Economics 205, Fall 2001: Suggested Answers to Quiz 2

Comments. There were 36 points total (9 points for each question). Class performance: Min: 13; Max: 36; Median: 30. Several people either misunderstood Part b of Question 1 or do not understand the relationship between directions and equations of lines and planes. On Question 2a, I made a deduction for people who asserted that the matrix wasn't diagonalizable because it did not have distinct eigenvalues and wasn't symmetric. Symmetry and distinct eigenvalues are sufficient for diagonalizability but not necessary as you can see from $\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \end{bmatrix}$.

1. Let $w = (1, 4, 0)$ and $v = (1, 1, 1)$.
   (a) $L(t) = w + tv$.
   (b) $v \cdot (x - w) = 0$ or $x_1 + x_2 + x_3 = 5$.
   (c) Here you must describe the set of x that satisfy both $x_1 + x_2 + x_3 = 5$ and $2x_1 + x_2 - x_3 = 0$. You can write the solution as, for example, $x_1 = -5 - 2x_3, x_2 = 10 - 3x_2$ or, parametrically: $L(t) = (-5, 10, 0) + t(2, -3, 1)$ (this formulation takes the solution and sets $t = x_3$). There are many equivalent answers to this question.

2. (a) Eigenvalue 2 (with multiplicity 2) (you can tell this by observation because the matrix is triangular). This matrix is not diagonalizable. If it were, then there exists P such that $P \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} P^{-1} = 2I$ (because $2I$ is the diagonal matrix with eigenvalues along the diagonal), which in turn implies that $\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} = 2I$, a contradiction.
   (b) This matrix is symmetric, so it must be diagonalizable. If you do the algebra, you find eigenvalues $1 + \sqrt{10}$ and $1 - \sqrt{10}$ with associated eigenvectors $(1, \sqrt{10} - 3)$ and $(3 - \sqrt{10}, 1)$. Since one eigenvalue is positive and the other is negative, this matrix is indefinite.
   (c) This matrix is symmetric, so it must be diagonalizable. Since only the diagonal element in row and column 1 is non-zero, you can see quickly that one eigenvalue is four and a corresponding eigenvector is $(1, 0, 0)$. You get the other eigenvalues by solving $(4 - \lambda)^2 - 4 = 0$ so $\lambda = 2$ or $\lambda = 6$. Corresponding eigenvectors are $(0, 1, -1)$ and $(0, 1, 1)$. Since this matrix has all positive eigenenvalues, it is positive definite.

3. Above. Notice that the question asked you to select a **symmetric** matrix from the previous question.

4. a_1, a_2, and a_3 are linearly independent. To see this, you can show that the determinant of the 3×3 matrix with columns a_i, $i = 1, 2, 3$ is zero, or directly show that the only way to solve $\sum_{i=1}^{3} \lambda_i a_i = 0$ is to set each $\lambda = 0$. In order to find a_4 in terms of the first three a_i you must solve for the equations: $a_4 = \sum_{i=1}^{3} \beta_i a_i$ for β_i. I get $\beta_1 = 3, \beta_2 = 1, \beta_3 = -3$. Any subset of 3 of the a_i is a basis for S, so there are alternative answers to this question.