

Economics 205, Fall 2010
Quiz III

September 10, 2010

Instructions. Try to answer all parts of both questions. Make your answers as complete and rigorous as possible. Informal and intuitive arguments are better than nothing, but please provide complete justification.

1. Let $K = \{(x, y) : x^2 + y^2 \leq 4\}$.
 - (a) Prove that K is convex.
 - (b) Show that $(3, 1) \notin K$.
 - (c) Find the equation of a hyperplane that separates K from $(3, 1)$.
 - (d) Show that $(x_0, y_0) = (0, 2)$ satisfies $x^2 + y^2 = 4$.
 - (e) Is it possible to solve the equation $x^2 + y^2 = 4$ for y as a differentiable function of x for (x, y) near $(0, 2)$. If so, write $y = Y(x)$ and find $Y'(0)$.
 - (f) Is it possible to solve the equation $x^2 + y^2 = 4$ for x as a differentiable function of y for (x, y) near $(0, 2)$. If so, write $x = X(y)$ and find $X'(2)$.

2. A monopoly firm can influence demand by advertising. If the firm buys a units of advertising, it can sell q units at the price $P(a, q) = a(15 - q)$. The price of a units of advertising is αa^2 dollars. It costs the monopolist βq^2 to produce q units.
 - (a) Write the profit function of the firm.
 - (b) Show that when $\alpha = 5$ and $\beta = 2.5$ the solution to the monopolist's profit maximization problem is to set $a = 5$ and $q = 5$.
 - (c) Is it possible to describe how the profit maximizing values of q and a change as α and β change (near the point in part (b))? If so, compute the derivatives of q and a as functions of α and β near $(q, a, \alpha, \beta) = (5, 5, 5, 2.5)$.
 - (d) If α increases to 5.01 and β decreases to 2.48 will the monopolist's output increase?