Economics 205, Fall 2007
Quiz III

September 21, 2007

Instructions. Try to answer all of the problems. (Read all of the questions now and start on the ones that seem easiest.) Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing.

Let $f(x_1, x_2) = x_1^6 + x_1 x_2$, $g(y_1, y_2) = y_1 + e^{y_1 y_2}$, $F(x_1, x_2) = (f(x_1, x_2), x_2)$ and $G(y_1, y_2) = (y_1, g(y_1, y_2))$.

1. Find the partial derivatives of f and g.
2. Find the critical points of the functions f and g.
3. Decide which of the critical points of these functions are local maxima, local minima, or neither.
4. Find the derivative of the function $F \circ G$.
5. Find the derivative of the function $G \circ F$.
6. Find the equation of a plane that is tangent to the graph of f at the point $(x_1, x_2) = (1, 1)$.
7. Consider the surface $g(y_1, y_2) = 0$. Which one of the following points is one the surface?
   (a) $(y_1, y_2) = (0, 0)$.
   (b) $(y_1, y_2) = (-1, 0)$.
   (c) $(y_1, y_2) = (0, -1)$.
8. Pick a point from the previous part that is on the surface. Call the point y^*. Is it possible to solve for y_1 as a differentiable function of y_2 in the neighborhood of y^*? If so, write an expression for the derivative of this function (at the point y^*).