Economics 205, Fall 2001: Suggested Answers to Quiz 3

Comments. There were 36 points total (12 points for each question). Class performance: Min: 15; Max: 36; Median: 27. Almost everyone did well on the first question, although many did not find all critical points. Most people who missed the second question wrote the equation for the tangent to the surface $f(x,y)=c$ rather than to the graph of f. These sets are different: the first lives in two dimensional space; the second in $\mathbf{R}^3$. On the third question, a few people wrote that DH was a single number. This is a serious mistake, please talk to me if you do not understand the answer below.

1. (a) $Df(x,y) = \begin{bmatrix} 2xy+y & x^2+3y^2+x \end{bmatrix}$.

   (b) $D_v f(1,1) = Df(1,1)v$. Here, $Df(1,1) = \begin{bmatrix} 3 & 5 \end{bmatrix}$ and so $D_v f(1,1) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$.

   (c) We need $Df(x,y)=0$ or both $(2x+1)y=0$ and $x^2+3y^2+x=0$. The first equation requires either $x=-\frac{1}{2}$ or $y=0$. If $x=-\frac{1}{2}$, the second equation yields $y=\sqrt{\frac{1}{12}}$ or $y=-\sqrt{\frac{1}{12}}$. If $y=0$, the second equation yields $x=0$ or $x=-1$. Hence there are four critical points $(-1,0),(0,0),(-\frac{1}{2},\sqrt{\frac{1}{12}}),(-\frac{1}{2},-\sqrt{\frac{1}{12}})$.

   (d) The matrix of second derivatives is $H(x,y) = \begin{bmatrix} 2y & 2x+1 \\ 2x+1 & 6y \end{bmatrix}$. At the critical points we have: $H(0,0) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $H(-1,0) = -H(0,0)$ (both indefinite), $H(-\frac{1}{2},\sqrt{\frac{1}{12}}) = -H(-\frac{1}{2},-\sqrt{\frac{1}{12}}) = \sqrt{\frac{1}{12}}I$. So $(-\frac{1}{2},\sqrt{\frac{1}{12}})$ is a local min, $(-\frac{1}{2},-\sqrt{\frac{1}{12}})$ is a local max, and the other two critical points are neither.

2. The equation of the plane is:

$$z - f(2,-1) = Df(2,-1)\begin{bmatrix} x-2 \\ y+1 \end{bmatrix} = -5x+9y+19.$$

   Therefore, since $f(2,-1)=-7$, the expression becomes:

$$-5x+9y-z=-12.$$

3. By the chain rule it suffices to compute $Df(g(0,10))Dg(0,10)$. Use the definition of g to find that $g(0,10)=(11,10)$. $Df(x,y) = \begin{bmatrix} \frac{1}{y} & -\frac{x}{y^2} \end{bmatrix}$. So $Df(11,10) = \begin{bmatrix} .1 & -.11 \end{bmatrix}$ and $Dg(0,10) = \begin{bmatrix} 4 & 1 \\ 0 & 0 \end{bmatrix}$. It follows that the derivative of the composite function is [.4 .1]. (The composite function is a real-valued function of two variables; say H(u,v). I have shown that $D_1H(0,10)=.4$ and $D_2H(0,10)=.1$.)