Economics 205, Fall 2007
Quiz I - Possible Answers

September 5, 2007

Instructions. Try to answer all four problems. (Read all of the questions now and start on the ones that seem easiest). Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing.

1. Consider the function $f(x) = x^3 - 3x + 1$.

   (a) Graph the function. Clearly label all of the local maxima and minima.

   The function increases from minus infinity, crosses zero between -2 and -1, has a local maximum at -1 (value 3), decreases, crosses zero between 0 and 1, has a local minimum at 1 (value -1), increases, and crosses zero between 1 and 2.

   (b) Find the maximum value of f over the set $[0, 10]$.

   The function has only one critical point, a local minimum, on the interval. Hence the only possible choices for the maximum value are $f(0)$ and $f(10)$. $f(10) = 971$ is clearly bigger.

   (c) Find the equation of the line tangent to the graph of $y = f(x)$ at the point $(x, y) = (2, 3)$

   The equation has slope $f'(2) = 9$ and passes through $(2, 3)$ so the equation is: $y - 3 = 9(x - 2)$ or $y = 9x - 15$.

   (d) Find the derivative of the function $g(x) = (f(x))^{10}$ evaluated at $x = 2$.

   By the chain rule, $g'(2) = 10\,(f(2))^9\, f'(2) = 90\,(3)^9$.

2. Find the indicated limits (if they exist). Justify your answers either by appealing to a general property of limits or by giving an $\epsilon - \delta$ proof.

   (a) $lim_{x \to 2}(3x + 5) = 11$ by continuity of linear functions.

   (b) $lim_{x \to 2}\left(\frac{x}{x+1}\right) = \frac{2}{3}$ by continuity of ratios of continuous functions.

   (c) $lim_{x \to 1}\left(\frac{(x-1)(3x-5)}{x^2-1}\right) = lim_{x \to 1}\left(\frac{(3x-5)}{x+1}\right) = -1$.

   The first equation comes from factoring and simplifying. The second by substitution (continuity of rational functions).

(d) $lim_{x\to 1}\left(\frac{1}{x-1}-\frac{3}{1-x^2}\right)=lim_{x\to 1}\left(\frac{-4-x}{1-x^2}\right)$.

The last limit does not exist (the ratio is positive and large when $x<1$ (but close to 1) and negative and large when $x>1$ (but close to 1)).

3. Determine whether the functions below are continuous at $x=0$.

(a) $f(x)=\begin{cases} 0 \text{ if } x<0 \\ x \text{ if } x\geq 0 \end{cases}$

This one is continuous. Either use the result I described in section or a direct proof $\delta=\epsilon$ works.

(b) $f(x)=\begin{cases} 0 \text{ if } x=0 \\ \frac{x^2+3}{x} \text{ if } x\neq 0 \end{cases}$

This one is not continuous (at zero). It is "obvious" that the ratio is far from zero when x is close to zero.