Economics 205, Fall 2002: Quiz 2, Possible Answers

Comments. 36 possible points, 12 points each question. High: 33; Low: 6; Median: 20. There were clear signs that you did not have enough time. Sorry. You will have more time time for Quiz 3. I promise that there will be much less time pressure for the final. Only three people had useful things to say on the first problem. I had hoped that you'd say: If $g(x) \neq 0$, then $\frac{f}{g}$ is well defined. The condition says that the derivative of $\frac{f}{g}$ is not zero, which means the ratio is either strictly increasing or strictly decreasing. But $f(0) = f(1) = 0$ means that the ratio must start and end at 0, so something must be wrong. I had hoped that the second and third problems were straightforward, but several people had trouble. A few wrote parametric equations of the plane in 2(a). This is correct, but I find the standard equation more useful; learn how to write it. A few were confused about what directions really meant: notice that if you understand the questions, parts b and c require no computation. There were lots of mistakes on the third question, which surprised me. If you lost points on this one, please review the answer and try to get a better understanding of the central ideas.

1. Assume $g(x) \neq 0$ for all $x \in (0,1)$. Since $f(x)g'(x) - g(x)f'(x) \neq 0$ for all $x \in [0,1]$ and $f(0) = f(1) = 0$, it must be that $g(0), g(1) \neq 0$. Consequently, $F(x) = \frac{f(x)}{g(x)}$ is continuous on $[0,1]$ and differentiable on $(0,1)$. By assumption, $F(0) = F(1) = 0$, so by Rolle's Theorem, there exists $x^* \in (0,1)$ such that $F'(x^*) = 0$, which is a contradiction because $F'(x) = \frac{f(x)g'(x) - g(x)f'(x)}{g^2(x)}$.

2. (a) The equation of the plane has the form $a \cdot (x - x_0) = 0$, where a is an orthogonal direction and x_0 is a point on the plane. We can take $x_0 = 0$ and w and v to be directions (since 0 is in the plane). So $a \cdot w = a \cdot v = 0$. Solving these equations gives $a = (1, -2, 1)$ (or non-zero multiples).

   (b) The direction of the line is a and w is a point on the line, so the equation is $w + ta = (1,2,3) + t(1,-2,1)$.

   (c) There are many answers to this one. Perhaps the most obvious is simply tw. The important point is to realize is that any direction in the plane is a potential direction for the line.

3. (a) The dimension is 3. A basis is $y^1 = (1,0,0,1), y^2 = (0,1,0,0), y^3 = (0,0,1,0)$. You need to show that $y^i \in S$ for each i (so that the span of the y^i is contained in S; each x^j can be written as a linear combination of the y^i (so that S contains the span of the y^i); and that the y^i are linearly independent. For the first, note $y^1 = .5x^2; y^2 = x^1 - .5x^2; y^3 = x^3 - x^1$; for the second, note $x^1 = y^1 + y^2; x^2 = 2y^1; x^3 = y^1 + y^2 + y^3; x^4 = y^1 + y^3$; and for the third note that $\lambda_1 y^1 + \lambda_2 y^2 + \lambda_3 y^3 = (\lambda_1, \lambda_2, \lambda_3, \lambda_1)$, which is equal to zero only if $\lambda_i = 0$ for $i = 1,2,3$.

   (b) No. From above, a general element of S looks like $(\lambda_1, \lambda_2, \lambda_3, \lambda_1)$, that is the first and last components are equal.

   (c) Yes (set $\lambda_1 = 0, \lambda_2 = \lambda_3 = 1$).