Econ 172A, Fall 2004: Problem Set 2

Due: November 16, 2004, in class (no late papers)

paragraphNote: Although this assignment is due on November 16, it covers material that will be on the second midterm (November 9). Plan accordingly.

1. A Mining Company owns two different mines that produce an ore which, after being crushed, is graded into three classes: high, medium and low-grade. The company has contracted to provide a smelting plant with 12 tons of high-grade, 8 tons of medium-grade and 24 tons of low-grade ore per week. The two mines have different operating characteristics as detailed below.

Mining Problem				
Mine	Cost Per Day ($1,000)	Production (tons per day)		
		High	Medium	Low
X	2	6	3	4
Y	4	1	1	6

   How many days per week should each mine be operated to fulfill the smelting plant contract at minimum cost? Formulate a linear programming problem to describe this problem. Make sure to define your variables in words and explain how your formulation describes all of the relevant constraints.

2. Solve Problem 1 using Excel (you could solve this one by hand, but you'll need the sensitivity report to answer the next question).

3. Answer the following questions about variations on the mine problem. It is possible to answer all of the questions by modifying the information you entered into Excel, but try to answer as many as possible without doing this. (I can ask similar questions on exams and you won't have the computer.)

   (a) How does the problem and its solution change if the mines can only be open five days per week?

   (b) What would happen to the solution and its value if the demand for high-quality ore doubled?

   (c) What would happen to the solution and its value if the demand for medium-quality ore was reduced by 2 tons per week?

   (d) What would happen to the solution and its value if the demand for low-quality ore increased by 2 tons per week?

   (e) What would happen to the solution and its value if the demand for low-quality ore increased to 30 tons per week.

   (f) What would happen to the solution and its value if the cost of operation of Mine Y decreased to $1,000 per day?

   (g) Mine Z becomes available. It produces five tons of low-quality ore each day at a cost of $2,000. How does the availability of this mine influence the production schedule?

4. Give an example of a linear programming problem of the form $\max x_0$ subject to a single constraint, which is an equation. Find an objective function for which the dual variable associated with the constraint is positive; find a second objective function for which the dual variable associated with the constraint is zero; find a third objective function for which the dual variable associated with the constraint is negative. (In order to answer the question, you must give one linear equation and then three different choices for x_0.) In each part you should confirm the value of the dual variable by running excel. Explain how you found your solution and what it means.

   This problem may look much more complicated than it is. Your equation can be very simple. Think about what dual variables mean before doing any complicated computations.