Econ 172A, Fall 2004: Problem Set 2

Possible Answers

1. Let x be number of days per week mine X is operated and y the number of days per week mine Y is operated.

   We need:

   (a) $6x + y \geq 12$ (high)

   (b) $3x + y \geq 8$ (medium)

   (c) $4x + 6y \geq 24$ (low)

   (d) $x, y \leq 7$ (week has only seven days)

   (e) $x, y \geq 0$ (week has non-negative number of days)

   The first three constraints guarantee that you make at least as much of each type of ore. The final two constraints make sure that the variables x and y really are capable of describing work weeks.

   The objective is to find x and x to minimize cost, $2x + 4y$.

2. The solution is to set $x = 6$ and $y = 0$ (cost \$12,000 per week)

3. (a) This changes the length of week constraint by lowering 7 to 5. The change is within the allowable range from Mine Y, but not for Mine X. Consequently, we must resolve the problem. Cost goes up to 12.667, with $x = 5$ and $y = .667$. (You really could have guessed this one: You operate Mine X to capacity, then use Mine Y to make sure you satisfy production demands.)

   (b) Nothing. This change is within the allowable range. You are already supplying 36 tons of high-quality ore. Only if demand exceeds 36 would you need to change the production plan.

   (c) Nothing. This change is within the allowable range.

   (d) This change is within the allowable range, but since the constraint is binding, you do change your production quantities: You must increase production in Mine X to meet the increased demand for low-quality ore. Your cost goes up by the change times the shadow price: a total of 1 (\$1,000 per week).

   (e) This increases the demand by six, which is outside of the allowable range. We know that cost will go up, but the basis will change and we need to resolve the problem to figure out how. (Actually, the problem is simple. You'll expand operations at Mine X until you reach capacity, and then use Y production to meet excess demand. Excel provides the solution: $x = 7$, $y = \frac{1}{3}$ with value 15.333.)

   (f) This change lowers costs, but is outside of the allowable range. The new solution (from Excel) is: $x = 1.3125, y = 4.125$ with cost equal to 6.75.

   (g) Five units of low-grade ore is worth \$2,500 per day (because the shadow price associate with low-grade ore is .5). Consequently, having the new mine lowers costs. The precise production plan requires solving another linear programming problem. I think that the solution involves $x = z = \frac{8}{3}$, $y = 0$ and cost

4. The simplest example probably is: $\max x_0$ subject to $x = 1$. If $x_0 = x_1$, the dual variable is 1; if $x_0 = 0$, the dual variable is 0; $x_0 = -x_1$, the dual variable is -1. (In general, if $x_0 = Ax_1$, the dual variable is A.) The dual variable tells you how much it is worth to you to change the right-hand side of the constraint. In this simple problem, the value is equal to the right-hand side.