

Economics 205, Fall 2009
Quiz I

August 28, 2009

Instructions. Try to answer all 3 problems. (Read all of the questions now and start on the ones that seem easiest. My guess is that you should save 1(c) and 3 until the end). Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing.

1. Let f be defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 2x - 1, & \text{if } x > 1 \end{cases}.$$

- (a) Identify the set of points in the interval $(0, 2)$ at which f is continuous.
- (b) Identify the set of points in the interval $(0, 2)$ at which f is differentiable.
- (c) Find a function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that the composite function $g \circ f$ is differentiable and onto. (Try to find one function g that leads to $g \circ f$ satisfying both properties. If you cannot do that, find two different functions.)
2. Let f be a differentiable function such that $f(x) > 0$ all x . Calculate the derivative of the function h defined in each of the problems below:
- (a) $h(x) = f(3e^2 + x)$.
- (b) $h(x) = \log(xf^2(x))$, for $x > 0$.
- (c) $h(x) = e^{f^2(x)}$.
3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy $f(x) = -f(-x)$ for all $x \neq 0$. Show that if f is continuous at 0, then $f(0) = 0$.