Cheap Talk

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SIMPLE COMMUNICATION MODEL

- Two agents.
- One (Sender) has private information, t. The other (Receiver) takes action, a.
- Nature picks $t \in [0, 1]$ from prior, $F(\cdot)$.
- Sender learns t. Receiver does not.
- Sender sends message $m \in M$ to Receiver.
- Receiver takes action $a \in \mathbb{R}$.

Preferences

$U^i(t, a)$, $i = R, S$.
Note (important): U^i does not depend on m.
Talk is cheap.

ASIDE: OTHER POSSIBLE ASSUMPTIONS

- Standard "Spence" signaling: $U^i(\cdot)$ depends on m.
  Normally assume single crossing.
- Verifiable information. $M(t)$ set of messages available to t.
  ($M(t) = t$, truth required. $M(t) = M$, cheap talk.)

WHAT IS THIS MODEL ABOUT?

1. Communication in everyday settings.
2. Avoiding inefficiency caused by incomplete information.
3. Advertising
4. Expert Advise
5. Legislative Decision Making

ASSUMPTIONS

$a^i(t)$ solves: $\max U^i(t, a)$.
$\bar{a}(t', t'')$ be the unique solution to $\max_a \int_{t'}^{t''} U^R(a, t) dF(t)$.
Leading example:
$U^R(t, a) = -(a - t)^2$ and $U^S(a, t) = -(a - t - b)^2$, $b > 0$.
Uniform prior.
More generally: U^i concave in a and with positive mixed partial so that a^i is increasing.
Assume: $a^S(t) > a^R(t)$.
So there is $\varepsilon > 0$ such that $a^S(t) - a^R(t) \geq \varepsilon$.
(In quadratic example, $a^S(t) = t + b$ and $a^R(t) = t$.)

CONFLICT OF INTEREST

- S and R have similar interests: $a^i(t)$ increases in t.
- S and R have different interests: $a^S(t) > a^R(t)$.
- Sometimes add parameter b, intuitively decreasing b decreases conflict.

STRATEGIES

Three elements:

1. Message for each type: $\mu : [0,1] \rightarrow M$ for S.
2. Action for each message: $\alpha : M \rightarrow \mathbb{R}$ for R.
3. Interpretation of message: $\beta(t \mid m)$.

EQUILIBRIUM CONDITIONS

1. $$\text{for each } t \in [0,1], \mu(t) \text{ solves } \max_m U^S(\alpha(m), t), \tag{1}$$
2. $$\text{for each } m \in M, \alpha(m) \text{ solves } \max_a \int_0^1 U^R(a,t)\beta(t \mid m)dt, \tag{2}$$
3. $\beta(t \mid m)$ is derived from μ and F from Bayes's Rule.

An equilibrium with strategies (μ^*, α^*) **induces** action a if $\{t : \alpha^*(\mu^*(t)) = a\}$ has positive prior probability.

SIMPLIFICATIONS

- R uses a pure strategy by concavity.
- M is finite and S's strategy pure.

Finiteness is a conclusion.
Most S types will have unique best response.

ON FINITENESS

Assume that $a < a'$ induced in equilibrium. Then

1. There exists t such that $U^S(t, a) = U^S(t, a')$.
2. $a^S(t) \in (a, a')$.
3. No $t' > t$ induces a.
4. $a \leq y^R(t)$
5. No $t' < t$ induces a'.
6. $a' \geq y^R(t)$
7. $a^R(t), a^S(t) \in [a, a']$.
8. $a' - a \geq \varepsilon$.

NECESSARY CONDITION FOR MEANINGFUL CHEAP-TALK

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This rules out:

1. Cheap Talk in Labor Market
2. Cheap Talk about Quality

ANOTHER NECESSARY CONDITION

Receivers and Senders have some common interest.
That is, R can learn t only if there exists a, such that $U^i(a^R(t), t, m) \leq U^i(a, t, m)$ for $i = R, S$ ($a^R(t)$ is R's best response to t).
Naively this rules out: Communication with Enemies

CHARACTERIZATION

Proposition

There exists N^* *such that for every* N *with* $1 \leq N \leq N^*$, *there exists an equilibrium in which the set of induced actions has cardinality* N *and there is no equilibrium which induces more than* N^* *actions. Equilibria are described by a partition* $t(N) = (t_0(N), \ldots, t_N(N))$ *with* $0 = t_0(N) < t_1(N) < \ldots < t_N(N) = 1$, *and signals* m_i, $i = 1, \ldots, N$, *such that for all* $i = 1, \ldots N - 1$

$$U^S(\bar{a}(t_i, t_{i+1}), t_i)) - U^S(\bar{a}(t_{i-1}, t_i), t_i)) = 0, \tag{3}$$

$$\mu(t) = m_i \text{ for } t \in (t_{i-1}, t_i], \tag{4}$$

and

$$\alpha(m_i) = \bar{a}(t_{i-1}, t_i). \tag{5}$$

PROPERTIES OF EQUILIBRIA

1. Unit interval partitioned.
2. Types in each partition element send the same message.
3. R best responds.
4. Incentive constraints determine edges of partition.
5. "Babbling" equilibrium always exists.
6. Typically multiple equilibria.

REGULARITY CONDITION

Definition

The cheap-talk game satisfies the Monotonicity (M) Condition if for any two solutions to (3), $\hat{t}$ and $\tilde{t}$ with $\hat{t}_0 = \tilde{t}_0$ and $\hat{t}_1 > \tilde{t}_1$, then $\hat{t}_i > \tilde{t}_i$ for all $i \geq 2$.

- Exactly one equilibrium partition for each $N = 1, \ldots, N^*$.
- Ex-ante equilibrium expected utility for both S and R is increasing in N.
- N^* decreasing in b.
- Ex-ante equilibrium expected utility for both S and R is decreasing in b for fixed N.

NITS

Definition

An equilibrium (μ^*, α^*) satisfies the *No Incentive to Separate* (NITS) Condition if $U^S(\alpha^*(\mu^*(0)), 0) \geq U^S(a^R(0), 0)$.

> *NITS states that the lowest type of Sender prefers her equilibrium payoff to the payoff she would receive if the Receiver knew her type (and responded optimally).*

NITS EXISTS

Proposition

If an N-step equilibrium fails to satisfy NITS, then there exists an $(N + 1)$-step equilibrium. Moreover, if an equilibrium satisfies NITS, then so will any equilibrium with a shorter first segment.

NITS MEANS MORE ACTIONS INDUCED

Proposition

If there is only one equilibrium partition with N induced actions for any $N \in \{1, \ldots, N^*\}$, then there exists $\hat{N} \in \{1, \ldots, N^*\}$ such that an equilibrium with N actions satisfies NITS if and only if $N \geq \hat{N}$.

NITS AND Condition (M)

Proposition

If a cheap-talk game satisfies (M), then only the equilibrium partition with the maximum number of induced actions satisfies NITS.

NITS in Single-Crossing Models

1. S's preferences are monotonic in R's actions.
2. R's action is monotonically increasing in S's type.
3. Costly signals.

   $t = 0$ satisfies NITS in any perfect bayesian equilibrium.

Restriction on Belief

Proposition

If there exists a message m^* such that R's beliefs given m^* are supported within $[0, t^*]$ where $t^* = 1$ if $U^S(\bar{a}(0,t),0) > U^S(a^R(0),0)$ for all $t \in (0,1)$ and t^* is the unique positive solution to $U^S(\bar{a}(0,t^*),0) = U^S(a^R(0),0)$, then NITS must hold because type 0 can always send m^*.

The restriction is a weak relative to Farrell's Credible Neologism.

Credible Neologisms

Definition

A credible neologism relative to a fixed equilibrium exists if there exists a set of types, T, such that precisely types in T prefer R's optimal response to T than the equilibrium payoff.

If NITS fails, then there is a credible neologism containing $t = 0$.
The problem with Credible Neologisms is that they typically destroy all equilibria.

Downward Verifiable Information

- Assume: Each type of S can prove that her type is no greater than her true type.
- Conclude:
  1. No equilibria created.
  2. NITS holds (so equilibria typically destroyed).
- The ability to avoid being pooled with higher types is typically unattractive.

Upward Verifiable Information

- Assume: Each type of S can prove that her type is no less that her true type.
- Conclude:
    1. New equilibria created.
    2. Unique outcome: separation due to unraveling at the top.

Chen

CS with perturbations (some S and R follow fixed strategies):

- $M = [0, 1]$
- Some S must send $m = t$.
- Some R must set $a = a^R(m)$.

NITS holds if there is positive probability of non-strategic types and strategic Senders use non-decreasing strategies. So limiting equilibria make selection in CS.

INTUITION

A proof by contradiction: Assume NITS fails.

1. Dishonest low Senders pool at $m = 0$.
   (Otherwise attractive deviation to 0.)
2. There exists a "small" positive message that is attractive to $t = 0$.
   (Any low message induces a nice response from non-strategic R. A simple argument also shows that one such message must also induce an attractive response from strategic R.)

Kartik

Kartik perturbs CS:
Sender payoffs: $U^S(a,t) - kC(m,t)$, *for* $k > 0$.
Assume:

- C is twice continuously differentiable.
- $C_1(t,t) = 0$.
- $C_{11}(m,t) > 0 > C_{12}(m,t)$.
- $C(t,t) = 0$.

Example: $C(m,t) = -(m-t)^2$.
Model approaches CS as k *goes to zero.*

Properties

- Look at pure-strategy equilibria in which S and R use weakly increasing strategies.
- Result: Limiting equilibria must satisfy NITS.

Intuition

A proof by contradiction: Assume NITS fails.

1. Low Senders pools at $m = 0$.
   (Otherwise attractive deviation to 0 - better action and less cost.)
2. There exists a "small" positive message that is attractive to $t = 0$.
   (Any low message induces a nice response from non-strategic R. A simple argument also shows that one such message must also induce an attractive response from strategic R.)
3. High types not in the pool.
   ($t = 1$ can deviate to $m = 1$ and receive a better action at less cost.)
4. The existence of another "on-path" action (step 3) and lying costs implies that pool must be small.

ELIMINATION OF BABBLING

In Chen and Kartik, $m = 1$ must be used in equilibrium. (Otherwise, $t = 1$ deviates.)

- Pooling Equilibria Must Pool at $m = 1$.
- $m = 0$ is a profitable deviation if NITS fails.

Veto Threats: Model

- Players: Chooser (C) and a Proposer (P).
- Quadratic Preferences.
  C's ideal point t.
  P's ideal point 0.
- t is private information.

Veto Threats: Game

- Chooser learns her type
- Chooser sends (cheap) message to Proposer.
- Proposer proposes a.
- Chooser rejects (final outcome 1) or accepts final outcome a.
- t is supported on $[\underline{t}, \overline{t}]$ with
  $\underline{t} < 1$ and $\overline{t} > \frac{1}{2}$.

Veto Threats: Equilibria

- Always Babbling.
- Sometimes size 2: C induces P's ideal or a compromise.
- Never more than two serious messages in equilibrium.

Veto Threats: NITS

NITS: $\underline{t}$ does at least as well in equilibrium as by revealing.

- 2 step satisfies NITS.
- If 1 step fails NITS, then 2 step exists.
- Both 1 and 2 step may satisfy NITS.

Sir Philip Sidney

- Preferences: $U^S(a,t) = (1-a)(1+ky) + a(t+r)$
  $U^R(a,t) = a(1+kt) + (1-a)(y+r)$.
- k degree of relationship.
- y fitness of "mother."
- Like CS, but not smooth and Sender likes lower value of a.
- Apply NITS at $t = 1$.
- Conflict of interest: self interest dominates.
- Common interest: if k is large and t is large, then both sides want a to be low.

Equilibrium

1. At most two actions induced in equilibrium.
2. Babbling Equilibrium Exists
3. $$y^* := \frac{y}{k} + 1 - \frac{1}{k}. \tag{6}$$

   The Receiver finds it uniquely optimal to set $a = 0$ if $\mathbb{E}[t|m] < y^*$, uniquely optimal to set $a = 1$ if $\mathbb{E}[t|m] > y^*$, and is indifferent over all a otherwise.

Results

- The babbling equilibrium satisfies NITS if and only if $\mathbb{E}[t] \geq y^*$.
- A two-step equilibrium exists if and only if

$$\mathbb{E}[t|t < 1 - k(1 - y)] \leq y^*. \quad (7)$$

- If a two-step equilibrium exists, it satisfies NITS.
- If the one-step equilibrium fails NITS, then a two-step equilibrium exists.
- If the one-step equilibrium satisfies NITS, a two-step equilibrium may or may not exist.