Econ 172A, Fall 2004: Midterm I-B

This page is an answer sheet.

Instructions

- Check to make sure that this page has your name and seat number on it.
- Separate this page from the rest of the examination.
- Read the instructions provided on the next page.
- Answer the exam questions.
- Use the form below to record your answers.
- Grading: There are 100 possible points. Question 1 is worth 36 points; you get 3 points for every correct choice that you circle; you get 3 points for every incorrect choice that you do not circle. (So, for example, if all of the choices for 1 (a) are correct, then you get 12 points for circling all of the choices and 6 points for circling any two.) Question 2 is worth 32 points (two points for each correct choice circled; 2 points for each incorrect choice not circled). Question 3 is worth 32 points. Each correct answer is worth four points.

ANSWERS

1. (a) Circle the correct choice or choices: i. ii. iii. iv.
   (b) Circle the correct choice or choices: i. ii. iii. iv.
   (c) Circle the correct choice or choices: i. ii. iii. iv.
2. (a) Circle the correct choice or choices: i. ii. iii. iv.
   (b) Circle the correct choice or choices: i. ii. iii. iv.
   (c) Circle the correct choice or choices: i. ii. iii. iv.
   (d) Circle the correct choice or choices: i. ii. iii. iv.
3. Circle the correct choice:

   (a) **TRUE** **FALSE**
   (b) **TRUE** **FALSE**
   (c) **TRUE** **FALSE**
   (d) **TRUE** **FALSE**
   (e) **TRUE** **FALSE**
   (f) **TRUE** **FALSE**
   (g) **TRUE** **FALSE**

Econ 172A, Fall 2004: Midterm I-B

Instructions.

1. The first page of the exam package is your answer sheet. Make sure that your name and seat number is on this sheet. It is the only page you will hand in.
2. You may use no books, notes, calculators or other electronic devices.
3. The examination has 3 questions. Answer them all by circling the appropriate choices on the answer sheet.
4. There may be 0, 1, 2, 3, or 4 correct choices for every part of the first two questions. Each part will be graded independently as explained on the answer sheet.
5. If you do not know how to interpret a question, then ask me.

1. The following is an array that describes a simplex-algorithm computation.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	1	-1	-10	-1	0	0	0	10
(1)	x_5	10	3	-4	1	1	0	0	45
(2)	x_6	2	-1	0	3	0	1	0	1
(3)	x_7	0	1	1	0	0	0	1	10

The following questions refer to the array above, which was obtained from transforming a problem: $\max x_0$ subject to $Ax \leq b, x \geq 0$ into a form that can be solved using the simplex algorithm.

Remember, any number of choices (from zero to four) can be correct. As described on the answer sheet, you receive credit for each correct choice you select and for each incorrect choice you do not select.

(a) Which of the following arrays would be a correct array for the next step of the computation? That is, which of the following arrays can be obtained from the one above from one pivot that follows all of the rules of the simplex algorithm?

i.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	$\frac{13}{3}$	0	$-\frac{34}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	0	0	25
(1)	x_2	$\frac{10}{3}$	1	$-\frac{4}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	0	15
(2)	x_6	$\frac{16}{3}$	0	$-\frac{4}{3}$	$\frac{10}{3}$	$\frac{1}{3}$	1	0	16
(3)	x_7	$-\frac{10}{3}$	0	$\frac{7}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	1	-5

ii.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	1	9	0	-1	0	0	10	110
(1)	x_5	10	7	0	1	1	0	4	85
(2)	x_6	2	-1	0	3	0	1	0	1
(3)	x_3	0	1	1	0	0	0	1	10

iii.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	1	0	-9	-1	0	0	1	20
(1)	x_5	10	0	-7	1	1	0	-3	15
(2)	x_6	2	0	1	3	0	1	1	11
(3)	x_2	0	1	1	0	0	0	1	10

iv.

Row	*Basis*	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Value*
(0)	x_0	0	-1.5	-10	-2.5	0	$-.5$	0	9.5
(1)	x_5	0	8	-4	-14	1	-5	0	40
(2)	x_1	1	$-.5$	0	1.5	0	.5	0	.5
(3)	x_7	0	1	1	0	0	0	1	10

(b) Which of the arrays in the previous question is a legitimate final array for the simplex algorithm?

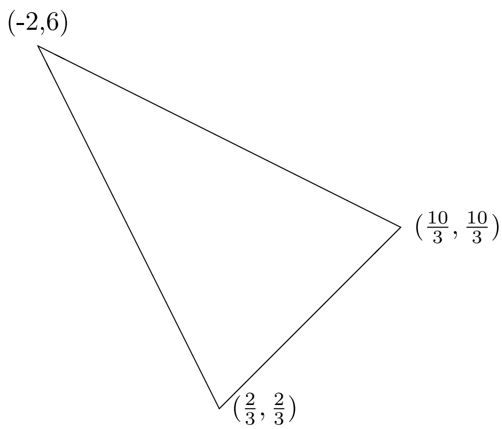
(c) Which of the following vectors $(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ satisfy the constraints of the problem described by the original simplex array?

i. $(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (20, 0, 10, 0, 0, 15, 11, 0)$

ii. $(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (10, 0, 0, 0, 0, 45, 1, 10)$

iii. $(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (100, 1, 1, 9, 0, 68, 0, 0)$

iv. $(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (20, 0, 0, 10, 0, 0, 1, 10)$



2. The following questions relate to the triangle above (and its interior), which I call S. Interpret S as the feasible set of a linear programming problem.

   Remember, any number of choices (from zero to four) can be correct. As described on the answer sheet, you receive credit for each correct choice you select and for each incorrect choice you do not select.

   (a) The triangle above (and its interior) is described by which of the following sets of linear inequalities.

   i. $$\begin{array}{rcrcr} x_1 & + & 2x_2 & \leq & 10 \\ x_1 & - & x_2 & \geq & 0 \\ 2x_1 & + & x_2 & \geq & 2 \end{array}$$

   ii. $$\begin{array}{rcrcr} x_1 & + & 2x_2 & \leq & 10 \\ x_1 & - & x_2 & \geq & 0 \\ 2x_1 & + & x_2 & \geq & 2 \\ x_1 & & & \geq & 0 \end{array}$$

   iii. $$\begin{array}{rcrcr} x_1 & + & 2x_2 & \leq & 10 \\ x_1 & - & x_2 & \leq & 0 \\ 2x_1 & + & x_2 & \geq & 2 \\ & & x_2 & \geq & 0 \end{array}$$

   iv. $$\begin{array}{rcrcr} x_1 & + & 2x_2 & \leq & 10 \\ x_1 & - & x_2 & \leq & 0 \\ 2x_1 & + & x_2 & \geq & 2 \end{array}$$

Consider the problem of finding x_1 and x_2 to solve the linear programming problem $\max x_0$ subject to $(x_1, x_2) \in S$, where x_0 is a linear function of x_1 and x_2 that is not constant (so $x_0 = Ax_1 + Bx_2$ and at least one of A and B is not zero). Call this problem **P**.

(b) For which of the following specifications of x_0 does the problem **P** have a **unique** solution?

i. $x_0 = x_1$

ii. $x_0 = x_1 + .5x_2$

iii. $x_0 = -2x_1 - x_2$

iv. $x_0 = x_1 - x_2$

(c) Suppose that x_0 is a function with the property that $(\frac{10}{3}, \frac{10}{3})$ solves **P**. For which of the following functions y_0 **must** it be the case that $(\frac{10}{3}, \frac{10}{3})$ solves $\max y_0$ subject to $(x_1, x_2) \in S$?

i. $y_0 = x_0 + x_2$

ii. $y_0 = x_0 + x_1$

iii. $y_0 = 5x_0$

iv. $y_0 = -x_0$

(d) For which of the following pairs of points is it possible to find a non-constant x_0 such that both points solve **P**?

i. $(\frac{10}{3}, \frac{10}{3})$ and $(0, 5)$

ii. $(\frac{10}{3}, \frac{10}{3})$ and $(\frac{2}{3}, \frac{2}{3})$

iii. $(\frac{2}{3}, \frac{2}{3})$ and $(-2, 6)$

iv. $(\frac{2}{3}, \frac{2}{3})$ and $(0, 5)$

3. For each of the statements below, circle **TRUE** on your answer sheet if the statement is always true, circle **FALSE** otherwise. No justification is required.

(a) If a linear programming problem is not feasible, then it will continue to be infeasible if the objective function changes.

The next six parts refer to the linear programming problem (P) written in the form:

$$\max c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

and its dual (D):

$$\min b \cdot y \text{ subject to } yA \geq c, y \geq 0$$

(b) If (P) has a unique solution, then its dual has a unique solution.

(c) If a linear programming problem has more constraints than variables it is not feasible.

(d) If x^* is a solution to (P), then rx^* will be a solution to

$$\max c \cdot x \text{ subject to } Ax \leq rb, x \geq 0 \quad \text{for any } r > 0.$$

(e) If (P) has a solution, then

$$\max c \cdot x \text{ subject to } Ax \leq \bar{b}, x \geq 0$$

has a solution ($\bar{b}$ may be different from b).

(f) If (P) has a solution and $c' \leq c$, then

$$\max c' \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

has a solution.

(g) If (P) has a solution, then the dual of the problem:

$$\max 4c \cdot x \text{ subject to } Ax \leq b, x \geq 0$$

is feasible.

(h) If (P) has a solution, x^*, then there exists a solution to (D), y^*, such that $y^* = x^*$.