

Econ 172A, Fall 2007: Problem Set 2, Suggested Answers

1. See excel spreadsheet. The numerical answers are there, but you should make sure that you can explain how to get them.
2. The proposed change is like changing the “right hand side” of a constraint. There are really two cases: if $x_1^* > 0$, then the constraint is not binding. This means that as long as $x_1^* \geq d$, neither the solution of the problem nor the value will change. If $x_1^* = 0$, then in general moving to $d > 0$ will change the solution and lower the value and moving to $d < 0$ will change the solution and raise the value. In either of these cases it is possible that the value won't change. (In the first case, you are losing the ability to set $x_1^* = 0$. In the second case, you are gaining the ability to let x_1^* be negative.) When the value changes, I would expect $x_1 = d$ to hold in the new solution, but otherwise I do not know what the solution will be. I do know something about the new value. The reduced cost plays the role of the shadow price (or dual variable) associated with a nonnegativity constraint. Hence the value will change by d times the reduced cost associated with the first constraint in the dual.
3. See the supplementary formulation answers for a formulation.
 - (a) You produce 133.33 cans of each mixture and make a profit of \$440.
 - (b) This change is within the allowable range, so the solution stays the same. Profit goes up by 10 cents for each cheap mixture sold, so it is now \$453.33.
 - (c) This change is outside of the allowable range (the coefficient goes down by 40 cents, which is greater than the allowable decrease of 39.09 cents). You know that profit will fall, but you don't know the details unless you solve the problem again. I solved the problem again using Excel and I got: produce 41.67 cans of the cheap mixture and 333.33 cans of the party mixture (and no deluxe). Profit is now \$387.50. (Check that if you didn't change your production plan profits would be \$386.67.)
 - (d) This change is within the allowable range, the shadow price of peanuts is 1.02, so profit falls by this much times 20, or profit falls by \$20.44 (to \$419.56).
 - (e) This change is within the allowable range, the shadow price is \$1.91 per pound, so the extra almonds are worth \$19.10.
 - (f) Almonds and cashews are both worth \$1.91 per pound, to make a twelve ounce can of alshews you need \$1.91 times .75 (12 ounces is three quarters of a pound) worth of material, so a bit more than \$1.43 per can is the break even point.
 - (g) The original solution does not satisfy this extra constraint, so we must resolve the problem. The solution (on Excel) is: 100, 206, and 84.8 cans of cheap, party, and deluxe mixes respectively. Profit is now: \$427. (I rounded these answers.)
 - (h) The original solution satisfies this extra constraint, so the solution (and value) does not change.
 - (i) There are several ways to approach this problem. One method is to introduce 3 new variables, y_i is the amount of miracle nut used to replace ingredient i . The sum of all of the y_i is the amount of miracle nut that you use. This means that the new problem has three new, non-negative variables; the sum of these variables must be no more than 100 pounds; and that the right-hand sides of the original constraints go up by y_i . If the allowable increase of the right-hand side variable with the largest shadow price was at least 100, then you could solve the problem without additional computation. (If you had just 15 pounds of miracle nut, then you would use it instead of almonds.) In fact, since the price of almonds and cashews is equal, you know that the first 28.289 pounds of the new nut will replace cashews and almonds.

After that, however, your production plan changes. So I used Excel. I learned that the solution involves making 155.5555556, 0, 377.7777778 cans of cheap, party, and deluxe mix respectively and using the miracle nut as a substitute for 65 pounds of cashews and 35 pounds of almonds. Profit becomes \$631.11. Notice that profit goes up by \$191.11, which is 100 times the shadow price of the most valuable resource. I was not able to predict that from the sensitivity and answer reports.