

Economics 205, Fall 2001

Quiz I

August 31, 2001

**Instructions.** Try to answer all three questions. (Read all of the questions now and start on the ones that seem easiest.) Make your answers as complete and rigorous as possible. In particular, give reasons for your computations and prove your assertions. Informal and intuitive arguments are (much) better than nothing.

1. Let  $f$  be a differentiable function. Calculate the derivative of the function  $h$  defined in each of the problems below:
  - (a)  $h(x) = f(x + 3)$
  - (b)  $h(x) = x^{f(x)}$  for  $x > 0$ .
  - (c)  $h(x) = \frac{3x-1}{x^2+1}$
  - (d)  $h(x) = [\log x]^2$ , for  $x > 0$
  
2. The greatest integer function,  $f(x) = [x]$ , assigns to each  $x$  the greatest integer less than or equal to  $x$ . (So, for example,  $[3] = 3$ ,  $[2.5] = 2$ ,  $[-2] = -2$ ,  $[-3.5] = -4$ .)
  - (a) Graph the function  $f(x) = [x]$ .
  - (b) Find the set of points where  $f(\cdot)$  is continuous. (Justify your answer.)
  - (c) Find the set of points where  $f(\cdot)$  is differentiable. (Justify your answer.)
  - (d) Compute  $f'(x)$  whenever it exists.
  
3. Let  $f : [0, 1] \rightarrow [0, 1]$  be continuous. Prove that there exists  $x^* \in [0, 1]$  such that  $f(x^*) = x^*$ . (For intuition, draw a picture.)