Econ 172A, Fall 2004: Problem Set 3
Possible Answers

1. (a) Rows reduced:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Ann	112	196	0	182
Bob	180	156	0	228
Cindy	39	260	0	91
Dave	105	390	0	315

Columns reduced:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Ann	73	40	0	91
Bob	141	0	0	137
Cindy	0	104	0	0
Dave	66	234	0	224

We can cover all of the zeros with three lines (accounting column; Bob and Cindy rows), so we do not have a solution. Forty (Ann's row and Clerical column) is the minimum non-covered value.

Adjust the table:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Ann	33	0	0	51
Bob	141	0	40	137
Cindy	0	104	40	0
Dave	26	194	0	184

You get this new table by first subtracting 40 from every entry and then adding 40 to any row or column that was covered in the earlier step. When you are done, you get an equivalent table. Now you can still cover all of the zeros with three lines (Cindy's row and the middle two columns). The minimum uncovered number becomes 26. Form another table:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Ann	7	0	0	25
Bob	115	0	40	111
Cindy	0	130	66	0
Dave	0	194	0	158

At this point we are done: Bob must be assigned to Public Relations (the only zero in his row). That means that Ann must be assigned to Accounting. Dave must do clerical and Cindy Administration. The total cost is 378 + 540 + 559 + 480 = 1957.

(b) This just adds 15 minutes to the total time, but does not change the assignment.

(c) This just adds 30 minutes to the total time, but does not change the assignment.

(d) We need to recompute the solution.

Rows reduced:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Alice	56	98	0	91
Bob	180	156	0	228
Cindy	39	260	0	91
Dave	105	390	0	315

(This is the same as before, except that the first row is cut is half.)
Columns reduced:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Alice	17	0	0	0
Bob	141	58	0	137
Cindy	0	162	0	0
Dave	66	292	0	224

We can cover all of the zeros with three lines (accounting column; Alice and Cindy rows), so we do not have a solution. Fifty eight (Bob's row and Clerical column) is the minimum non-covered value.
Adjust the table:

Person	*Clerical*	*PublicRelations*	*Accounting*	*Administration*
Alice	17	0	58	0
Bob	83	0	0	79
Cindy	0	162	58	0
Dave	8	234	0	166

At this point we have a matching: Alice- Administration, Bob-PR, Cindy-Clerical, Dave-Accounting, with cost $280 + 540 + 507 + 375 = 1702$.

2. (a) The pure strategy security level of Row is 2; for Column it is a payoff of 4 (to Row). The game therefore has a mixed-strategy equilibrium. The equilibrium strategy for Row is to play up with probability $\frac{4}{7}$ and down with probability $\frac{3}{7}$; column plays left with probability $\frac{2}{7}$ and right with probability $\frac{5}{7}$. The value of the game is $\frac{20}{7}$.

   (b) The pure strategy security level of Row is 1; for Column it is a payoff of 3 (to Row). Therefore the equilibrium is in mixed strategies. We can ignore Column's left strategy; it is dominated. You can compute the value as 2.8. The equilibrium mixed strategy for Row is $(.2, .8)$ the equilibrium mixed strategy for Column is $(0, .1, .9)$. (Notice that the value is between the pure-strategy security levels.)

   (c) Once again, there is no pure-strategy equilibrium. Row's security level is 0; Column's is 4. No strategies are dominated. We can still figure out what Row's mixed strategy security level is. If Row plays UP with probability p, he will get (no less than) the minimum of $\{5p, 2p + 4(1 - p), 5(1 - p)\}$. When $p > .5$, the smallest of these is $5(1 - p)$; when $p < .5$ the smallest of these is $5p$. To maximize the minimum, Row picks $p = .5$ and guarantees the value 2.5. (You may see this more easily from a carefully drawn graph.) Notice that Column does not use her middle strategy when she holds Row down to the minimum. Hence her equilibrium mixture will only involve Left and Right. It is not hard to check that an equal mixture does the job. Summary: value 2.5; Row's strategy: $(.5, .5)$. Column's strategy $(.5, 0, .5)$.

3. (a) I used excel and obtained the strategy $(0.124137931, 0.158620690.717241379, 0)$ for Column, with a value of 8.393103448. (Notice that the way that I set up the problem, I computed the strategy of the column player.) You can resolve to get the row player's strategy, but Excel's solution shows that the player one won't use her top strategy, because it has a payoff that is strictly less than all other payoffs.

   (b) Doubling the entries changes the value, but it does not change the equilibrium strategies. It is just a change of units without strategic consequences.

   (c) Increasing the payoffs is as if player 2 must pay player 1 a fixed fee to play the game. It doesn't change equilibrium strategies (but increases the value of the game by 5).

   (d) Increasing the payoff in Row 1 by a small amount will not change the value or the equilibrium. As it stands, player one gets only 5.793103448 by playing his top strategy. Since the value of the game exceeds this by more than 2, there is no way that increasing top-row entries will matter unless one cell goes up by more than two.

   (e) Now the change could make a difference. In the attached spreadsheet, I increased the row 3, column 3 entry by 2 and the equilibrium strategies and values changed.

   (f) The answers to (b) and (c) are general (multiplying payoffs by a positive constant or adding a constant to all payoffs does not change equilibrium strategies for the reasons indicated). As for (d), if you just increase one payoff, eventually the other player will avoid the strategy that enables his opponent to get the payoff.

   (g) If K is greater than the value of the original game, then Row will play it with probability one. Column can continue to use the equilibrium strategy from the given game. If K is less than on equal to the value of the original game, then the value and the equilibrium strategies remain unchanged. These claims follow from the definition of security levels and of value.