Economics 205, Fall 2010: Quiz II, Possible Answers
September 3. 2010

Comments. 100 points possible, range 39-99, median 82, mean 76.

1. Some people did not know the definition of concavity. Apparently some people claimed that $f^{''} < 0$ implies that f is monotonically increasing. Nope (try $-x^2$). Concave functions typically increase and then decrease (graphs look like an upside-down "U." The third party is probably easiest if you just use the definition.

2. Remember that when you have an e-value for multiplicity k you need to find k linearity independent associated e-vectors to diagonalize.

3. Answers to part (c) were not good, apparently due to time pressure.

1. (a) False. Pick a function that is strictly concave and decreasing, for example $f(x) = 1 - x^2$. On $[0, 1]$ this function attains its unique minimum at $x = 1$.
   (b) True. In fact, it will be a global maximum.
   (c) True. By strict concavity, $f(1) > .5f(0) + .5f(1)$.

2. (a) Eigenvalues are -1 and 1, the multiplicity of the eigenvalue 1 is two.
   (b) Two linearly independent eigenvectors associated with the eigenvalue 1 are: $(1, 0, 0)$ and $(0, 1, 1)$. An eigenvector associated with the eigenvalue -1 is: $(0, 1, -1)$.
   (c) A is diagonalizable (symmetric).
   (d) One possible P is the matrix with columns equal to normalized eigenvectors: $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$. In this case, $P^{-1} = P^t = P$ and, if $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then $A = PDP^{-1}$.
   (e) Quadratic Form is Indefinite Since it has positive and negative eigenvalues.

3. Let $w = (1, 4, 0)$ and $v = (1, 0, 2)$.
   (a) Find the equation of the line that passes through the point w in the direction v.
   Point: $w = (1, 4, 0)$; Direction: $v = (1, 0, 2)$. Equation: $w + tv$.
   (b) Find the equation of a hyperplane that contains the point w and contains the line you found in part a.

Point: w; Orthogonal direction: Anything orthogonal to v. For example: $u = (0, 1, 0)$.

Equation: $u \cdot (x - w) = 0$ or $x_2 = 4$. There are lots (infinitely many) of alternative solutions.

(c) Find an equation of a line that is contained in the hyperplane that you found in part b, contains the point w, and is orthogonal to the line you found in part a.

Point: w; Direction: must be orthogonal to both v and u. That is, if the direction is $p = (p_1, p_2, p_3)$, then $p \cdot v = 0$ (this guarantees that the line is orthogonal to the line in part a) and $p \cdot u = 0$ (this guarantees that the line is in the plane described in part b). Hence $p_1 + 2p_3 = 0$ and $p_2 = 0$, so a direction is $p = (2, 0, -1)$ and equation for line is:

$$w + tp$$