfies \( w = m + w' \) for a transfer \( m \) satisfying \( m_1 + m_2 \leq 0 \) and a continuation value \( w' \in W \),

2. and this offer is accepted and the players make transfer \( m \),

then the equilibrium continuation from the production phase achieves value \( w' \).

The second agreement consistency axiom is Pareto External Agreement Consistency (PEAC), which deems an agreement \( w \) credible if it is Pareto optimal among values that are supported within any equilibrium satisfying IAC and NFD.

**Axiom 4** (Pareto External Agreement Consistency, or PEAC). Let \( W \) be the set of equilibrium continuation values that maximize the joint payoff from the start of the production phase, across all histories of all subgame perfect equilibria satisfying IAC and NFD. An equilibrium satisfies PEAC if, for any history to a given period, if
1. the offer $w$ satisfies $w = m + w'$ for a transfer $m$ satisfying $m_1 + m_2 \leq 0$ and a continuation value $w' \in \overline{W}$,

2. and this offer is accepted and the players make transfer $m$,

then the equilibrium continuation from the production phase achieves value $w'$.

PEAC gives the parties the ability to discuss which equilibrium to play. They can select any equilibrium that is optimal among those that accord with how they expect their future negotiations to play out—namely, any equilibrium that satisfies IAC and NFD, and will not itself be renegotiated to an even better such equilibrium. Like IAC, it relates to external or “strong” renegotiation-proofness notions (Bernheim and Ray 1989; Farrell and Maskin 1989; Asheim 1991), but by allowing the players to select Pareto-optimal equilibria rather than requiring them to do so.

### 3.3 Contractual equilibrium

The NFD, IAC, and PEAC axioms together define contractual equilibrium.

**Definition 1.** A SPE of Model 4 is a **contractual equilibrium** if it satisfies NFD, IAC, and PEAC.

A contractual equilibrium has a specific structure, which we can deduce directly from the axioms. Consider the negotiation phase at some history $h$. By NFD, there is a specific disagreement value $w$ that the parties know will arise if they fail to agree. By IAC, they can agree to obtain any payoff vector that is attained at any other history starting from the production phase in the same equilibrium, and augment it with an up-front transfer $m$. So if there is any production phase history $h'$ at which the continuation value $w'$ adds up to a higher joint payoff than that of their current disagreement point, then the proposer (player $i$) can offer a continuation value of $w = m + w'$. If $w$ Pareto dominates the disagreement value $\overline{w}$, then the responder (player $j$) will be willing to accept. Since the proposer makes an ultimatum offer, in equilibrium the proposer will extract all the surplus by offering a continuation payoff that gives the responder $w_j$ and gives the proposer $w'_i + w'_j - w_j$.

Among all equilibria with this structure, by PEAC the proposer will optimally choose as $w'$ a $\overline{w} \in \overline{W}$ that maximizes the joint payoff among all equilibria satisfying IAC and NFD.

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8This is why PEAC considers only Pareto optimal agreements credible. Contradictions would arise if a Pareto-dominated agreement were also considered credible, since there would be histories at which the players would be required to carry out their Pareto-dominated agreement and yet also have the opportunity to renegotiate out of it to a Pareto-superior equilibrium.
So in any given period, under these axioms there is a straightforward bargaining process arising from a standard Ultimatum Game analysis: there is a disagreement point (which may be below the Pareto frontier), and the proposer chooses a continuation value on the Pareto frontier (of what can be attained under IAC and NFD), augmented with a transfer that gives the responder his disagreement payoff and gives the rest to the proposer.

From the perspective of the start of the period, before the proposer is selected, the players’ expected continuation value will depend on \( \pi_1 \) and \( \pi_2 \), the proposer selection probabilities. Specifically, in expectation they split the bargaining surplus in proportion to their probabilities of being selected to propose, which we can now interpret as bargaining weights.

Since the proposer offers a continuation value \( \bar{w} \) from the production phase that is not sensitive to the history, it follows that the payoff set under agreement lies in a straight line of slope \(-1\) through \( \bar{w} \). The *contractual equilibrium value (CEV)* set \( W^* \) is the convex hull of the expected payoffs from the start of the period, across all histories. We call the joint value \( \bar{w}_1 + \bar{w}_2 \) the *level* of the CEV set, written as \( \text{level}(W^*) \). In the context of our simple principal-agent setting, if the parties are sufficiently patient then \( \text{level}(W^*) \) is attained by exerting a particular level of effort \( \bar{e} \), which depends on \( \delta \), whenever they agree. Given our specific functional form assumptions, it is the case that \( \text{level}(W^*) = \bar{e} \). If the parties are not sufficiently patient, they will not be able to sustain any effort in equilibrium, and should disengage each period. In what follows we focus on the case in which they are sufficiently patient, and we will note what constitutes sufficient patience.

The parties’ joint value \( \bar{e} \) is constrained by the strength of incentives available in the equilibrium. The incentive for the agent to engage and exert effort in the production phase of the current period is provided by transitioning to a high continuation value for the agent within \( W^* \) if she exerts the right effort; but to her worst continuation value in \( W^* \), which we call \( z^1 \), if she deviates. Depending on whether further incentives are needed to induce the principal to engage, the best continuation value for the agent, which we call \( z^2 \), may be used either to reward the agent or to punish the principal. (The superscript indicates which player is being punished.) The larger is the difference between these two, the higher-powered are the incentives.\(^9\) We call this difference the *span* of \( W^* \), computed as \( \text{Span}(W^*) = z^2 - z^1 \).

The foregoing chain of logic indicates that the level of the CEV set is closely linked with its span, where a longer span enables a higher level.

Consider a history to the start of a period at which \( z^i \) is the continuation value. By NFD, there is a particular disagreement value \( w^i \) to anchor their negotiation process. Given \( w^i \),

\(^9\)The parties are deterred from disengaging similarly: whomever deviates by disengaging is punished by their worst continuation value. If both simultaneously deviate, it doesn’t matter how they continue, so for concreteness say they continue with \( z^2 \).
$z^i$ must deliver joint payoff $\text{Level}(W^*)$ and split the surplus in proportion to the players’ bargaining weights. Specifically,

$$z^i = w^i + \pi \cdot (\text{Level}(W^*) - w^1 - w^2),$$

where $\pi \equiv (\pi_1, \pi_2)$. Accordingly, to characterize the CEV set it suffices to characterize $w^1$, $w^2$, and $\text{Level}(W^*)$.

This characterization is accomplished for general games by Theorem 5 in Miller and Watson (2013). In Proposition 4, in the Appendix, we specialize the characterization to Model 4. Proposition 4 provides a recipe for computing the CEV set, by simultaneously solving three inter-related optimization problems. Each optimization problem involves finding a stage game action profile and a transfer of continuation value such that the action profile is enforced.

### 3.4 Modeling a relationships as a hybrid game

Contractual equilibrium is what results when axioms on endogenous meaning are applied to a non-cooperative repeated game model with a cheap-talk bargaining phase at the start of each period. But it turns out that the equilibria that satisfy these axioms are outcome-equivalent (in terms of expected payoffs) to what arises in a hybrid model that replaces the non-cooperative cheap-talk bargaining phase in each period with a cooperative bargaining model. That is, the bargaining phase is modeled as a joint decision, reached according to the Nash bargaining solution (Nash 1950).

**Model 4 (Hybrid).** In each period, the agent and principal interact in the following two phases, in the order shown:

- **Negotiation phase:** The players jointly bargain over a continuation plan and a balanced monetary transfer $m = (m_1, m_2)$, where $m_1 + m_2 = 0$. The vector of exogenous bargaining weights is $\pi = (\pi_1, \pi_2)$, where $\pi_1, \pi_2 \geq 0$ and $\pi_1 + \pi_2 = 1$. If they fail to agree, they continue without a transfer and follow the plan they most recently agreed on in a prior period, or the default plan if they have never agreed. If they agree, the transfer is instantiated immediately.

- **Production phase (stage game):** The players simultaneously decide whether to disengage in the current period. If either player elects to disengage, then there is no production in the period and the players receive their outside-option payoffs given by
vector $u$. If they are engaged, the agent selects an effort level $e \geq 0$ and the players receive stage-game payoff vector $u = (-e^2, e + e^2)$.

The negotiation phase is resolved according to the Nash bargaining solution, where the bargaining set is the set of equilibrium continuation values and the disagreement point is the value of disagreement play. The production phase is resolved according to recursive Nash equilibrium, where the stage game is augmented with conditional expected continuation values. Any equilibrium value set consistent with these conditions will correspond to an equilibrium of Model 4 under the IAC and NFD axioms. The equilibrium value set with the highest level is the CEV set. While we have outlined the approach only informally here, the formal details are laid out in Watson, Miller, and Olsen (2020). While the hybrid model is not widely used in the literature, we recommend it as a simpler reduced form of the non-cooperative model when employing contractual equilibrium. Conveniently, the CEV set is characterized by a recursive formulation along the lines of Abreu, Pearce, and Stacchetti (1990); see Watson (2021) for an overview.

4 Contractual equilibrium in the principal-agent model

To illustrate the interesting properties that arise in contractual equilibrium, in this section we describe contractual equilibrium for the special case of $u = (0, 0)$. The mathematical details are reserved to the appendix. In this special case, we can ignore the decision over whether to disengage, since disengagement yields the same payoff as engagement with zero effort.\textsuperscript{10} Remarkably, the axioms suffice to identify behavior that is essentially unique, in the sense that there is a unique strategy profile that the parties will play in any continuation game following their first agreement and first transfer. The behavior is also interesting and intuitive. An important property is that the level of effort sustainable in equilibrium is $\pi_1 \delta / (1 - \delta)$, which is proportional to the agent’s bargaining weight.

Effort $\bar{e}$ that gives the level of the CEV set is increasing in the span of the CEV set, because the agent’s incentive to exert effort is maximized by using the full span of continuation values to reward and punish her. Specifically, play of $\bar{e}$ is rewarded with continuation value $z^2$ from the next period, whereas any deviation would be punished with continuation value $z^1$. Thus, $\bar{e}$ is the largest effort $e$ satisfying $(1 - \delta)e \leq \delta \text{Span}(W^*) = z^2 - z^1$, implying $\bar{e} = \sqrt{(z^2 - z^1)} / (1 - \delta)$. Points along the line between $z^1$ and $z^2$ are achieved by having player 1 select $\bar{e}$ with the rewards and punishments just described, preceded by a

\textsuperscript{10}This special case avoids needing to deter the principal from disengaging when under disagreement. A full analysis with arbitrary $u$ is sufficiently rich as to be outside the scope of this paper.
transfer in the current period to shift utility from one player to the other.

Next we characterize the endpoints of the CEV set, $z^1$ and $z^2$. The key to the analysis—in fact, the key to understanding the contractual equilibrium construction overall—is to recognize how the disagreement value in one period can depend on the agent’s effort in the prior period. Recall that $z^1$ is the value of negotiation from disagreement point $w^1$, and likewise $z^2$ is the value of negotiation from disagreement point $w^2$. Thus $w^1$ should be the value that most favors player 2 and $w^2$ should be the value that most favors player 1, among the possible continuation values under disagreement.

Suppose the agent exerted the desired effort in the prior period and the parties fall into disagreement in the current period, with continuation value $w^2$. To make this continuation value most favorable the agent, the equilibrium specifies that the agent should not exert any effort (or, equivalently, the parties should disengage for the period), and starting in the next period the agent gets her highest equilibrium payoff. In this scenario, the disagreement value for the negotiation phase is $w^2 = (1 - \delta)(0, 0) + \delta z^2$. At the start of the negotiation phase, the parties expect to bargain up to the equilibrium-path effort, splitting the additional surplus generated in proportion to their bargaining weights. That is, $z^2 = w + \pi (\text{Level}(W^*) - w^2 - w^2)$, which implies $z^2 = \pi \text{Level}(W^*)$.

Next suppose that the agent deviated in the prior period and the parties fall into disagreement in the current period. The disagreement value in this case, $w^1$, entails the agent selecting $\bar{e}$ in current period and being rewarded with continuation value $z^2$ from the next period, whereas she would be punished for any deviation by reverting to continuation value $z^1$ from the next period. This means $w^1 = (1 - \delta)(-\bar{e}^2, \bar{e}^2) + \delta z^2$, which is equal to $(1 - \delta)0 + \delta z^1$ due to the agent’s binding incentive condition, and thus $z^1 = 0$. Note that there is no negotiation surplus with this disagreement point, and so $w^1 = z^1$. As a result, the same behavior is called for under both agreement and disagreement. This conclusion stands in stark contrast to the assumption in Levin (2003) that disagreement immediately causes disengagement.

The preceding analysis shows that effort $\bar{e}$ and the endpoints of the CEV set satisfy $\bar{e} = \sqrt{(z^2_1 - z^1_1)\delta/(1 - \delta)}$, $z^1_1 = \pi_1 \bar{e}$, and $z^1_1 = 0$. These equations imply $\bar{e} = \pi_1 \delta/(1 - \delta)$, and therefore $z^1 = (0, \pi_1 \delta/(1 - \delta))$ and $z^2 = \pi \cdot \pi_1 \delta/(1 - \delta)$. The CEV set is pictured in Figure 2. The endpoints $z^1$ and $z^2$ are shown along with their corresponding disagreement points $w^1$ and $w^2$.

In equilibrium, in the period after the agent deviates, she gets an expected payoff of zero (at $z^1$), whereas if she did not deviate then she gets a $\pi_1$ share of the surplus (at $z^2$). This also stands in stark contrast to the Levin (2003) conclusion that the entire relationship surplus
can be used to motivate the agent. There are two keys to understanding the distinction. First is that because the familiar approach, exemplified by Levin (2003), does not model active contracting, bargaining power does not play a role. Instead, the familiar models give none or all of the surplus to the agent depending on whether the agent deviated. In contrast, by modeling active contracting, contractual equilibrium incorporates bargaining power, so each party receives their share of the surplus at every successful negotiation. Second, Levin (2003) assumes that the outside option is triggered automatically in case of disagreement. In the Levin (2003) analysis this doesn’t matter, as we showed in Proposition 1. But if that assumption were imposed here—as an alternative theory of disagreement in place of the NFD axiom—then no effort could be sustained at all. Positive effort is sustainable in contractual equilibrium only if the behavior under disagreement differs depending on whether the agent is being rewarded or punished.

5 Conclusion

We have contrasted the contractual equilibrium approach, based on axioms that address how players interpret bargaining outcomes that would otherwise be mere cheap talk, with the standard approach of selecting an equilibrium on the Pareto frontier. Under the standard approach, various modeling features—like ultimatum contract offers, promised bonuses, enforceable wages, and automatic disengagement upon disagreement—have no effect on
the set of attainable equilibrium values, leaving the equilibrium selection problem just as much in the hands of the analyst as they would be without these features. Contractual equilibrium, in contrast, identifies essentially unique equilibrium behavior in many settings, including the canonical principal-agent game that we use as an example.

Under contractual equilibrium, we find that bargaining power matters, as does play under disagreement. In the principal-agent game the incentive power given to the agent is limited to the agent’s bargaining share of the relationship surplus. Incentives are supported by playing differently under disagreement depending on whether the agent is being rewarded or punished, so that the agreements formed relative to these disagreement outcomes feature high and low transfers to the agent, respectively.

In constructing a contractual equilibrium for the principal-agent game, we limited our attention to the case in which the outside option yields the same payoff vector that would arise if the agent exerted zero effort. Extending the analysis to allow for higher outside options is not particularly difficult under the standard approach (as detailed in Levin 2003), but under contractual equilibrium there are new subtleties to consider. While these are outside the scope of this paper, which is intended to be heuristic, they are quite interesting. Briefly, the issue is that while an attractive outside option will constrain what is enforceable under disagreement if the parties do engage, it also provides the additional possibility of disengagement. To determine whether it is optimal to disengage under disagreement when rewarding or punishing the agent will require a detailed comparison that will naturally depend on the nature of the outside option. This is an interesting question for future research, since understanding the effect of outside options can help us understand how relational contracts between two parties are influenced by the broader environment in which such relationships are formed and maintained.

Contractual equilibrium can, of course, be applied to much broader class of economic settings than the complete-information principal-agent model we analyzed here. Both of the more general models considered by Levin (2003)—one with imperfect monitoring, and one with private information—fall within the class of imperfect public monitoring models considered by Miller and Watson (2013). Team production and endogenous monitoring (as in Gjertsen, Groves, Miller, and Watson 2010) are two additional areas of application, among many others. Miller and Watson (2013) also showed how to handle heterogeneous discount factors, along with a simple way to address relational contracts among more than two players. More recently, Watson, Miller, and Olsen (2020) extend the scope of contractual equilibrium to allow for external enforcement of arbitrary long-term contracts. Kostadinov (2019) further extends the scope (including long-term contracts) to allow the agent to be
risk averse, so that monetary transfers are not synonymous with transfer of utility. With external enforcement of long-term contracts, the contract signed in one period determines the environment in which the parties will interact if they disagree in the following period and fail to renegotiate the contract.

Looking forward, contractual equilibrium can be applied to understanding relational contracts in non-stationary environments, such as with accumulation of capital, innovation, limited liability for the agent, or financing constraints for the principal. Such environments naturally feature equilibrium multiplicity, and contractual equilibrium provides a disciplined equilibrium selection. It is also important to understand contractual relationships in their larger context, whether that be a matching market where parties can search for new relationships upon separation, or a network of overlapping relationships. In some cases a large player may be able to design or at least influence the environment, such as a platform that connects agents and principals to form matches. Bernstein (2016) explains how a large firm with many suppliers can overcome its own hold-up incentives by connecting its suppliers to each other, facilitating collective punishments—even while each relationship between the firm and a given supplier is governed by a bespoke contract. In such an environment, the disagreement point in a given bilateral relationship could depend on the state of the multilateral relationship.

The axiomatic underpinnings of contractual equilibrium are modular, in the sense that No-Fault Disagreement could be replaced with a different theory of disagreement, and the agreement consistency axioms IAC and PEAC could be replaced with different notions of agreement consistency. We have shown that the theory of disagreement should not be too permissive (Proposition 3), but it also should not be too restrictive. If play under disagreement were anchored to single stage game outcome regardless of the history, then under reasonable agreement consistency axioms the parties would always negotiate to the same agreement, and there would be no scope for incentives. Moreover, there are several different approaches in the renegotiation-proofness literature regarding whether/how to disqualify equilibria with Pareto-dominated punishments (e.g., Pearce 1987; Asheim 1991); one could translate these approaches into an alternative set of agreement consistency axioms. It would also be interesting to incorporate more nuance into the negotiation protocol, to allow for negotiating costs (Blume 1994), infrequent renegotiation (Goldlücke and Kranz

\[11\]Safronov and Strulovici (2018) study a more permissive theory of disagreement that allows deviant proposals to be punished, and find that while some low payoff vectors are ruled out, typically both Pareto efficient and inefficient payoff vectors are attainable under their refinement.

\[12\]Ramey and Watson (2002) develop a theory of this sort to model how an external and costly dispute-resolution system can help enforce contracts without being able to verify which party deviated to initiate a dispute. Klimenko, Ramey, and Watson (2008) is an application to international trade.
2017), or an explicit account of the court enforcement process.

References


A Appendix

A.1 Characterization of the CEV set

In this appendix, we provide details for how to compute the CEV set, by specializing Theorem 5 in Miller and Watson (2013) here to Model 4. For simplicity we restrict attention to pure strategies in the production phase, which in this particular model is without loss of generality.

To account for both engagement and effort decisions in the same stage game payoff function, let $f \in \{0, 1\}^2$ be the pair of indicators for whether the agent and principal, respectively, chose to engage; let $F \in \{0, 1\}$ be the indicator for whether they are engaged. Then (with some abuse of notation) we define $u(e, f) = F u(e) + (1 - F) u$. The parties’ interaction starting in the production phase and then continuing with a value from the CEV set next period can be summarized by augmenting their production phase payoffs in the current period with a transfer of continuation value $\eta$:

$$R_+ \times \{0, 1\}^2 \rightarrow [0, \text{Span}(W^*)]$$

from principal to agent, so for purposes of verifying incentives their payoffs can be normalized to $(1 - \delta)u_1(e, F) + \delta \eta(e, f)$, (2)

and we say it is individually rational (IR) under $\eta$ if $f = (0, 0)$, or if $f = (1, 1)$ and

$$\begin{align*}
(1 - \delta)u_1(e, (1, 1)) + \delta \eta(e, (1, 1)) &\geq (1 - \delta)u_1(0) + \delta \eta(0, (1, 1)), \\
(1 - \delta)u_2(e, (1, 1)) - \delta \eta(e, (1, 1)) &\geq (1 - \delta)u_2 - \delta \eta(e, (1, 0)).
\end{align*}$$

Proposition 4. Let $W^*$ be the CEV set.

1. $\text{Span}(W^*)$ is equal to the maximal fixed point of $\Gamma \equiv \gamma_1 + \gamma_2$, where

$$\begin{align*}
\gamma_1(d) &\equiv \max_{e_1, f_1, \eta_1} \left( \pi_1 u_2(e_1, f_1) - \pi_2 u_1(e_1, f_1) - \frac{\delta}{1 - \delta} \eta_1(e_1, f_1) \right) \\
\gamma_2(d) &\equiv \max_{e_2, f_2, \eta_2} \left( \pi_2 u_1(e_2, f_2) - \pi_1 u_2(e_2, f_2) + \frac{\delta}{1 - \delta} \eta_2(e_2, f_2) - d \right)
\end{align*}$$

Without loss of generality, we restrict attention to strategy profiles that call upon both players to take the same engagement/disengagement action. Without loss of generality, our incentive compatibility condition focuses on deterring the agent from exerting zero effort. If $\eta$ suffices to deter deviating from $e$ to zero effort, then setting $\eta(e') = \eta(0)$ for all $e' \neq e$ deters all other effort deviations without violating any of the constraints. We ignore the incentive conditions for both players to disengage when $F = 0$, since if both are called on to disengage, then each is indifferent over whether to do so.
where for each \( i \in \{1, 2\} \), \( \eta_i : \mathbb{R}_+ \times \{0, 1\} \to [0, d] \), and each maximization is subject to \((e_i, f_i)\) is incentive compatible and individually rational under \( \eta_i \):

2. \( \text{Level}(W^*) = \max_{\bar{e}, \bar{f}, \bar{\eta}} (\pi_1 u_1(\bar{e}, \bar{f}) + u_2(\bar{e}, \bar{f})), \) where \( \bar{\eta} : \mathbb{R}_+ \times \{0, 1\} \to [0, \text{Span}(W^*)] \), subject to \((\bar{e}, \bar{f})\) is incentive compatible and individually rational under \( \bar{\eta} \);

3. The endpoints of \( W^* \) are

\[
\begin{align*}
z^1 &= (-1, 1) \gamma_1(\text{Span}(W^*)) + \pi \text{Level}(W^*), \quad (7) \\
z^2 &= (1, -1) \gamma_2(\text{Span}(W^*)) + \pi \text{Level}(W^*). \quad (8)
\end{align*}
\]

This proposition provides an algorithm for computing the CEV set: first solve part 1 to compute the span, then solve part 2 to compute the level, and then calculate part 3 to find the endpoints. In part 1, \( \gamma_1(d) \) finds the disagreement behavior that supports an agreement that maximally punishes the agent. The first two terms in the objective function, \( \pi_1 u_2(e_1, f_1) - \pi_2 u_1(e_1, f_1) \), value pushing the stage game payoff in a direction (favoring the principal) that is perpendicular to the direction in which the parties will negotiate. The third term, \( \frac{\delta}{1-\delta} \eta_1(e_1, f_1) \), values transferring as much continuation value to the principal as possible. The constraint that \( \eta_1 : \mathbb{R}_+ \times \{0, 1\} \to [0, d] \) ensures that, whatever is the outcome of the stage game, the transfer of continuation value is feasible if the span of the set of continuation values is \( d \). Similarly, \( \gamma_2(d) \) finds the disagreement behavior that supports an agreement that maximally punishes the principal. Together, \( \Gamma(d) = \gamma_1(d) + \gamma_2(d) \) is the span of continuation values supportable from the start of the current period if the span available starting in the next period is \( d \). When \( d = \Gamma(d) \), the span of continuation values is “self-generating” (Abreu et al. 1990) and consistent with IAC and NFD.

Once \( \text{Span}(W^*) \) is known, it is straightforward to find \( \text{Level}(W^*) \): simply find the stage game behavior that maximizes the sum of the parties’ payoffs, subject to the constraint that it must be incentivized by transfers of continuation value that are feasible given \( \text{Span}(W^*) \). This is why \( \text{Span}(W^*) \) is the largest fixed point of \( \Gamma \)—it provides the widest scope for incentives, allowing joint payoffs to be maximized subject to IAC and NFD, thereby satisfying PEAC. Finally, knowing the span, the level, and the behavior under disagreement at each extreme of the CEV set, the endpoints of the CEV set are identified.

A.2 Constructing a contractual equilibrium

Because we assume \( u = (0, 0) \), we can assume that both parties engage after every history, since zero effort gives the same payoff as disengagement. The first step is to characterize
\( \gamma_1(d) \), which determines play under disagreement when punishing the agent.

**Lemma 1.** \( \gamma_1(d) = \pi_1 \sqrt{\frac{\delta}{1-\delta}} d. \)

**Proof.** To support action profile \((e_1, (1, 1))\), incentive compatibility requires that

\[
\eta_1(e_1, (1, 1)) - \eta_1(0, (1, 1)) \geq \frac{1 - \delta}{\delta} e_1^2. \tag{9}
\]

Since the objective function of \( \gamma_1(d) \) is decreasing in \( \eta_1(e_1, (1, 1)) \), for a given \( e_1 \) at best we can set

\[
\eta_1(e_1, (1, 1)) = \frac{1 - \delta}{\delta} e_1^2 \geq \eta_1(0, (1, 1)) = 0 \tag{10}
\]

(ignoring the constraint that \( \eta_1(e_1, (1, 1)) \leq 0 \)), in which case the objective function simplifies to

\[
\gamma_1(d) = \pi_1 e_1. \tag{11}
\]

Evidently it is optimal to maximize \( e_1 \), subject to the constraint that \( \eta_1(1, (1, 1)) \leq d \). This is solved at \( e_1 = \sqrt{\frac{\delta}{1-\delta}} d \), yielding

\[
\gamma_1(d) = \pi_1 \sqrt{\frac{\delta}{1-\delta}} d. \tag{12}
\]

The next step is to characterize \( \gamma_2(d) \), which determines play under disagreement when rewarding the agent.

**Lemma 2.** \( \gamma_2(d) = 0. \)

**Proof.** To support action profile \((e_2, (1, 1))\), incentive compatibility requires that

\[
\eta_2(e_2, (1, 1)) - \eta_2(0, (1, 1)) \geq \frac{1 - \delta}{\delta} e_2^2. \tag{13}
\]

Since the objective function \( \gamma_2(d) \) is increasing in \( \eta_2(e_2, (1, 1)) \), at best we can set

\[
\eta_2(e_2, (1, 1)) = d \geq \eta_2(0, (1, 1)) = d - \frac{1 - \delta}{\delta} e_2^2 \tag{14}
\]
(ignoring the constraint that $\eta_2(0, (1, 1)) \geq 0$), in which case the objective function simplifies to

$$\gamma_2(d) = -\pi_1 e_2 - e_2^2.$$  \hfill (15)

Evidently it is optimal to minimize $e_2$, which does not violate the ignored constraint. The solution is to set $e_2 = 0$, yielding $\gamma_2(d) = 0$ for all $d$.

Next we compute the span of $W^*$, which equals the largest fixed point of $\Gamma(d) = \gamma_1(d) + \gamma_2(d)$.

**Lemma 3.** $\text{Span}(W^*) = \pi_1 \frac{\delta}{1 - \delta}$.

**Proof.** We have that $\Gamma(d) = \pi_1 \sqrt{\frac{\delta}{1 - \delta}} d$. Its maximal fixed point is computed as follows:

$$d = \pi_1 \sqrt{\frac{\delta}{1 - \delta}} d \implies d = \pi_1^2 \frac{\delta}{1 - \delta}.$$ \hfill (16)

The next step is to compute the level of $W^*$.

**Lemma 4.** $\text{Level}(W^*) = \frac{\delta}{1 - \delta} \pi_1$.

**Proof.** To support action profile $(\bar{e}, (1, 1))$, incentive compatibility requires that

$$\bar{\eta}(\bar{e}, (1, 1)) - \bar{\eta}(0, (1, 1)) \geq \frac{1 - \delta}{\delta} \bar{e}^2.$$ \hfill (17)

Since the objective function is invariant to $\bar{\eta}$, it is optimal to maximize $\bar{e}$ subject to

$$\text{Span}(W^*) \geq \bar{\eta}(\bar{e}, (1, 1)) \geq \bar{\eta}(\bar{e}, (1, 1)) - \frac{1 - \delta}{\delta} \bar{e}^2 \geq \bar{\eta}(0, (1, 1)) \geq 0.$$ \hfill (18)

This is solved at $\bar{e} = \sqrt{\frac{\delta}{1 - \delta} \text{Span}(W^*)}$, yielding

$$\text{Level}(W^*) = \sqrt{\frac{\delta}{1 - \delta} \text{Span}(W^*)} = \frac{\delta}{1 - \delta} \pi_1.$$ \hfill (19)

Finally, we characterize the CEV set.

**Lemma 5.**
Proof. We compute the endpoints as follows:

\[ z^1 = (-1, 1)\pi_1^2 \frac{\delta}{1 - \delta} + \pi \frac{\delta}{1 - \delta} \pi_1 = \left( 0, \frac{\delta}{1 - \delta} \pi_1 \right), \tag{20} \]

\[ z^2 = \pi \frac{\delta}{1 - \delta} \pi_1 = \left( \frac{\delta}{1 - \delta} \pi_1^2, \frac{\delta}{1 - \delta} \pi_1 \pi_2 \right). \tag{21} \]