

Economics 205, Fall 2002: Quiz 2

Instructions. Try to answer all three problems. (Read all of the questions now and start on the ones that seem easiest.) Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing. Please think before you do computations (thinking may enable you to avoid needless computations).

1. Let $f(\cdot)$ and $g(\cdot)$ be continuously differentiable functions from \mathfrak{R} to \mathfrak{R} such that $f(x)g'(x) - g(x)f'(x) \neq 0$ for all $x \in [0, 1]$. Prove that if $f(0) = f(1) = 0$, then there exists $x^* \in (0, 1)$ such that $g(x^*) = 0$. Hint: If $g(x) \neq 0$ for all $x \in (0, 1)$, then show that $\frac{f(x)}{g(x)}$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Next use a theorem to reach a contradiction.
2. Let $w = (1, 2, 3)$ and $v = (4, 5, 6)$.
 - (a) Find the equation of the plane that passes through the points w , v and the origin.
 - (b) Find the equation of a line that passes through w and is orthogonal to the plane that you found in part a.
 - (c) Find an equation of a line that passes through w and is contained in plane that you found in part a.
3. Consider the vectors $x^1 = (1, 1, 0, 1)$, $x^2 = (2, 0, 0, 2)$, $x^3 = (1, 1, 1, 1)$, $x^4 = (1, 0, 1, 1)$. Denote by S the span of $\{x^1, x^2, x^3, x^4\}$.
 - (a) Find the dimension of S . Confirm your answer by exhibiting a basis for this set.
 - (b) Is $(1, 0, 0, 2)$ in S ?
 - (c) Is $(0, 1, 1, 0)$ in S ?