

Linear Programming Notes IV: Solving Linear Programming Problems Using Excel

1 Introduction

Software that solves moderately large linear programming problems is readily available. You can find programs on the internet. Software comes with many textbooks. Standard spreadsheet programs often can solve linear programming problems. In this course, we will use the solver tool found on Excel to solve problems. I request that you use this program. If you would strongly prefer to use other software, then talk to me.

You can find Excel on the machines in the computation lab in Econ 100. You may have access to Excel (it comes bundled with Microsoft Office). If you do, then use it. The capacity to solve linear programs is only available if you install the solver tool. If you find that the solver is not available on your machine, you must either re-install Excel or use the computation lab.

These notes explain how to use Excel to solve linear programming problems. I will assume that you know the basics of using a computer in the computation lab. That is, you must know how to log on, locate and start Excel, save your work, print your work, and log out. (One pointer: If you use the machines in Econ 100, you must know that your account is on iacs5 in order to log on.)

2 Creating a Spreadsheet

This section explains how to set up a spreadsheet to solve LPs. On the class webpage there are companion .pdf files that contain an Excel spreadsheet that I have created to illustrate the procedure. You should refer to that spreadsheet when you read these notes. The webpage also contains the same spreadsheet in .xls format. You can copy this and use it as a template spreadsheet for class assignments or your own experimentation. In any event, you need not prepare a new starting spreadsheet for each program. (When you do a homework assignment, make sure to type in the assignment number, the problem you are solving, and your name at the top of the page. If you use my template, it would be a good idea to erase my name before handing in the assignment.)

I will describe how to set up and solve the basic linear programming example that I solved in class. This problem has four variables. The variables are constrained to be non-negative and there are three additional constraints. The

problem is to maximize $2x_1 + 4x_2 + 3x_3 + x_4$ subject to:

$$\begin{array}{rccccrcr} 3x_1 & + & x_2 & + & x_3 & + & 4x_4 & \leq & 12 \\ x_1 & - & 3x_2 & + & 2x_3 & + & 3x_4 & \leq & 7 \\ 2x_1 & + & x_2 & + & 3x_3 & - & x_4 & \leq & 10 \\ & & & & & & x & \geq & 0 \end{array}$$

I set up the sample template to deal with a problem of this size (four variables and three resource constraints). You would need to make adjustments to deal with smaller or larger problems. The boxes that have dark borders are the ones in which you enter numbers. The coefficients of the objective function (the c_j) go in cells E5 through H5. The resource constraints (the b_j) go in K13 through K15. The technological coefficients (the matrix A) go in the block from E to H and 13 to 15. In addition, the cells E8 through H8 are reserved for values of the variables. The program will find the best values for these x_j and put them in these cells. B18 contains a formula. It is the formula for the value of the objective function. If you did not have four variables you would need to adjust the range of entries in the formula so that it would properly compute $\sum_{j=1}^n c_j x_j$. For reference, I wrote the formula that appears in cell B18 on the spreadsheet (starting in A20). A20 does not enter into any of the computations; it is just a comment. Similarly, I13 through I15 contain formulas. These formulas compute the left hand side of a resource constraint. I wrote the formula that appears in I13 for reference in A21. I14 and I15 contain similar formulas. You need not retype them. If you copy I13 by dragging downward, clever Excel makes the necessary changes in the formula. (The dollar signs in this formula tell Excel not to adjust the cell number when it copies.) If you had additional resource constraints, you would just use more rows in the spreadsheet. If you had a different number of variables, you would need to adjust the range of the variables that appear in the formulas. Finally, L13 through L15 contain a formula that gives you the difference (in absolute value) between the right and left hand side of each constraint. When you solve the problem, this difference must be zero for equality constraints. Otherwise, a strictly positive number corresponds to a non-binding constraint. I have not discussed J13-15. This column reports the way in which you compare the left-hand side of the resource constraint to the right-hand side ($=$, \leq , or \geq). The computer program lets you enter any of the three comparison operators (and different constraints can use different comparisons).

Once you have a template, you can create copies of it. Go to the pull down Edit menu at the top of the Excel page. Click on Move or Copy Sheet . . . Under To Book: Book1 (or some other destination). Under Before Sheet: Highlight Template (or whatever you call your template). Put a check mark in Copy. Click OK. This creates a copy of the template. Excel calls it Template (2). The name of the sheet is on a tab at the bottom of the sheet. You can change the name by putting the cursor on the tab, right clicking, clicking on Rename . . . , and entering the new name.

I renamed the copy Example 1. The spreadsheet is now in perfect shape to solve a problem with 4 variables and 3 resource constraints. If your problem has more constraints, you can enter them below the ones in the problem (after line 15). Make sure to copy the formulas for the left-hand side and the slack. If your problem has more variables, then you need to adjust all of the formulas to include extra terms in the sums.

A comment about spreadsheets. Cells in Excel spreadsheets can contain words (like A1). I use these as comments. The cells can contain numbers. Some of these cells contain the basic data of the problem (the A , b , and c). Some of the cells contain formulas. You enter the formula; Excel inserts a value into the cell using the formula. To enter a formula in a cell put the cursor on the cell and then type something that starts with an equal sign. The formulas that you use will compute linear functions - products of given coefficients and variables. Excel does this using the “sumproduct” function. For example, when you write

$$= \text{SUMPRODUCT}(E5:H5,E8:H8)$$

Excel computes:

$$(E5)(E8) + (F5)(F8) + (G5)(G8) + (H5)(H8),$$

where (E5) is the number in cell E5 of the spreadsheet. Excel will find some way to whine at you if the formula is improperly configured (for example, if there are four cells to the left of the comma and three cells to the right; or if the entry in any one of the cells is not a number). Numbers should appear in cells that contain formulas. These numbers are the value obtained if you plug in current referenced values from other parts of the spreadsheet.

3 Solving the Problem

On this spreadsheet I entered the data from the example. You may also enter guesses for the variables in E8 through H8; if you do not put anything there, Excel will assign the initial value of zero to each variable.

Once you have entered the data (and adjusted the formulas, if necessary), you are ready to solve.

Go to the Tools column on the menu bar at the top of the page. Click once to bring down the menu. Click on Solver A Solver Parameter box should appear.

In the Set Target Cell box: Insert B18. (You can do this by clicking on cell B18 of the spreadsheet.) In general, what goes into the Set Target Cell box is the cell in your spreadsheet that has the formula for your objective function.

Put the bullet in the circle to the left of max (of course, if you were solving a minimization problem, you would put the bullet to the left of min).

By changing cells: enter E8:H8 (in general, indicate the cells that you have reserved for the variables).

To the right of the “Subject to the Constraints” box, click on Add An Add Constraint Box should appear. Enter each resource constraint. For this

problem, I typed "I13:15" in the left hand box (under cell reference); I made sure that the middle cell showed the operator "<=" (that is, \leq); I typed "K13:15" in the right hand box (below and to the right of "Constraint:"). I then clicked OK. I then clicked add again and entered the non-negativity constraints by putting "E8:H8" in the left; ">=" in the center; and "0" in the right. I clicked OK again. There are other ways to enter this information. The important thing to note is that you should enter each constraint and that the Add Constraint Box lets you designate the constraint as an equation or either kind of inequality.

You need to tell Excel to treat the program as a linear model. To do this, click on Options Put a check mark next to "Assume Linear Model" (you can ignore the rest of the box). Click OK.

Now you can try to get the solution to your problem by clicking on Solve.

The Solver Results box now appears. You should place a bullet to the left of "Keep Solver Solution" and highlight "Sensitivity" in the Reports list of options on the right. Click on OK.

You now have a spreadsheet that contains the answer to the problem and another sheet that has sensitivity results. You should be able to interpret the answer page: the spreadsheet now tells you the solution (x_* in E8 through H8) and the value of the problem (in B18). The spreadsheet also tells you which of the constraints were binding and the amount of slack in the non-binding constraints. (This information is in L11:13.) The sensitivity report contains a lot of useful information that you'll learn about later.

At this point you can print out and save your results. Sometimes the output looks nicer if you print in using the landscape option rather than the portrait option. Click: File, then Page Setup, then highlight the bullet to the left of Landscape if you think that your results would look better in that format. Similarly by going through: File, Page Setup, Sheet, you can check boxes that will (or will not) print the grid lines and the row and column headings.

If all has gone well, then you can reproduce the answer to the Example that we found using direct computation. My spreadsheet (Example 1) does contain the data from the problem and its solution. You can read off the values of the variables (in dark bordered box under x_1, \dots, x_4). You can read off the value and the slack as well. Note that the computer decided that the slack in the first and third constraints was not quite zero. This is due to rounding error. $2.17E - 12$ means 2.17×10^{-12} , which is closer to zero than most of us will ever get.

4 Loose Ends

I have described how to use Excel to solve linear programming problems. There are a couple of details.

4.1 Infeasible Problems

If your feasible set is empty, then Excel will tell you. After you ask Excel to find a solution, the Solver Results Box appears with the statement that “Solver could not find a feasible solution.” Provided that there are no mistakes, this means that the problem is not feasible. There is no solution to find and there is nothing else to do.

4.2 Unbounded Problems

If your problem is unbounded, then Excel will tell you. The Solver Results Box appears with the statement that “The Set Target Cell Values do not converge.” The spreadsheet then provides you with a finite guess for all of the variables, but the guess will have the property that you can increase (at least) one of the variables without bound; maintain feasibility by varying only the variables that have positive values; and increase the objective function without bound.

4.3 Mysterious Failures

Almost certainly something will go wrong at some point. Maybe it is the hardware. Maybe it is the software. Probably you made an incorrect assumption about how the program interprets the data you give it or you just typed a formula or cell address incorrectly. Identifying the problem is frustrating and often time consuming. If you obtain a numerical answer, then I urge you to examine it to see whether it looks sensible. It could be the answer to the wrong problem. If Solver does not work, then go over the data and formulas slowly and carefully. Usually you will find a mistake.

4.4 Sensitivity Analysis

At this point you can generate a report on the sensitivity of your solution, but you cannot interpret this report. Sensitivity analysis is a major topic of the course. I will teach you what this report means soon.