Economics 205, Fall 2007
Quiz III Answers

Comments. Let $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

1. $D_1 f(x) = 6x_1^5 + x_2$,$D_2 f(x) = x_1$; $D_1 g(y) = 1 + y_2 e^{y_1 y_2}$,$D_2 g(y) = y_1 e^{y_1 y_2}$.

2. Solving the systems, you get $Df(x) = (0, 0)$ if and only if $x = (0, 0)$. Similarly, $y = (0, -1)$ is the only critical point of g.

3. Examine the matrix of second partial derivatives of the two functions. For f, we have: $D^2 f(x) = \begin{bmatrix} 30x_1^4 & 1 \\ 1 & 0 \end{bmatrix}$ therefore, $D^2 f(0,0) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Check that the eigenvalues of $D^2 f(0,0) = -1, 1$, hence this matrix is indefinite and the critical point is neither a minimum nor a maximum. For g, we have $D^2 g(0,-1) = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Check that the eigenvalues of $D^2 g(0,-1) = \frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$. So, once again matrix is indefinite so that the critical point is neither min nor max.

4. $$DF(x) = \begin{bmatrix} 6x_1^5 + x_2 & x_1 \\ 0 & 1 \end{bmatrix}$$

   and
   $$DG(y) = \begin{bmatrix} 1 & 0 \\ 1 + y_2 e^{y_1 y_2} & y_1 e^{y_1 y_2} \end{bmatrix}.$$

   $D\left(F \circ G\right)(y) = DF(G(y))G(y) =$
   $$\begin{bmatrix} 6x_1^5 + x_2 + (1 + y_2 e^{y_1 y_2})x_1 & (y_1 e^{y_1 y_2})x_1 \\ 1 + y_2 e^{y_1 y_2} & y_1 e^{y_1 y_2} \end{bmatrix},$$

   where $x = (x_1, x_2) = (y_1, g(y_1, y_2)) = (y_1, y_1 + e^{y_1 y_2})$. So
   $D\left(F \circ G\right)(y) =$
   $$\begin{bmatrix} 6y_1^5 + y_1 + e^{y_1 y_2} + (1 + y_2 e^{y_1 y_2})y_1 & (y_1 e^{y_1 y_2})y_1 \\ 1 + y_2 e^{y_1 y_2} & y_1 e^{y_1 y_2} \end{bmatrix}.$$

5. $D\left(G \circ F\right) = DG(F(x))F(x) =$
   $$\begin{bmatrix} 6x_1^5 + x_2 & x_1 \\ (1 + y_2 e^{y_1 y_2})(6x_1^5 + x_2) & (1 + y_2 e^{y_1 y_2})(x_1) + y_1 e^{y_1 y_2} \end{bmatrix},$$

where $y = (y_1, y_2) = (x_1^6 + x_1x_2, x_2)$. So $D\,(G \circ F)\,(x) =$

$$\begin{bmatrix} 6x_1^5 + x_2 & x_1 \\ (1 + x_2e^{(x_1^6+x_1x_2)x_2})(6x_1^5 + x_2) & (1 + x_2e^{(x_1^6+x_1x_2)x_2})(x_1) + (x_1^6 + x_1x_2)e^{y_1x_2} \end{bmatrix}.$$

6. $Df(1,1) = \begin{bmatrix} 7 & 1 \end{bmatrix}$ so the equation is $(z_3 - f(1,1)) = (7,1) \cdot [(z_1, z_2) - (1,1)]$, since $f(1,1) = 2$, this simplifies to: $7z_1 + z_2 - z_3 = 6$.

7. Only the second choice is on the surface. If we write $h(y_2)$ for the function that satisfies: $g(h(y_2), y_2) = 0, h(0) = -1$, then, by the chain rule, $D_1g(-1,0)h'(0)+D_2g(-1,0) = 0$, where $D_1g(-1,0) = 1$ and $D_2G(-1,0) = -1$. Since $D_1g(-1,0) \neq 0$ it is possible to write y_1 as a function of y_2 and the derivative $h'(0) = 1$.