Econ 172A, Fall 2004: Midterm II-A

Instructions.

1. Please check to see that your name is on this page. If it is not, then you are in the wrong seat.
2. The examination has four questions. Answer them all.
3. If you do not know how to interpret a question, then ask me.
4. No justification is required for the first question (each part will be graded on a "right" or "wrong" basis – no partial credit will be awarded). You must justify your answers to the last three questions. Please read and answer these questions carefully.
5. The table below indicates how points will be allocated on the exam.

	Score	Possible
I		10
II		21
III		33
IV		36
Exam Total		100

1. Consider the linear programming problem **P**: $\max c \cdot x$ subject to $Ax \leq b$, $x \geq 0$, where A is a matrix with two columns and three rows. Answer the questions below. You do not need to justify your answers. No partial credit will be awarded of this problem.

   (a) How many variables does the dual of **P** have?

   (b) Suppose that the solution x^* to **P** satisfies $x_1^* > 0$ and the first constraint is binding.

      i. Which constraints **must** bind in the corresponding solution to the dual?

      ii. Which dual variables **must** be positive in the corresponding solution to the dual?

2. Consider the linear programming problem:

   Find x_1 and x_2 to solve **P**:

$$\begin{array}{rrcrcl} \max & x_1 & + & x_2 & & \\ \text{subject to} & x_1 & + & 2x_2 & \leq & 4 \\ & 2x_1 & - & x_2 & \leq & 4 \\ & x_1 & + & x_2 & \leq & 3 \\ & & & x & \geq & 0 \end{array}$$

   You must provide justifications for your answers to the questions below. In particular, say what you need to do to check for feasibility and the basis for your inferences in part (c).

   (a) Write the dual of the problem **P**.

   (b) Verify that $(x_1, x_2) = (2, 1)$ is feasible for **P**.

   (c) Assuming that $(2, 1)$ is a solution to **P**, use Complementary Slackness to determine a candidate solution to the dual.

   (d) Is $(2, 1)$ a solution to **P**? Explain.

3. The following is a formulation problem that appeared on an old midterm.

   The Pallo Winery produces three varieties of wine – red, white, and blue. These products sell for \$15, \$20, and \$25 per case, respectively. Each case of red wine costs \$10 to produce and requires one hour to process and six hours to bottle. Each case of white wine costs \$12 to produce, needs two hours of processing time and eight hours of bottling time. Blue wine costs \$21 per case to produce, uses three hours of processing time, and eight hours in bottling time. Each week there are 220 hours of processing time and 480 hours of bottling time available in the Winery's factories.

   Pallo stores its wine in a warehouse with a capacity of 3000 cubic feet. Each week they ship out their entire inventory. White and red wine occupy 1.5 cubic feet per case. The blue wine occupies two cubic feet per case. Pallo has a contract with a local wine seller to deliver at least 60 cases of red wine each week.

   Here is a formulation of the problem as a linear programming problem. I denote by x_i the number of cases of each variety of wine produced in a week, for i = red, white, and blue. Taking into account the costs of production, the objective function is

   $$\max 5x_R + 8x_W + 4x_B.$$

   In addition to non-negativity constraints, the problem has four other constraints:

   - $x_R + 2x_W + 3x_B \leq 220$ (processing time)
   - $6x_R + 8x_W + 8x_B \leq 480$ (bottling time)
   - $1.5x_R + 1.5x_W + 2x_B \leq 3000$ (warehouse capacity)
   - $-x_R \leq -60$ (contracted delivery of red wine)

   Use this information to answer the questions on the following page.

(a) Write the dual of this problem.

(b) Provide an interpretation of the dual. Your interpretation **must** include: a definition of the dual variables in words, including a specification of the units in which they are measured; an economic interpretation of the meaning of these variables; and interpretations of both the constraints and objective function of the dual.

(c) Does the Pallo wine problem have a solution? (Answer this question as completely as possible. If you can demonstrate that the problem does (or does not) have a solution, then do so. If you need additional information, describe what information you need.)

4. A local bakery makes three different kinds of bread. A loaf of whole wheat bread uses one pound of whole wheat flour and an ounce of yeast. A loaf of oatmeal-rye bread uses three quarters of a pound of white flour, one quarter pound of rye flour, one quarter pound of oatmeal, and an ounce of yeast. A loaf of white bread uses three quarters of a pound of white flour and two ounces of yeast. The bakery can sell a loaf of whole wheat bread for \$2.00, a loaf of oatmeal-rye bread for \$2.50, and a loaf of white bread for \$1.50. Each day the bakery has available 120 pounds of whole wheat flour, 100 pounds of white flour, 50 pounds of rye flour, 30 pounds of oatmeal, and 140 ounces of yeast. In addition, its ovens are able to bake at most 125 loaves each day.

   The bakery wishes to know how many loaves of each type of bread to produce in order to maximize profits subject to the constraints above. In order to formulate the problem, I defined the variables:

   WHOLE = number of loaves of whole wheat bread produced.

   OATRYE = number of loaves of oatmeal-rye bread produced.

   WHITE = number of loaves of white bread produced.

   The bakery's problem is then: find values for WHOLE, OATRYE, and WHITE to solve:

$$
\begin{array}{lrcrcrcr}
\text{max} & 2\text{WHOLE} & + & 2.5\text{OATRYE} & + & 1.5\text{WHITE} & & \\
\text{subject to} & \text{WHOLE} & & & & & \leq & 120 \\
 & & & 0.75\text{OATRYE} & + & 0.75\text{WHITE} & \leq & 100 \\
 & & & 0.25\text{OATRYE} & & & \leq & 50 \\
 & & & 0.25\text{OATRYE} & & & \leq & 30 \\
 & \text{WHOLE} & + & \text{OATRYE} & + & 2\text{WHITE} & \leq & 140 \\
 & \text{WHOLE} & + & \text{OATRYE} & + & \text{WHITE} & \leq & 125
\end{array}
$$

$$
\text{WHOLE, OATRYE, WHITE} \geq 0.
$$

   I solved this problem using Excel. The output follows this problem. Use the output to answer the questions on the next page. Answer the questions independently (so that a change described in one part applies only to that part). You must justify your answers by providing brief (but complete) descriptions of how you arrived at them.

(a) What is the bakery's profit maximizing output? How much does it earn?

(b) What is the most that the bakery would be willing to pay for an additional pound of white flour?

(c) What is the most that the bakery would pay for another pound of oatmeal?

(d) Would the baker produce more oat-rye bread if the price of oat-rye bread doubled?

(e) How much would the profits of the bakery decrease if it were only able to bake 120 loaves each day?

(f) The baker notices that loaves of white bread are smaller than the others, and decides that he could fit two of these loaves in the oven where he could only place one of the other loaves. As a result, the last resource constraint above changes to:

$$\text{WHOLE} + \text{OATRYE} + .5\text{WHITE} \leq 125$$

Is it worthwhile for the baker to produce white bread now?

(g) The baker invents a new recipe for all-grain bread. A loaf of this bread contains one ounce of yeast, and .25 pounds of whole wheat flour, white flour, rye flour, and oatmeal. It requires the same amount of oven space as the other types of bread. How much would the baker need to charge for all-grain bread for it to be profitable to produce it?