Economics 205, Fall 2002: Quiz 3

Instructions. Try to answer all three problems. (Read all of the questions now and start on the ones that seem easiest.) Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing. Please think before you do computations (thinking may enable you to avoid needless computations).

1. Let $f(x) = x^3$ and $g(y_1, y_2) = y_1 + e^{y_1 y_2}$.
   (a) Find the partial derivatives of f and g.
   (b) Find the derivative of the function $f \circ g$.
   (c) Which of the functions f, g, and $f \circ g$ are homogeneous. For any function that is homogeneous, find its degree of homogeneity and verify Euler's Theorem.
   (d) Find the equation of a plane (or translation of a plane) that is tangent to the graph of g at the point (y_1, y_2).
2. For the functions below, find all critical points and decide whether the critical points are local minima, local maxima, or neither.
   (a) $f(x, y, z) = xy + z$.
   (b) $f(x, y) = e^{x-y^2}$.
3. Let $D^2 f(1, 1, 1)$ be the second-derivative of the function f from part a of question 2. Find the eigenvalues and associated eigenvectors of $D^2 f$, state whether $D^2 f$ is diagonalizable. If $D^2 f$ is diagonalizable, find a matrix P such that $P^{-1} D^2 f(1, 1, 1) P$ is a diagonal matrix.