Economics 205, Fall 2001: Quiz 2

Instructions. Try to answer all four problems. (Read all of the questions now and start on the ones that seem easiest.) Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are better than nothing. Please think before you do computations (thinking may enable you to avoid needless computations).

1. Let $w = (1, 4, 0)$ and $v = (1, 1, 1)$.
   (a) Find the equation of the line that passes through the point w in the direction v.
   (b) Find the equation of the hyperplane that contains the point w and is orthogonal to the line you found in part a.
   (c) Find an equation of a line that is contained in the intersection of the plane with equation $2x_1 + x_2 - x_3 = 0$ and the plane you found in part b.

2. State which of the matrices below are diagonalizable. You need not diagonalize the matrices, but you must justify your answer.
   (a) $\begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$
   (b) $\begin{bmatrix} 4 & 1 \\ 1 & -2 \end{bmatrix}$
   (c) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

3. Pick one of the symmetric matrices above and state whether it is positive-definite, negative-definite, positive semi-definite, negative semi-definite, or indefinite.

4. Let S be the set spanned by a_1, a_2, a_3, a_4, where $a_1 = (1, 2, -2)$, $a_2 = (0, -1, 1)$, $a_3 = (0, 0, -5)$ and $a_4 = (3, 5, 10)$.
   (a) Select a basis from a_1, a_2, a_3, a_4.
   (b) Express the remaining vectors in a_1, a_2, a_3, a_4 as a linear combinations of the chosen basis.