

Economics 205, Fall 2007  
Quiz II

**Instructions.** Try to answer all four problems. (Read all of the questions now and start on the ones that seem easiest.) Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing.

1. Let  $x = (4, 4, 0)$ ,  $y = (1, 1, 1)$ , and  $z = (1, 1, 0)$ .
  - (a) Find an equation of the plane that passes through the points  $x$  and  $y$  (and  $(1, 1, 0)$ ).
  - (b) Find an equation of a line that passes through  $x$  and is orthogonal to the plane that you found in part a.
  - (c) Find an equation of a plane that passes through  $(0, 1, 1)$  and does not intersect the plane that you found in part a.
  
2. State which of the matrices below are diagonalizable. You need not diagonalize the matrices, but you must justify your answer. For each part determine at least one eigenvector and an associated eigenvalue of the matrix.
  - (a)  $\begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$
  - (b)  $\begin{bmatrix} 4 & 1 \\ 1 & -2 \end{bmatrix}$
  - (c)  $\begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$
  
3. Determine whether the matrix  $\begin{bmatrix} 4 & 1 \\ 1 & -2 \end{bmatrix}$  is positive (semi-)definite, negative (semi-)definite, or indefinite.
  
4. Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  be three times continuously differentiable. Assume that  $f(0) = f(1) = 0$ . Find an upper bound for  $f(.5)$  under the following conditions:
  - (a)  $f'(x) \geq 0$  for all  $x \in (0, 1)$ .
  - (b)  $f''(x) \geq 0$  for all  $x \in (0, 1)$ .
  - (c)  $f'(x) \in (-1, 2)$  for all  $x \in (0, 1)$ .

You may express your bound in terms of  $f'(x)$  and  $f''(x)$ .

You should try to find the best possible upper bound in each case. An ideal answer would state a bound, prove that the bound is greater than or equal to  $f(.5)$  under the stated conditions, **and** exhibit a function  $f$  that satisfies the conditions that attains the stated bound. Answer the three parts independently.