Economics 205, Fall 2001: Suggested Answers to Quiz 1

Comments. There were 36 points total (12 points for each question). Most people could handle the differentiation, although 1(b) caused several people to claim that the derivative of $x^{f(x)}$ was something like $f'(x)x^{f(x)-1}$, which is a big mistake (it treats x being raised to a power as if it were a constant). Most people had the right idea on question 2, although few wrote complete answers. On this problem several people claimed that the function was continuous on sets of the form $[n, n+1)$, for integer n. But the union of these sets is the entire line. The function is not continuous at any integer. Finally, a few people drew suggestive pictures for question 3, but only two wrote sensible proofs. Range of scores: High 33, low 8, median 23. Scores of roughly 15 and above demonstrate basic proficiency. I am a bit concerned about lower scores.

1. Let f be a differentiable function. Calculate the derivative of the function h defined in each of the problems below:

   (a) $h'(x) = f'(x+3)$. This follows from the chain rule ($f(x+3)$ is the composition of $f(y)$ and $y = x+3$).

   (b) $h'(x) = x^{f(x)}[f'(x)\log x + \frac{f(x)}{x}]$. This follows from the definition of exponential ($h(x) = e^{f(x)\log x}$), the chain rule, and the rule for differentiating products.

   (c) $h'(x) = \frac{-3x^2+2x+3}{(x^2+1)^2}$. This follows from the rule for differentiating products (or quotients) and some routine simplification.

   (d) $h'(x) = 2\frac{\log x}{x}$, by the chain rule.

2. $f(x) = [x]$ is differentiable (and therefore continuous) for all x that are not integers and discontinuous at every integer. If n is an integer, and $n < x < n+1$, then $f(y) = n$ for all $n < y < n+1$. Hence, letting $\delta = \min\{x-n, n+1-x\} > 0$, if $|x-y| < \delta$, then $f(y) - f(x) = 0$. It follows that f is continuous at x and that f is differentiable at x and $f'(x) = 0$. When x is an integer, $f(x) \geq f(y)+1$ for all $y < x$, so f cannot be continuous at x (the definition of continuity fails for all $\epsilon < 1$).

3. Let $g(x) = f(x)-x$. $g(\cdot)$ is continuous on $[0,1]$ (because it is the difference of continuous functions). Because $f(\cdot)$ takes values in $[0,1]$, $g(0) = f(0) - 0 \geq 0$ and $g(1) = f(1) - 1 \leq 0$. It follows from the intermediate value theorem than there exists $x^* \in [0,1]$ such that $g(x^*) = 0$. It follows from the definition of $g(\cdot)$ that $f(x^*) = x^*$.